Dimensional reduction on curved manifolds: low energy, swampland, de Sitter

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Introduction

Motivation: **connection of string theory to pheno.**
(cosmology, particle physics, gravitational waves)
Context: string compactifications: 10d $\to$ 4d

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Truncation + 4d theory: **low energy??**

Consider 6d $\mathcal{M}$ which are **curved/non-Ricci flat**.
Away from the lamppost, different regions of the landscape, new appealing phenomenological features:
- stabilize some specific moduli
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$\mathcal{M}$ = Lie group / lattice , compact.
Underlying Lie algebra:
$[V_a, V_b] = f^c{}_{ab} V_c \leftrightarrow \mathrm{d}e^a = -\frac{1}{2} f^a{}_{bc} e^b \wedge e^c$
$\hookrightarrow \mathcal{M}$: $\mathrm{d}s_6^2 = \delta_{ab} e^a e^b$, $e^a = e^a{}_m \mathrm{d}x^m$, $f^a{}_{bc} = 2\, \omega_{[b}{}^a{}_{c]}$.

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Known de Sitter, Minkowski, anti-de Sitter solutions with $\mathcal{M}$=group manifold (and fluxes, D_p/O_p).
$\Rightarrow$ dimensional reduction $\rightarrow$ fluctuations + truncation ?

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3. Recent de Sitter swampland criterion.

Laplacian spectrum on a nilmanifold

Nilmanifold geometry

3d Heisenberg Lie algebra:
$[V_1, V_2] = -\mathtt{f}\, V_3 \ , \quad [V_1, V_3] = [V_2, V_3] = 0, \qquad \mathtt{f} = -f^3{}_{12}$
$\mathrm{d}e^3 = \mathtt{f}\, e^1 \wedge e^2 \ ; \quad \mathrm{d}e^1 = 0 \ ; \quad \mathrm{d}e^2 = 0$

$\hookrightarrow$ 3d nilmanifold M.
Build 6d $\mathcal{M} = M \times M'$, $M \times T^3$.

Coordinates with (positive) radii $r^{m=1,2,3}$:
$e^1 = r^1 \mathrm{d}x^1 \ ; \quad e^2 = r^2 \mathrm{d}x^2 \ ; \quad e^3 = r^3 \left(\mathrm{d}x^3 + N x^1 \mathrm{d}x^2\right)$
$N = \frac{r^1 r^2}{r^3} \mathtt{f} \in \mathbb{Z}^*$
$V = \int \mathrm{d}^3x \sqrt{g} = r^1 r^2 r^3$
Geometry: **fibration of a S^1_3 on a base $T^2_{1,2}$.**

Laplacian spectrum

Eigenmodes		Eigenvalues
Scalars	$v_{p,q}$	$-\mu^2_{p,q}$
	$u_{k,l,n}$	$-M^2_{k,l,n}$
Exact one-forms	$\mathrm{d}v_{p,q}$ $(pq \neq 0)$	$-\mu^2_{p,q}$
	$\mathrm{d}u_{k,l,n}$	$-M^2_{k,l,n}$
Co-closed one-forms	$B^{p,q}_{\pm}$	$Y^{p,q}_{\pm}$
	$B^{k,l,n}_{\pm}$	$Y^{k,l,n}_{\pm}$

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$$(\Delta + \mu^2_{p,q})v_{p,q} = 0\ , \quad (\Delta + M^2_{k,l,n})u_{k,l,n} = 0$$
$$\Delta \mathrm{d}v_{p,q} = -\mu^2_{p,q}\, \mathrm{d}v_{p,q}\ , \quad \Delta \mathrm{d}u_{k,l,n} = -M^2_{k,l,n}\, \mathrm{d}u_{k,l,n}$$

$\mathbf{v_{p,q}(x^1, x^2)}$

$\mathbf{u_{k,l,n}(x^1, x^2, x^3)}$

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$$\mathbf{v_{p,q}(x^1, x^2)} = \frac{1}{\sqrt{V}}\, e^{2\pi \mathrm{i} p x^1} e^{2\pi \mathrm{i} q x^2} \ , \quad p, q \in \mathbb{Z}$$
$$\mu^{\mathbf{2}}_{\mathbf{p,q}} = p^2 \left(\frac{2\pi}{r^1}\right)^2 + q^2 \left(\frac{2\pi}{r^2}\right)^2$$

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$$\boldsymbol{\mu^2_{p,q}} = p^2 \left(\frac{2\pi}{r^1}\right)^2 + q^2 \left(\frac{2\pi}{r^2}\right)^2 \ , \ \mathbf{M^2_{k,l,n}} = k^2 \left(\frac{2\pi}{r^3}\right)^2 + (2n+1)|k|\, \frac{2\pi |\mathfrak{f}|}{r^3}$$

$$\mathbf{u_{k,l,n}(x^1, x^2, x^3)} =$$
$$\sqrt{\frac{r^2}{|N| V}}\, \frac{1}{\sqrt{2^n n! \sqrt{\pi}}}\, e^{2\pi \mathrm{i} k (x^3 + N\, x^1 x^2)} e^{2\pi \mathrm{i} l x^1} \sum_{m\in\mathbb{Z}} e^{2\pi \mathrm{i} k m x^1}\, \Phi^{\lambda}_n(w_m)$$
where $\Phi^{\lambda}_n(z) = |\lambda|^{\frac{1}{4}}\, e^{-\frac{1}{2}|\lambda| z^2} H_n(|\lambda|^{\frac{1}{2}} z)$, $\quad k \in \mathbb{Z}^* \ , \ n \in \mathbb{N}$

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$$p^2+q^2 \neq 0: \quad \mathbf{B_{\pm}^{p,q}} = D\, v_{p,q} \left(Q e^1 - P e^2 + \frac{\Upsilon_{\pm}^{p,q} - (P^2+Q^2)}{i\mathtt{f}} e^3 \right)$$

$$p=q=0: \quad B_1^{0,0} = v_{0,0}\, e^1 \ , \ B_2^{0,0} = v_{0,0}\, e^2 \quad (Y_-^{0,0} = 0)$$

$$B_3^{0,0} = v_{0,0}\, e^3 \quad (Y_+^{0,0} = \mathtt{f}^2)$$

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$$\mathbf{Y_{\pm}^{p,q}} = P^2 + Q^2 + \frac{\mathtt{f}^2}{2} \pm \sqrt{\left(P^2+Q^2+\frac{\mathtt{f}^2}{2}\right)^2 - (P^2+Q^2)^2}$$

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$$\mathbf{B^{k,l,n}_{\pm}} = \sum_{a=1}^{3} \varphi_a^{k,l,n}(x^m)\, e^a$$

$$\varphi_1^{k,l,n} = \frac{1}{2} c\sqrt{2^n n!} \left(u_{k,l,n} - \frac{\sqrt{(n+1)(n+2)}\mathtt{f}^2}{\alpha_n + 2|\lambda| + (n+2)\mathtt{f}^2}\, u_{k,l,n+2} \right)$$

$$\varphi_2^{k,l,n} = -\frac{\mathrm{i}}{2}\mathrm{sgn}(\lambda) c\sqrt{2^n n!} \left(u_{k,l,n} + \frac{\sqrt{(n+1)(n+2)}\mathtt{f}^2}{\alpha_n + 2|\lambda| + (n+2)\mathtt{f}^2}\, u_{k,l,n+2} \right)$$

$$\varphi_3^{k,l,n} = -\frac{1}{2|\lambda|^{\frac{1}{2}}} c\sqrt{2^n n!}\ \frac{\mathtt{f}\sqrt{2(n+1)}(\alpha_n + 2|\lambda|)}{\alpha_n + 2|\lambda| + (n+2)\mathtt{f}^2}\, u_{k,l,n+1} \ ,$$

$$\mathbf{Y^{k,l,n}_{\pm}} = k^2 \left(\frac{2\pi}{r^3}\right)^2 + (2n+3)\frac{2\pi}{r^3}|k\mathtt{f}| + \frac{1}{2}\mathtt{f}^2$$

$$\pm\sqrt{\left(\frac{2\pi}{r^3}|k\mathtt{f}| + \frac{1}{2}\mathtt{f}^2\right)^2 + 2(n+1)\frac{2\pi}{r^3}|k\mathtt{f}|\mathtt{f}^2}$$

Truncations and the swampland

Consistent truncations and gauged supergravity

Standard truncation on group manifolds:
Scherk–Schwarz truncation:
keep the left-invariant/Maurer–Cartan forms: constant $\times\ e^a$.
(compare to previous spectrum: $v_{p,q} \times e^{1,2,3}$, $v_{0,0} \times e^{1,2,3}$)
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Example with an $SU(3)$ structure:
keep finite set of forms on $\mathcal{M}$: $1, \omega_I, \alpha_K, \beta^K, \tilde{\omega}^I, \mathrm{vol}_6$, on which fields are developed
+ build SU(3) structure forms J, Ω, analogous to CY
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Scherk–Schwarz truncations are **consistent truncations**:
select set of "independent" or decoupled modes $\rightarrow$ 4d theory.
However: **not low energy** truncation/not low energy theory:
light modes a truncated, heavy modes are kept.

Low energy truncation on the nilmanifold

$-f^3{}_{12} = \mathtt{f} = \frac{r^3 N}{r^1 r^2}$
$\hookrightarrow$ consider approximation: $r^3 \leqslant |N| r^3 \ll r^1,\ r^2$
(**small fiber/large base**)

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$$1, \quad e^1,\ e^2,\ e^3, \quad e^2 \wedge e^3,\ e^3 \wedge e^1,\ e^1 \wedge e^2, \quad e^1 \wedge e^2 \wedge e^3$$

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Surprise: low energy truncation matches Scherk–Schwarz trunc.
(on the Laplacian spectrum, in this regime)
Specific to nilmanifold w.r.t. to other group manifolds.

Testing the (refined) swampland distance conjecture

In field space, move from ϕ_0 to $\phi_0 + \Delta\phi$:
a tower of modes of mass $m(\phi)$ becomes light:

$$m(\phi_0 + \Delta\phi) = m(\phi_0)\ f(\phi_0, \Delta\phi)\ e^{-\alpha \frac{\Delta\phi}{M_p}}$$

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Test this here on towers of Laplacian eigenmodes:

points in field space:	$r^3 \ll r^{1,2}$ •	$\xleftrightarrow{\text{dist.}}$	•	$\xleftrightarrow{\text{dist.}}$	•
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masses of light modes:	$0,\ \lvert\mathtt{f}\rvert$				

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masses of light modes:	0, $\lvert\mathtt{f}\rvert$				0, $\sqrt{Y_-^{p,q}}$

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Distance? Compute kinetic terms...

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If at ϕ_0: ✓ effective theory ⇒ ruined at $\phi_0 + \Delta\phi$.

Test this here on towers of Laplacian eigenmodes:

points in field space:	$r^3 \ll r^{1,2}$ •	⟵ dist. ? ⟶	$r^3 \sim r^{1,2}$ •	⟵ dist. ? ⟶	$r^3 \gg r^{1,2}$ •
number of light modes:	finite		finite		infinite
masses of light modes:	$0, \lvert\mathtt{f}\rvert$		0		$0, \sqrt{Y_-^{p,q}}$

Distance? Compute kinetic terms...

Testing the (refined) swampland distance conjecture

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Counterexample of the conjecture?
String effects? Quantum effects?
Due to the nilmanifold geometry, away from the lamppost...

De Sitter swampland criterion

David ANDRIOT

Consider gravity minimally coupled to scalar fields with potential $V(\phi_i)$:

$$\mathcal{S} = \int \mathrm{d}^4 x \sqrt{|g_4|}\, (\mathcal{R}_4 + \text{kin. terms} - V)$$

Solutions with constant scalars:

$$\mathcal{R}_{\mu\nu} - \frac{g_{\mu\nu}}{2}\mathcal{R}_4 = -\frac{g_{\mu\nu}}{2} V \Rightarrow \mathcal{R}_4 = 2V \ , \quad \partial_{\phi_i} V = 0$$

Extrema of potential, value $V|_0$: maximally symmetric 4d space-time, cosmological constant $\Lambda = \frac{1}{2} V|_0$, de Sitter $V|_0 > 0$.

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$\Rightarrow$ extremum: $V|_0 \leqslant 0 \Rightarrow$ **no de Sitter solution**
(+ cosmological consequences)

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$\hookrightarrow$ a natural criterion:

$$\exists\ b_i \in \mathbb{R}, c_i \in \mathbb{R}_+ \text{ such that}$$
$$V + \sum_i b_i\ \phi_i \partial_{\phi_i} V + \sum_i c_i\ \phi_i^2 \partial^2_{\phi_i} V \leqslant 0$$

$$\text{Solution: } V|_0 + \sum_i c_i\ (\phi_i^2 \partial^2_{\phi_i} V)|_0 \leqslant 0$$

$\Rightarrow$ **no stable de Sitter solution**, tachyonic de Sitter sol. ✓.

$\Rightarrow$ checks? Cosmological implications?

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in type IIA/B with intersecting D_p/O_p, on group manifolds

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↪ which de Sitter swampland criterion is valid? (if any)

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- Found a low energy truncation that matches Scherk–Schwarz truncation; probably only for nilmanifolds
- Used the spectrum to “test” (refined) swampland distance conjecture, maybe a counterexample

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Thank you for your attention!