# Cosmology of Fibre Inflation Models



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Based on papers written with: Burgess, Ciupke, Diaz, de Alwis, Guidetti, Mayrhofer, Muia, Pedro, Piovano, Quevedo, Shukla

# Challenges for string inflation

- Conditions for viable string inflation:
  - (i) approximate symmetry to control quantum corrections

inflaton is a pseudo NG boson

(ii) full moduli stabilisation to control orthogonal directions and fix all energy scales
(iii) global CY embedding to check theoretical consistency
(iv) understand post-inflationary cosmology to check phenomenological consistency

make trustable predictions

- n<sub>s</sub> and r depend on:
  - i)  $N_e$  which depends on post-inflation: reheating:  $T_{re}$ ?  $w_{re}$ ? moduli domination:  $N_{mod}$ ?  $N_e + \frac{1}{4}N_{mod} + \frac{1}{4}(1 - 3w_{re})N_{re} \approx 57 + \frac{1}{4}\ln r + \frac{1}{4}\ln\left(\frac{\rho_*}{\rho_{end}}\right)$

ii) fix n<sub>s</sub> by matching observations and then predict r BUT Planck value of n<sub>s</sub> depends on priors, e.g.  $\Delta N_{eff} = 0$   $\longrightarrow$  n<sub>s</sub> = 0.966 ± 0.006  $\Delta N_{eff} = 0.39$   $\longrightarrow$  n<sub>s</sub> = 0.983 ± 0.006 can get different r!  $\longrightarrow$  Compute amount of extra dark radiation  $\Delta N_{eff}$ ultra-light axions?

# Moduli stabilisation

• General Swiss-cheese form of the CY volume

$$\mathcal{V} = \frac{1}{6} \sum_{i,j,k=1}^{N_{\text{large}}} k_{ijk} t_i t_j t_k - \frac{1}{6} \sum_{s=1}^{N_{\text{small}}} k_{sss} t_s^3 \qquad \qquad N_{\text{large}} + N_{\text{small}} = h^{1,1}$$

 EFT coordinates: Kahler moduli
 τ<sub>i</sub> = Re(T<sub>i</sub>) = ∂ν/∂t<sub>i</sub>
 Leading order: α' + non-perturbative effects

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right) \qquad W = W_0 + \sum_{s=1}^{N_{\text{small}}} A_s \ e^{-a_s T_s}$$

LVS models: fix V + N<sub>small</sub> del Pezzo moduli

$$V \simeq e^{a_s \tau_s} \gg 1$$
  $\tau_s \simeq g_s^{-1} > 1$   $\forall s = 1, ..., N_{\text{small}}$   $\longrightarrow$   $N_{\text{flat}} = h^{1,1} - N_{\text{small}} - 1$  flat directions!

Flat directions lifted by perturbative corrections



- Good inflaton candidates:
- 1) Inflaton naturally lighter than H
- 2) Flatness protected by approximate rescaling shift symmetry

#### **Perturbative corrections**



Higher derivatives: α<sup>'3</sup> F<sup>4</sup> terms from 10D R<sup>2</sup> G<sup>4</sup> term [Ciupke, Louis, Westphal]

$$V_{F^4} = -\left(\frac{g_s}{8\pi}\right)^2 \frac{3^4 \lambda W_0^4}{g_s^{3/2} \mathcal{V}^4} \,\Pi_i t_i$$

$$\Pi_m = \int_X c_2 \wedge \hat{D}_m$$

Dependence on all t-moduli!  $\longrightarrow$  fix all LVS flat directions for arbitrary CY if  $\lambda < 0$ [MC,Ciupke, de Alwis, Muia] ( $\lambda < 0$  with  $|\lambda| \sim 10^{-3}$  from dimensional reduction)

[Green, Mayer, Weissenbacher]

# **Global CY embedding**

• General requirements for successful global embedding: 1) Search through the Kreuzer-Skarke list for toric CY 3-folds with: (i) fibration structure (ii) at least 1 rigid blow-up mode  $\longrightarrow N_{small} \ge 1$ (iii) at least 1 flat direction  $N_{flat} = h^{1,1} - N_{small} - 1 \ge 1$  $h^{1,1} \ge 2 + N_{small} \ge 3$ 

2) Choose and orientifold involution and a D3/D7 brane setup which satisfy tadpole cancellation

- 3) Fix all Kahler moduli inside Kahler cone
- 4) Generate  $g_s$  and  $F^4 \alpha'$  terms can drive inflation
- 5) Turn on gauge fluxes on D7s for a chiral visible sector

6) Get an explicit dS vacuum \_\_\_\_ use T-branes! [MC, Quevedo, Valandro]

- For  $h^{1,1} = 3$  cannot satisfy (5) and (6) since FI-terms lift flat directions
- For h<sup>1,1</sup> = 4 can potentially get a global CY embedding with chirality and dS

[MC, Ciupke, Diaz, Guidetti, Muia, Shukla]

See Guidetti's talk

[MC,Muia,Shukla]

# Explicit CY models

- h<sup>1,1</sup> = 3: explicit brane set-up + moduli stabilisation without chirality
- CY volume:

$$\mathcal{V} = t_1 t_2^2 + t_3^3 = \sqrt{\tau_1} \tau_2 - \tau_3^{3/2}$$

- Oguiso theorem: if v is linear in  $t_1$ 
  - $\tau_1$  is a K3 or a T<sup>4</sup> fibre over a P<sup>1</sup> base t<sub>1</sub>
    - $\rightarrow$   $\tau_1$  is the inflaton with V constant
- $h^{1,1} = 4$ : explicit brane set-up + moduli stabilisation + chirality
- CY volume:

$$\mathcal{V} = t_1 t_2 \tilde{t}_2 + t_3^3 = \sqrt{\tau_1 \tau_2 \tilde{\tau}_2} - \tau_3^{3/2}$$

- V is linear in  $t_1$ ,  $t_2$  and  $\tilde{t_2} \longrightarrow 3$  K3 fibrations
- Visible sector on  $\tau_1$ ,  $\tau_2$  and  $\tilde{\tau}_2$
- Turn on gauge fluxes  $\longrightarrow$  FI-term = 0 fixes  $\tau_2 \sim \tilde{\tau}_2$  $\longrightarrow$  reduce to  $h^{1,1} = 3$  case
- String loops + F<sup>4</sup> terms give inflation without tuning + chiral visible sector!



[MC,Muia,Shukla]

[MC,Ciupke,Diaz,Guidetti,Muia,Shukla]

#### Inflation



#### **Geometrical bounds**

- Compact inflaton moduli space due to Kahler cone
  - Upper bounded inflaton range!
- h<sup>1,1</sup> = 3: checked for all toric LVS vacua
   i) Δφ > M<sub>p</sub> only for K3 fibred examples
   ii) agreement with weak gravity conjecture
- $h^{1,1} > 3$ : conjecture for volume of reduced moduli space  $\mathcal{M}_r$

$$\operatorname{Vol}(\mathcal{M}_r) \lesssim \left[\ln\left(\frac{M_p}{\Lambda}\right)\right]^{\dim(\mathcal{M}_r)}$$

$$\Lambda \sim M_p / \mathcal{V}^{2/3}$$
 (KK scale)

Structureless:

$$\frac{\Delta \phi}{M_p} \le c \, \ln \mathcal{V} \qquad \qquad c \sim \mathcal{O}(1)$$
[MC, Ciupke, Mayr



 $\mathcal{V} = f_{3/2}(\tau_1, \tau_2) - \beta \tau_s^{3/2}$ 

2)  $n_{\text{ddP}} = 1$  and  $n_{\text{K3f}} = 1$ : K3 fibration

3 classes of LVS models for  $h^{1,1} = 3$ :

1)  $n_{\rm ddP} = 2$  and  $n_{\rm K3f} = 0$ : Strong Swiss cheese

$$\mathcal{V} = \alpha \sqrt{\tau_f} \, \tau_b - \beta \, \tau_s^{3/2}$$

 $\mathcal{V} = \alpha \, \tau_{\rm h}^{3/2} - \beta_1 \, \tau_{s_1}^{3/2} - \beta_2 \, \tau_{s_2}^{3/2}$ 

3)  $n_{\rm ddP} = 1$  and  $n_{\rm K3f} = 0$ : 2 subcases for  $\tau_{\rm s} \rightarrow 0$ 

Strong Swiss cheese-like:  $\mathcal{V} = \alpha \tau_b^{3/2} - \beta_1 \tau_s^{3/2} - \beta_2 (\gamma_1 \tau_s + \gamma_2 \tau_*)^{3/2}$ 

See Shukla's talk

Mavrhofer, Shuklal

# Scanning results

[MC, Ciupke, Mayrhofer, Shukla]

- Analytical proof + scanning results
- Scan of LVS geometries for h<sup>1,1</sup> = 2, 3, 4

$h^{1,1}$	$n_{ m CY}$	$n_{ m LVS}$	%	$n_{\rm ddP} = 1$	$n_{\rm ddP}=2$	$n_{\rm ddP}=3$
2	39	22	56.4%	22	_	_
3	305	132	43.3%	93	39	_
4	1997	749	37.5%	464	261	24

Classes of LVS models for h<sup>1,1</sup> = 3

$h^{1,1}$	$n_{ m CY}$	$n_{ m LVS}$	SSC	K3 fibred	SSC-like	structureless
3	305	132	39	43	36	14

Right ballpark

to match  $\delta \rho / \rho$ 

• Scan of reduced moduli space size for  $h^{1,1} = 3$ ,  $V = 10^5$  and  $g_s = 0.1$ 



#### Bound on tensor modes

[MC, Ciupke, Mayrhofer, Shukla]

• Generic LVS inflationary model

$$V \simeq V_0 \left( 1 - c_1 e^{-c_2 \phi} \right) \longrightarrow \epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2 \simeq \frac{1}{2} c_1^2 c_2^2 e^{-2c_2 \phi}$$

• For  $\epsilon(\phi_{\text{end}}) \simeq 1$  and  $r(\phi_*) = 16 \epsilon(\phi_*)$ 

$$N_e = \int_{\phi_{\text{end}}}^{\phi_*} \sqrt{\frac{8}{r(\phi)}} \, \mathrm{d}\phi \qquad \longrightarrow \qquad \frac{\Delta\phi}{M_p} \simeq \frac{N_e}{2} \sqrt{\frac{r(\phi_*)}{2}} \ln\left(\frac{4}{\sqrt{r(\phi_*)}}\right)$$

• Combine with upper bound  $\Delta \phi/M_p \le c \ln V$  for  $N_e = 50$ 



# Reheating

• End of inflation: transfer inflaton energy to SM dof



#### SM and dark radiation

- Where is the SM?
- Ultra-light bulk axions from inflaton decay contribute to ΔN<sub>eff</sub>
- Observational constraint:  $\Delta N_{eff} \leq 1$



Dark radiation overproduction!

SM on D7s wrapping inflaton cycle to increase branching ratio into visible dof

# Reheating and dark radiation

[MC, Piovano]

- SM con D7s wrapping bulk cycles  $\tau_b$  and  $\tau_f$   $\longrightarrow$  desequestering
- $M_{soft} \sim m_{3/2} \sim 10^{14} \text{ GeV} \gg m_{\Phi} \sim 10^{12} \text{ GeV}$  inflaton cannot decay to SUSY particles
- Unsuppressed inflaton decay to SM Higgs + would-be GBs + massless gauge bosons

$$\Gamma_{\Phi \to AA} = N_g \Gamma_0 \qquad N_g = 12$$

$$\Gamma_{\Phi \to \text{Higgs}} = f(\alpha, \beta) \frac{z^2}{16} \Gamma_0 \qquad f(\alpha, \beta) = 3\sin^2(2\beta) + \sin^2(2\alpha)$$

$$K_{\text{matter}} = \tilde{K}_{H_u} \bar{H}_u H_u + \tilde{K}_{H_d} \bar{H}_d H_d + z \frac{\sqrt{\tau_f}}{\mathcal{V}} (H_u H_d + h.c.)$$

• Dark radiation predictions

$$\Delta N_{\rm eff} = \frac{43}{7} \frac{5}{2} \left( \frac{10.75}{g_*(T_{\rm rh})} \right)^{1/3} \frac{1}{c_{\rm vis}} \qquad c_{\rm vis} = 12 + f(\alpha, \beta) \frac{z^2}{16}$$

• Reheating temperature:

$$T_{rh} \simeq m_{\phi} \sqrt{\frac{m_{\phi}}{M_{P}}} \sim 10^{9} \text{ GeV} \longrightarrow g_{*}(T_{rh}) = 106.5 \longrightarrow N_{e} \simeq 52$$
  

$$\tan \beta = 1 \quad \beta = \alpha = \frac{\pi}{4} \longrightarrow \Delta N_{eff} = \frac{114.4}{192 + 5z^{2}} \simeq 0.6 \text{ for } z = 0$$
  

$$\longrightarrow \Delta N_{eff} \approx 0.6 \text{ as a prior for Planck} \longrightarrow n_{s} = n_{s}(N_{e}, \mathcal{R}) = n_{s}(\mathcal{R}) \simeq 0.99$$
  

$$\longrightarrow \text{ fix } \mathcal{R} \longrightarrow \text{ prediction: } r = r(N_{e}, \mathcal{R}) = r(\mathcal{R}) \simeq 0.01$$

# **PBH DM**

• What is the origin of DM?



• PBHs form when large and rare fluctuations re-enter the horizon:

$$M = \gamma \frac{4\pi}{3} \frac{\rho_{tot}}{H^3} \bigg|_f = 4\pi \gamma \frac{M_p^2}{H_f}$$

• E-foldings-mass relation:

$$\Delta N_{CMB}^{PBH} = 18.4 - \frac{1}{12} \ln \left( \frac{g_*}{g_{*0}} \right) + \frac{1}{2} \ln \gamma - \frac{1}{2} \ln \left( \frac{M}{M_{\odot}} \right) \qquad \text{low-mass region:} \quad \Delta N_{CMB}^{PBH} \simeq 34.5$$

#### **PBH** formation



#### **PBH** abundance

- PBHs form when  $\zeta \geq \zeta_c$  re-enter the horizon:
- Collapse fraction:

$$\beta_f(M) = \frac{\rho_{PBH}(M)}{\rho_{tot}} \bigg|_f = \int_{\zeta_c}^{\infty} \frac{1}{\sqrt{2\pi\sigma_M}} e^{-\frac{\zeta^2}{2\sigma_M^2}} d\zeta \simeq \frac{\sigma_M}{\sqrt{2\pi\zeta_c}} e^{-\frac{\zeta_c^2}{2\sigma_M^2}}$$
$$\sigma_M^2 \simeq \langle \zeta\zeta \rangle \simeq P_k \ll \zeta_c$$

- Exponentially sensitive to critical value  $\zeta_c \simeq 1$
- PBHs redshift as matter \_\_\_\_\_ present abundance

$$\beta_{o}(M) = \Omega_{DM} \left. \frac{\rho_{PBH}(M)}{\rho_{DM}} \right|_{o} = 0.26 f_{PBH}(M) \qquad \longrightarrow \qquad \beta_{f}(M) \simeq 10^{-8} \sqrt{\frac{M}{M_{\odot}}} f_{PBH}(M)$$

• 100% of DM in PBHs in the low-mass region:

 $f_{PBH}(M) \simeq 1$   $M \simeq 10^{-15} M_{\odot}$   $\longrightarrow$   $\beta_f(M) \simeq 3 \times 10^{-16}$  $\longrightarrow$   $P_k \big|_{PBH} \simeq 10^7 \times P_k \big|_{CMB} \simeq 10^{-2}$   $\frac{\delta \rho}{\rho} \simeq 0.1$  Perturbation theory under control?

#### **PBHs from inflation**





Need to solve Mukhanov-Sasaki equation for rescaled curvature perturbations:

$$u_{k}''(\tau) + \left(k^{2} - z'' / z\right)u_{k}(\tau) = 0 \qquad \qquad \zeta = u / z \qquad \qquad z \equiv \sqrt{2\varepsilon a}$$
$$\frac{z''}{z} = \left(aH\right)^{2} \left[2 - \varepsilon + \frac{3}{2}\eta - \frac{1}{2}\varepsilon\eta + \frac{1}{4}\eta^{2} + \frac{1}{2}\eta\kappa\right] \qquad \qquad \eta = \frac{\dot{\varepsilon}}{\varepsilon H} \qquad \qquad \kappa = \frac{\dot{\eta}}{\eta H}$$

#### Ultra slow-roll

Klein-Gordon eq. in expanding universe:

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{\varphi} = 0$$

[Martin, Motohashi, Suyama] [Motohashi,Starobinsky,Yokoyama]

**CD** 

 $\alpha - 3$ 

i) Constant velocity: 
$$\ddot{\varphi} \approx 0$$
  
 $\rightarrow 3H\dot{\varphi} \approx -V_{\varphi}$  Slow roll  
ii) Near inflection point:  $V_{\varphi} \approx 0$   
 $\phi \approx -3H\dot{\varphi}$  Ultra slow-roll  $\phi$  deceleration  
 $\varepsilon = \frac{1}{2}\frac{\dot{\varphi}^2}{H^2} \ll 1$   $\eta = 2\left(\frac{\ddot{\varphi}}{H\dot{\varphi}} + \varepsilon\right) \approx -6$ 

Power spectrum for super-horizon scales:

$$P_k \propto H^{|\lambda|-1} a^{\lambda+|\lambda|} \qquad \lambda \equiv 3+2\alpha \qquad \ddot{\varphi} \equiv -(3+\alpha)H\dot{\varphi} \qquad \alpha \equiv 0 \qquad \text{USR}$$

i) Constant mode for  $\lambda < 0 \iff -3 \le \alpha < -3/2$ 

ii) Growing mode for  $\lambda > 0 \iff \alpha > -3/2$ 

#### Ultra slow-roll + growing mode help to produce PBHs

# **PBHs from Fibre Inflation**

[MC, Diaz, Pedro]

- Original Fibre inflation potential not rich enough to generate PBHs
- Embedding in explicit CY threefolds with O3/O7, D3/D7, moduli stabilisation, tadpole cancellation and chiral matter
  - more general structure of corrections

$$\delta V_{\rm W} = W_0^2 \, \frac{\tau_{\rm K3}}{\mathcal{V}^4} \left( D_{\rm W} - \frac{G_{\rm W}}{1 + R_{\rm W} \frac{\tau_{\rm K3}^{3/2}}{\mathcal{V}}} \right)$$

 $D_{\rm W} \sim G_{\rm W} \sim R_{\rm W} \sim \mathcal{O}(1)$ 

Term responsible for near inflection-point

potential rich enough to tune a near-inflection point:



#### Superhorizon evolution and power spectrum



#### **Open issues**



# Outlook

- Goal: understand PBH production from strings: generic mechanism, preferred PBH masses
- To do:
  - i) Redo the analysis with  $\zeta_c \simeq 0.5$ , quantum diffusion + non-Gaussianities
  - ii) Consider more general Fibre Inflation potential
  - iii) Find other single-field potentials from strings, axions? [Ozsoy, Parameswaran, Tasinato, Zavala]
  - iv) Consider matter domination due to light moduli (axions) at horizon re-entry
  - v) Find curvaton-like mechanism for PBH production, axions? [Ando,Inomata,Kawasaki,Mukaida,Yanagida]
  - vi) Study oscillon (oscilloton) collapse into BHs at the end of inflation

[Antusch,Cefalà,Krippendorf,Muia,Orani,Quevedo] [Helfer,Marsh,Clough,Fairbairn,Lim,Becerril]

much smaller and lighter BHs ----- reheating and DM from evaporation?

[Lennon, March-Russell, Petrossian-Byrne, Tillim]

# A geometrical instability?

- Spectator fields during inflation: heavy (m<sub>heavy</sub> >> H) + light (m<sub>light</sub> << m<sub>inf</sub> << H)</li>
- Effective mass of isocurvature pert.

$$m_{eff}^{2} = V_{\perp\perp} - \Gamma_{\perp\perp}^{i} V_{i} + \left(\varepsilon R + 3\eta_{\perp}^{2}\right) H^{2} \qquad \eta_{\perp} = \frac{\phi}{H} \kappa^{-1}$$

- Geometrical destabilisation during inflation in a non-linear sigma model?
- Dangerous even for heavy fields with geodesic trajectories ( $\eta_{\perp}=0$ ) when R < 0 since

$$m_{eff}^{2} = V_{\perp \perp} - \Gamma_{\perp \perp}^{i} V_{i} - \varepsilon \left| R \right| H^{2} < 0 \quad \text{if} \quad \left| R \right| = M_{p} / M \gg 1 \quad \text{[Renaux-Petel, Turzinsky]}$$

• NB: R < 0 generic in supergravity since  $K = -3\ln(T + \overline{T}) \longrightarrow R = -8/3$ 

BUT  $m_{eff}^2 > 0$  if computed on attractor background trajectory with  $\eta_{\perp} \neq 0$  ! [MC,Guidetti,Pedro,Vacca]

• Real issue for ultra-light fields with  $V_{\perp \perp} = 0$  when R < 0

$$m_{eff}^{2} = -\Gamma_{\perp\perp}^{i} V_{i} - \varepsilon \left| R \right| H^{2} + 3\eta_{\perp}^{2} H^{2}$$

1) Bending trajectory with  $\eta_{\perp} \neq 0$  can give  $m_{e\!f\!f}^2 > 0$ 

- 2) Geodesic trajectory with  $\eta_{\perp}=0$ 
  - $m_{e\!f\!f}^2 = -\Gamma_{\perp\perp}^i V_i \mathcal{E} \left| R \right| H^2 < 0 \qquad \text{if} \quad \Gamma_{\perp\perp}^i V_i > 0 \qquad \text{E.g.: } \mathbf{T}_1 \text{ axion in Fibre Inflation}$

Isocurvature pert. grow perturbation theory breaks down

- non-perturbative analysis (numerical)
- Intuition: kick along ultra-light direction and backreaction from  $\eta_{\perp} \neq 0$ ?

See Pedro's talk

#### Conclusions

- 1) Type IIB Fibre Inflation models: natural inflationary directions
- 2) Moduli stabilisation: non-perturbative +  $\alpha'$  effects + string loops + F<sup>4</sup> terms
- 3) Effective symmetry: non-compact rescalings
- 4) Plateau-like inflation with large tensors:  $0.005 \le r \le 0.01$
- 5) Global CY embedding:  $h^{1,1} = 3$  case without chirality + chirality for  $h^{1,1} \ge 4$
- 6) Compact reduced moduli space with  $\Delta \phi/M_p \le c \ln V$  with  $\Delta \phi > M_p$  only for K3-fibrations
- 7) General prediction:  $r \le 0.01$   $\longrightarrow$  agreement with weak gravity conjecture
- 8) Reheating: visible sector on bulk cycles due to generic  $\Delta N_{eff} \neq 0$
- 9)  $N_e \approx 52$  and  $\Delta N_{eff} \approx 0.6$   $\longrightarrow$   $n_s \approx 0.99$  and  $r \approx 0.01$
- 10) Potential rich enough to have a plateau + near inflection point
- 11) Power spectrum enhancement due to ultra slow roll + growing mode
- 12) PBHs in the low-mass region as 100% of DM
- 13) Are ultra-light fields stable during inflation?