D-brane-Anti D-brane Effective Actions, Universality of All Order Corrections

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based on:

1707.06669 [EH,JHEP]

1601.06667,1211.5538 [EH,JCAP]

1506.08802 [EH ,JHEP]

Outline

- Motivation

- D-branes

- Effective action DBI+new Wess-Zumino

- Brane-Anti brane effective actions

- Conclusion

Based on string amplitudes, we talk about a universal conjecture that holds even brane-anti brane.

Using scattering of Strings, we point out how to look for effective actions for D-brane and anti D-brane

Some new Wess-Zumino couplings with their corrections will be presented.

These new actions/couplings are neither inside Myers‘ terms nor within pull-back/Taylor expansions.

Motivations

1) To obtain **Universality** for all-order alpha-prime corrections to BPS/non-BPS systems

2) It seems that, description of world volume dynamics of D-brane is still lacking at fundamental level.

3) Holographic QCD Models, Cosmology,…

4) Working out with Mathematical Structures behind Scattering amplitudes (world-sheet integrals)

BPS Dp-branes in II

For stable Dp-branes **(p is even in IIA , odd in IIB)** and the only difference with non BPS branes is the absence of Tachyon.

Stability, Supersymmetry, conserved (RR) charge and having no tachyons are, all properties of these type II branes.

The charge of a Dp-brane is Ramond-Ramond (p+1) field.

The world-volume theory of a Dp-brane involves a massless U(1) vector, 9-p real massless scalars, fermions.

At leading order, the low-energy action is DBI.

There are higher $\alpha' = l_s^2$ order corrections.

When derivatives of the Field Strength are small on string scale, the action takes BI form.

$$S_{\rm BI} = -T_p \int d^{p+1}\sigma \, {\rm STr}\left(e^{-\phi} \sqrt{-\det\left(P\left[E_{ab} + E_{ai}(Q^{-1}-\delta)^{ij}E_{jb}\right] + \lambda F_{ab}\right) \det(Q^i{}_j)} \right),$$

with

$$E_{ab} = G_{ab} + B_{ab} \qquad , \qquad Q^i{}_j \equiv \delta^i{}_j + i\lambda\,[\Phi^i, \Phi^k]\,E_{kj},$$

where $\lambda = 2\pi\ell_s^2$, T_p is the brane tension, P[...] indicates pull-back of background metric and NSNS two-form (a,b = 0, .., 9), F_{ab} is the field strength of gauge field and STr(...) is symmetric trace prescription.

To find interactions, we expand the action and set all background field to zero, working on flat space.

The second part is the **Wess-Zumino** action, contains the coupling of the U(N) massless world volume vectors to RR field

Effective Field Theory on the World-Volume

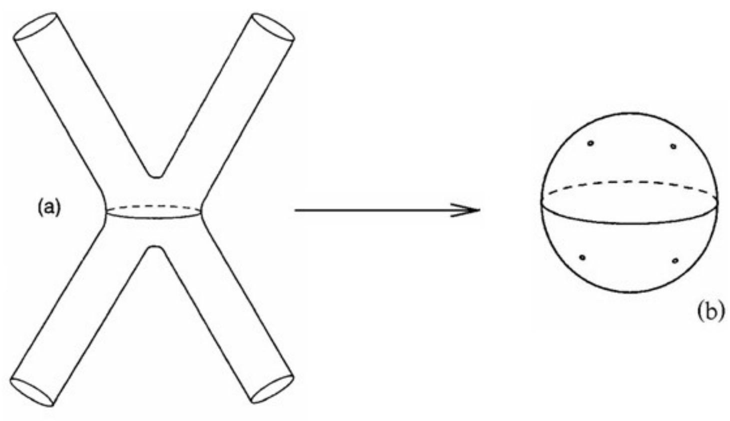
The states in our S-matrix are Gauge , Scalars and Tachyons from DBI action and RR field from the WZ action.

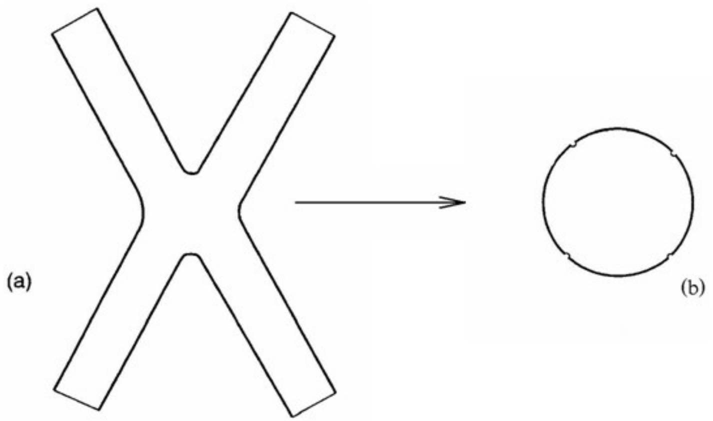
$$S_{CS} = \mu_p \int P\left[\sum C^{(n)}\, e^{B}\right] e^{2\pi\ell_s^2 F}$$

$$S_{WZ} = \mu_p \int \mathrm{STr}\left(P\left[e^{i\lambda\, i_\Phi i_\Phi}\left(\sum C^{(n)}\right)\right] e^{\lambda F}\right) \; .$$

Four Point amplitude

We calculate scattering amplitudes of strings by CFT methods





From CFT, one evaluates the correlation functions of all fields

Vertex Operators

Form of every vertex is calculated by using **the conformal invariance of S-matrix**.

$$V_{RR}^{(-1)}(z,\bar{z}) \;=\; (P_{-}\not{H}_{(n)}M_p)^{\alpha\beta}e^{-\phi(z)/2}S_\alpha(z)e^{ip\cdot X(z)}e^{-\phi(\bar{z})/2}S_\beta(\bar{z})e^{ip\cdot D\cdot X(\bar{z})}\otimes\sigma_3\sigma_1$$

$$V_T^{(-1)}(y) \;=\; e^{-\phi(y)}e^{2ik\cdot X(y)}\lambda\otimes\sigma_2$$

$$\not{H}_{(n)} = \frac{a_n}{n!}H_{\mu_1\ldots\mu_n}\gamma^{\mu_1}\ldots\gamma^{\mu_n}$$

$$V_A^{(0)}(x) \;=\; \xi_i\Big(\partial X^i(x)+2ik\cdot\psi\psi^i(x)\Big)e^{2ik.X(x)}\lambda\otimes I \qquad k^2=0, \qquad k.\xi=0$$

λ is the external CP matrix in the U(N) group.

$$I = \int_{\mathcal{H}^+} d^2z |1-z|^a |z|^b (z-\bar{z})^c (z+\bar{z})^d,$$

where $d = 0, 1, 2$ and a, b, c should be computed in terms of the Mandelstam variables. Since we are talking about disk level amplitude the integrations must be done on upper half plane. The necessary conditions for these integrals must be taken into account as

$$a + b + c \leq -2$$
$$a + b + d \leq -2$$

To remove integrals on x, y we may use the following definitions

$$|z|^b = \frac{1}{\Gamma(-\frac{b}{2})} \int_0^\infty du\, u^{-\frac{b}{2}-1} e^{-u|z|^2},$$
$$|1-z|^a = \frac{1}{\Gamma(-\frac{a}{2})} \int_0^\infty ds\, s^{-\frac{a}{2}-1} e^{-s|1-z|^2}.$$

$$\int d^2z|1-z|^a|z|^b(z-\bar{z})^c(z+\bar{z})^d = (2i)^c 2^d \pi \frac{J_1+J_2}{\Gamma(-\frac{a}{2})\Gamma(-\frac{b}{2})\Gamma(d+2+c+\frac{a+b}{2})}.$$

where

$$J_1 = \frac{1}{2}\Gamma(d+\frac{b+c}{2})\Gamma(d+\frac{a+c}{2})\Gamma(-d-\frac{a+b+c}{2})\Gamma(\frac{1+c}{2})$$

$$J_2 = \Gamma(d+1+\frac{b+c}{2})\Gamma(1+\frac{a+c}{2})\Gamma(-1-\frac{a+b+c}{2})\Gamma(\frac{1+c}{2}).$$

Universality in all-order α' higher derivative corrections of non-BPS and BPS branes

Regularity in the higher derivative expansions.

1st we gain the S-matrix element of desired amplitudes.

2nd, using Mandelstam variables, we rewrite the amplitudes such that all poles can be seen in a clear way.

The 3rd step is to explore leading couplings from tachyonic DBI action.

The last step is to express the symmetric trace in term of ordinary trace and applying the higher derivative corrections on them.

where

$$\mathcal{D}_{nm}(EFGH) \equiv D_{b_1}\cdots D_{b_m}D_{a_1}\cdots D_{a_n}EFD^{a_1}\cdots D^{a_n}GD^{b_1}\cdots D^{b_m}H,$$
$$\mathcal{D}'_{nm}(EFGH) \equiv D_{b_1}\cdots D_{b_m}D_{a_1}\cdots D_{a_n}ED^{a_1}\cdots D^{a_n}FGD^{b_1}\cdots D^{b_m}H.$$

The crucial step seems to extract the symmetric trace in term of ordinary trace and applying the higher derivative corrections $\mathcal{D}_{nm}, \mathcal{D}'_{nm}$ on them.

1st Ex : 2 Tachyons and two scalars, and also 4T's on [E.H,JCAP,JHEP,1601.06667,1707.06609]

$$2T_p(\pi\alpha')^3\mathrm{STr}\left(m^2T^2(D_a\phi^iD^a\phi_i)+\ D^\alpha TD_\alpha TD_a\phi^iD^a\phi_i-2D_a\phi^iD_b\phi_iD^bTD^aT\right)$$

the all-order vertices turned out to be

$$\mathcal{L} \;=\; -2T_p(\pi\alpha')(\alpha')^{2+n+m}\sum_{n,m=0}^{\infty}(\mathcal{L}_1^{nm}+\mathcal{L}_2^{nm}+\mathcal{L}_3^{nm}+\mathcal{L}_4^{nm})$$

where

$$\begin{aligned}
\mathcal{L}_1^{nm} &= m^2\mathrm{Tr}\Big(a_{n,m}[\mathcal{D}_{nm}(T^2D_a\phi^iD^a\phi_i)+\mathcal{D}_{nm}(D_a\phi^iD^a\phi_iT^2)]\\
&\quad +b_{n,m}[\mathcal{D}'_{nm}(TD_a\phi^iTD^a\phi_i)+\mathcal{D}'_{nm}(D_a\phi^iTD^a\phi_iT)]+h.c.\Big)\\
\mathcal{L}_2^{nm} &= \mathrm{Tr}\Big(a_{n,m}[\mathcal{D}_{nm}(D^\alpha TD_\alpha TD_a\phi^iD^a\phi_i)+\mathcal{D}_{nm}(D_a\phi^iD^a\phi_iD^\alpha TD_\alpha T)]\\
&\quad +b_{n,m}[\mathcal{D}'_{nm}(D^\alpha TD_a\phi^iD_\alpha TD^a\phi_i)+\mathcal{D}'_{nm}(D_a\phi^iD_\alpha TD^a\phi_iD^\alpha T)]+h.c.\Big)\\
\mathcal{L}_3^{nm} &= -\mathrm{Tr}\Big(a_{n,m}[\mathcal{D}_{nm}(D^\beta TD_\mu TD^\mu\phi^iD_\beta\phi_i)+\mathcal{D}_{nm}(D^\mu\phi^iD_\beta\phi_iD^\beta TD_\mu T)]\\
&\quad +b_{n,m}[\mathcal{D}'_{nm}(D^\beta TD^\mu\phi^iD_\mu TD_\beta\phi_i)+\mathcal{D}'_{nm}(D^\mu\phi^iD_\mu TD_\beta\phi_iD^\beta T)]+h.c.\Big)\\
\mathcal{L}_4^{nm} &= -\mathrm{Tr}\Big(a_{n,m}[\mathcal{D}_{nm}(D^\beta TD^\mu TD_\beta\phi^iD_\mu\phi_i)+\mathcal{D}_{nm}(D^\beta\phi^iD^\mu\phi_iD_\beta TD_\mu T)]\\
&\quad +b_{n,m}[\mathcal{D}'_{nm}(D^\beta TD_\beta\phi^iD^\mu TD_\mu\phi_i)+\mathcal{D}'_{nm}(D_\beta\phi^iD_\mu TD^\mu\phi_iD^\beta T)]+h.c.\Big)
\end{aligned}$$

We also found new couplings which are neither inside Myers' terms nor within pull back/Taylor expansions.These couplings like

$$S_3 = \frac{\lambda^3\mu_p\pi}{12}\int d^{p+1}\sigma\frac{1}{(p-3)!}(\epsilon^v)^{a_0\cdots a_p}\left(\frac{\alpha'}{2}\right)$$
$$\times C^{(p-3)}_{a_0\cdots a_{p-4}}\mathrm{Tr}\left(F_{a_{p-3}a_{p-2}}(D^aD_a)\Big[D_{a_{p-1}}\phi^iD_{a_p}\phi_i\Big]\right)$$

and

$$S_4 = \frac{\lambda^3\mu_p\pi}{6}\int d^{p+1}\sigma\,(\alpha')\,\mathrm{Tr}\left(C_{p-3}\wedge D^{b_1}F\wedge D_{b_1}\Big[D\phi^i\wedge D\phi_i\Big]\right)$$

can be found just by S-matrix and their coefficients should be set by applying Scattering approach not any other tools [see 1302.5024,EH.NPB]

Perturbative string theory

This action was proposed by S-Matrix method which is a generalization of DBI action for gauge fields on WV of branes.

$$L = -V(T)\sqrt{-det(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} + 2\pi\alpha'\partial_\mu T\partial_\nu T)}, \quad V(T) = 1 - \frac{T^2}{4} + O(T^4)$$

where for non –BPS branes in II ST this potential will produce tachyon's mass on branes very precisely.

In order to have consistency with S-Matrix method we have to generalize tachyonic action so that it reproduces all desired couplings in non-BPS branes and brane –anti brane.

D-brane-Anti D-brane Effective action

This effective action *in* ***IIA(IIB)*** *theory is given by some* extension of the DBI action and the WZ terms which include the tachyon fields.

$$S_{DBI} = -\int d^{p+1}\sigma \mathrm{Tr}\left(V(T)\sqrt{-\det(\eta_{ab}+2\pi\alpha' F_{ab}+2\pi\alpha' D_a T D_b T)}\right)$$

The trace in the above action should be completely symmetric between all matrices of the form

$$F_{ab}, D_a T, \text{ and individual } T$$

$$F_{ab} = \begin{pmatrix} F_{ab}^{(1)} & 0 \\ 0 & F_{ab}^{(2)} \end{pmatrix}, \quad D_a T = \begin{pmatrix} 0 & D_a T \\ (D_a T)^* & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & T \\ T^* & 0 \end{pmatrix}$$

Consistency with S-Matrix imposed

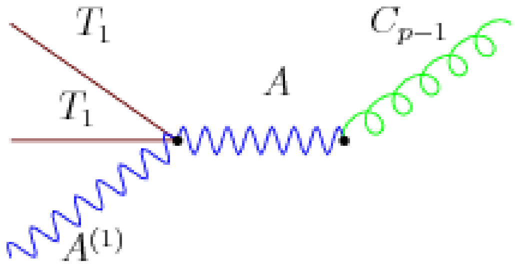
If one uses ordinary trace,instead, the above action reduces to the action proposed by A.Sen after making the kinetic term symmetric and performing the trace. This latter action is not consistent with S-matrix calculation. The tachyon potential which is consistent with S-matrix element calculations has

$$V(|T|) = 1 + \pi\alpha' m^2 |T|^2 + \frac{1}{2}(\pi\alpha' m^2 |T|^2)^2 + \cdots$$

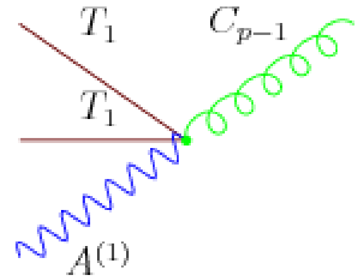
consistent with the tachyon potential of BSFT

The terms of the above action which have contribution to the S-matrix **CTTA**

$$
\begin{aligned}
\mathcal{L}_{DBI} = & -T_p(2\pi\alpha')\left(m^2|T|^2 + DT\cdot(DT)^* - \frac{\pi\alpha'}{2}\left(F^{(1)}\cdot F^{(1)} + F^{(2)}\cdot F^{(2)}\right)\right) + T_p(\pi\alpha')^3 \\
& \times\left(\frac{2}{3}DT\cdot(DT)^*\left(F^{(1)}\cdot F^{(1)} + F^{(1)}\cdot F^{(2)} + F^{(2)}\cdot F^{(2)}\right)\right. \\
& + \frac{2m^2}{3}|\tau|^2\left(F^{(1)}\cdot F^{(1)} + F^{(1)}\cdot F^{(2)} + F^{(2)}\cdot F^{(2)}\right) \\
& \left. - \frac{4}{3}\left((D^\mu T)^* D_\beta T + D^\mu T(D_\beta T)^*\right)\left(F^{(1)\mu\alpha}F^{(1)}_{\alpha\beta} + F^{(1)\mu\alpha}F^{(2)}_{\alpha\beta} + F^{(2)\mu\alpha}F^{(2)}_{\alpha\beta}\right)\right)
\end{aligned}
$$



(a)



(b)

$$
\begin{aligned}
\mathcal{L}_{DBI} = & -T_p(2\pi\alpha')\left(m^2|T|^2 + DT\cdot(DT)^* - \frac{\pi\alpha'}{2}\left(F^{(1)}\cdot F^{(1)} + F^{(2)}\cdot F^{(2)}\right)\right) + T_p(\pi\alpha')^3 \\
& \times\left(\frac{2}{3}DT\cdot(DT)^*\left(F^{(1)}\cdot F^{(1)} + F^{(1)}\cdot F^{(2)} + F^{(2)}\cdot F^{(2)}\right)\right. \\
& +\frac{2m^2}{3}|\tau|^2\left(F^{(1)}\cdot F^{(1)} + F^{(1)}\cdot F^{(2)} + F^{(2)}\cdot F^{(2)}\right) \\
& \left.-\frac{4}{3}\left((D^\mu T)^* D_\beta T + D^\mu T(D_\beta T)^*\right)\left(F^{(1)\mu\alpha}F^{(1)}_{\alpha\beta} + F^{(1)\mu\alpha}F^{(2)}_{\alpha\beta} + F^{(2)\mu\alpha}F^{(2)}_{\alpha\beta}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
V_b(A^{(1)},A^{(1)},T_1,T_1) = & \frac{4i}{3}T_p(\pi\alpha')^3 k_b\left[(s+1/4)(2k_2\cdot\xi) + (t+1/4)(2k_3\cdot\xi)\right] + 2iT_p(\pi\alpha')^3\times \\
& \times\left[k_{2b}(t+1/4)(2\xi.k_3) + k_{3b}(s+1/4)(2\xi.k_2) - \xi_b(s+1/4)(t+1/4)\right] \\
V_b(A^{(2)},A^{(1)},T_1,T_2) = & \frac{2i}{3}T_p(\pi\alpha')^3 k_b\left[(s+1/4)(2k_2\cdot\xi) + (t+1/4)(2k_3\cdot\xi)\right] + 2iT_p(\pi\alpha')^3\times \\
& \times\left[k_{2b}(t+1/4)(2\xi.k_3) + k_{3b}(s+1/4)(2\xi.k_2) - \xi_b(s+1/4)(t+1/4)\right]
\end{aligned}
$$

So just this Lagrangian could consistently produce CTTA amplitude.

Note that the term $F^{(1)} \cdot F^{(2)}$ in the tachyon DBI action is necessary for the above consistency

It can not be derived by field redefinition of fields nor by Sen's action.

A proposal by A. Sen for brane anti brane effective action is

$$S = -\int d^{p+1}\sigma V(|\tau|)\left(\sqrt{-\det \mathbf{A}^{(1)}} + \sqrt{-\det \mathbf{A}^{(2)}}\right),$$

where

$$\mathbf{A}^{(n)}_{\mu\nu} = \eta_{\mu\nu} + 2\pi\alpha' F^{(n)}_{\mu\nu} + \pi\alpha' \left(D_\mu\tau(D_\nu\tau)^* + D_\nu\tau(D_\mu\tau)^*\right)$$

On D-brane anti D-brane actions, their corrections to all orders in alpha-prime [1601.06667,EH,JCAP]

Discovering all higher derivative corrections to produce all scalar poles of <V_C V_{\phi} V_T V_T>.

we explore the presence of new term involving

$$D\phi^{i(1)}.D\phi_i^{(2)}$$

in DBI action of brane anti brane systems where this new coupling and its all order α' higher derivative corrections can be discovered just by applying S-matrix method of this paper. Having set the tachyon to zero, both ordinary and symmetric trace effective actions become equivalent.

The symmetric trace effective action has a non-zero coupling (like above) while this coupling does not exist in ordinary trace action.

The only consistent effective action for D-brane anti D-brane systems, **based on direct S-matrix computations** of **CATT, CTTTT**

was appeared in **[1211.5538,1601.06607,1707.06609, EH, JCAP,JCAP,JHEP].**

We have shown that there were non-zero couplings between F ^(1) .F^(2), D\phi^(1).D\phi^(2) and others and we found all their **higher derivative corrections.**

Note that for **ordinary trace prescription**, these coupling do not exist. Sen's action is not consistent with S-matrix computations and symmetrized trace works out for superstring computations.

The reason :

By using it not only we are unable to produce all the tachyon poles , but also the structure and forms of some of the new couplings like F ^(1).F^(2) (confirmed by S-matrix) have been overlooked.

Conclusion

We analyzed all the 3,4, 5 point functions of BPS, Non-BPS, D-brane anti-D-brane system.

It is not clear how to produce all contact terms of string amplitude. Possibly pull back may need modification.

We found universality in all order higher derivative corrections of brane-anti-brane system.

Fermionic amplitudes and universality in Type IIA?

The supersymmetric generalization of DBI action is still unknown.

Thank you for your attention