Dimensional reduction on curved manifolds: low energy, swampland, de Sitter

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Laplacian spectrum

Truncation

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Summary

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Motivation: connection of string theory to pheno. (cosmology, particle physics, gravitational waves) Context: string compactifications: $10d \rightarrow 4d$

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Find a 10d string background:
 4d space-time (de Sitter, Minkowski)×6d compact manifold M

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1. Find a 10d string background:

4d space-time (de Sitter, Minkowski)×6d compact manifold \mathcal{M}

Here: classical (perturbative) backgrounds: solutions of 10d type IIA/B supergravities with D_p -branes/orientifolds O_p -planes

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- 2. Dimensional reduction: fluctuations around background

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2. Dimensional reduction:

fluctuations around background, infinite (towers) of them

- \hookrightarrow truncation to a finite set
- $\rightarrow \int_{6d} \rightarrow 4d$ theory \leftrightarrow observations?

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Truncation + 4d theory: low energy??

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Consider 6d \mathcal{M} which are curved/non-Ricci flat. Away from the lamppost, different regions of the landscape, new appealing phenomenological features:

- stabilize some specific moduli
- (some classical) de Sitter solutions require $\mathcal{R}_6 < 0$

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with different behaviors/counterexamples w.r.t. swampland conjectures?

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which different behaviors/counterexamples w.r.t. swampland conjectures?

Prime example: \mathcal{M} being a group manifold: most are curved, easy to handle.

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Prime example: \mathcal{M} being a group manifold:

most are curved, easy to handle.

 $\mathcal{M} = \text{Lie group / lattice}$, compact.

Underlying Lie algebra:

$$[V_a, V_b] = f^c{}_{ab}V_c \leftrightarrow de^a = -\frac{1}{2}f^a{}_{bc}e^b \wedge e^c$$

$$\hookrightarrow \mathcal{M}: ds_6^2 = \delta_{ab} e^a e^b , e^a = e^a{}_m dx^m , f^a{}_{bc} = 2 \omega_{[b}{}^a{}_{c]}.$$

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Consider 6d \mathcal{M} which are curved/non-Ricci flat.

- (some classical) de Sitter solutions require $\mathcal{R}_6 < 0$ \longrightarrow different behaviors/counterexamples w.r.t.

Prime example: \mathcal{M} being a group manifold:

Away from the lamppost, different regions of the landscape,

Known de Sitter, Minkowski, anti-de Sitter solutions with \mathcal{M} =group manifold (and fluxes, D_p/O_p).

 \Rightarrow dimensional reduction \rightarrow fluctuations + truncation?

Plan of the talk

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Plan of the talk

- 1. Fluctuations: $\Phi(x^{\mu}, x^m) = \sum_I \phi^I(x^{\mu}) U_I(x^m)$ \hookrightarrow find an appropriate basis $\{U_I\}$
- **2**.
- **3.**

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Plan of the talk

- 1. Fluctuations: $\Phi(x^{\mu}, x^m) = \sum_I \phi^I(x^{\mu}) U_I(x^m)$
- \hookrightarrow find an appropriate basis $\{U_I\}$: Laplacian $\Delta_{\mathcal{M}}$ eigenmodes
- \Rightarrow spectrum of Laplacian on $\mathcal{M}=$ nilmanifold.
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- 1. Fluctuations: $\Phi(x^{\mu}, x^m) = \sum_I \phi^I(x^{\mu}) U_I(x^m)$
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- 2. Truncation(s): consistent truncations / low energy / relation to swampland distance conjecture.
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- \hookrightarrow find an appropriate basis $\{U_I\}$: Laplacian $\Delta_{\mathcal{M}}$ eigenmodes
- \Rightarrow spectrum of Laplacian on $\mathcal{M}=$ nilmanifold.
- **2.** Truncation(s): consistent truncations / low energy / relation to swampland distance conjecture.
- 3. Recent de Sitter swampland criterion.

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Laplacian spectrum on a nilmanifold

Nilmanifold geometry

3d Heisenberg Lie algebra:

$$[V_1, V_2] = -\mathbf{f} V_3$$
, $[V_1, V_3] = [V_2, V_3] = 0$, $\mathbf{f} = -f^3_{12}$
 $de^3 = \mathbf{f} e^1 \wedge e^2$; $de^1 = 0$; $de^2 = 0$

 \hookrightarrow 3d nilmanifold M.

Build 6d $\mathcal{M} = M \times M'$, $M \times T^3$.

Coordinates with (positive) radii $r^{m=1,2,3}$: $e^1 = r^1 dx^1$; $e^2 = r^2 dx^2$; $e^3 = r^3 (dx^3 + Nx^1 dx^2)$

$$N = \frac{r^1 r^2}{r^3} \mathbf{f} \in \mathbb{Z}^*$$

$$V = \int \mathrm{d}^3 x \sqrt{g} = r^1 r^2 r^3$$

Geometry: fibration of a S_3^1 on a base $T_{1,2}^2$.

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Laplacian spectrum

Eigenmodes		Eigenvalues
Scalars	$v_{p,q}$	$-\mu_{p,q}^2$
	$u_{k,l,n}$	$-M_{k,l,n}^2$
Exact one-forms	$dv_{p,q} \ (pq \neq 0)$	$-\mu_{p,q}^2$
	$\mathrm{d}u_{k,l,n}$	$-M_{k,l,n}^2$
Co-closed one-forms	$B^{p,q}_{\pm}$	$Y^{p,q}_{\pm}$
	$B^{k,l,n}_{\pm}$	$Y_{\pm}^{k,l,n}$

- $2\mbox{-}$ and $3\mbox{-} forms obtained by Hodge star on <math display="inline">1\mbox{-} forms$ and scalars
- \Rightarrow complete spectrum of Δ_M .

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2- and 3-forms obtained by Hodge star on 1-forms and scalars \Rightarrow complete spectrum of Δ_M .

$$\mathbf{v_{p,q}(x^1,x^2)}$$

$$\mathbf{u_{k,l,n}}(\mathbf{x^1},\mathbf{x^2},\mathbf{x^3})$$

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2- and 3-forms obtained by Hodge star on 1-forms and scalars \Rightarrow complete spectrum of Δ_M .

$$\begin{split} & \left(\Delta + \mu_{p,q}^2 \right) v_{p,q} = 0 \;, \quad \left(\Delta + M_{k,l,n}^2 \right) u_{k,l,n} = 0 \\ & \Delta \mathrm{d} v_{p,q} = - \mu_{p,q}^2 \, \mathrm{d} v_{p,q} \;, \quad \Delta \mathrm{d} u_{k,l,n} = - M_{k,l,n}^2 \, \mathrm{d} u_{k,l,n} \end{split}$$

$$\begin{aligned} \mathbf{v_{p,q}}(\mathbf{x^1}, \mathbf{x^2}) &= \frac{1}{\sqrt{V}} e^{2\pi \mathrm{i} p x^1} e^{2\pi \mathrm{i} q x^2} \ , \quad p, q \in \mathbb{Z} \\ \mu_{\mathbf{p,q}}^2 &= p^2 \left(\frac{2\pi}{r^1}\right)^2 + q^2 \left(\frac{2\pi}{r^2}\right)^2 \end{aligned}$$

$$\mathbf{u_{k,l,n}}(\mathbf{x^1},\mathbf{x^2},\mathbf{x^3})$$

Laplacian spectrum

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2- and 3-forms obtained by Hodge star on 1-forms and scalars \Rightarrow complete spectrum of Δ_M .

$$(\Delta + \mu_{p,q}^2) v_{p,q} = 0 , \quad (\Delta + M_{k,l,n}^2) u_{k,l,n} = 0$$

$$\Delta dv_{p,q} = -\mu_{p,q}^2 dv_{p,q} , \quad \Delta du_{k,l,n} = -M_{k,l,n}^2 du_{k,l,n}$$

$$\mathbf{v_{p,q}}(\mathbf{x^1}, \mathbf{x^2}) = \frac{1}{\sqrt{V}} e^{2\pi i p x^1} e^{2\pi i q x^2} , \quad p, q \in \mathbb{Z}$$

$$\mu_{\mathbf{p,q}}^2 = p^2 \left(\frac{2\pi}{r^1}\right)^2 + q^2 \left(\frac{2\pi}{r^2}\right)^2 , \quad \mathbf{M_{k,l,n}^2} = k^2 \left(\frac{2\pi}{r^3}\right)^2 + (2n+1)|k| \frac{2\pi |\mathbf{f}|}{r^3}$$

$$\mathbf{u_{k,l,n}}(\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3) = \sqrt{\frac{r^2}{|N|V}} \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} e^{2\pi \mathrm{i} k(x^3 + N x^1 x^2)} e^{2\pi \mathrm{i} l x^1} \sum_{m \in \mathbb{Z}} e^{2\pi \mathrm{i} k m x^1} \Phi_n^{\lambda}(w_m)$$

where $\Phi_n^{\lambda}(z) = |\lambda|^{\frac{1}{4}} e^{-\frac{1}{2}|\lambda|z^2} H_n(|\lambda|^{\frac{1}{2}}z), \quad k \in \mathbb{Z}^*$, $n \in \mathbb{N}$

$\Delta B = YB$

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 $\Lambda B = YB$

$$\begin{split} p^2 + q^2 \neq 0: \quad \mathbf{B}_{\pm}^{\mathbf{p},\mathbf{q}} &= D \, v_{p,q} \left(Q e^1 - P e^2 + \frac{\Upsilon_{\pm}^{p,q} - (P^2 + Q^2)}{\mathrm{if}} e^3 \right) \\ p &= q = 0: \quad B_1^{0,0} &= v_{0,0} \, e^1 \; , \; B_2^{0,0} &= v_{0,0} \, e^2 \quad (Y_-^{0,0} = 0) \\ & B_3^{0,0} &= v_{0,0} \, e^3 \quad (Y_+^{0,0} = \mathbf{f}^2) \\ \left(P = p \frac{2\pi}{r^1} \; , \; Q = q \frac{2\pi}{r^2} \right) \end{split}$$

 $\mathbf{Y}_{\pm}^{\mathbf{p},\mathbf{q}} = P^2 + Q^2 + \frac{\mathbf{f}^2}{2} \pm \sqrt{\left(P^2 + Q^2 + \frac{\mathbf{f}^2}{2}\right)^2 - (P^2 + Q^2)^2}$

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$$\Delta B = YB$$

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$$p^{2} + q^{2} \neq 0: \quad \mathbf{B}_{\pm}^{\mathbf{p},\mathbf{q}} = D \, v_{p,q} \left(Q e^{1} - P e^{2} + \frac{\Upsilon_{\pm}^{p,q} - (P^{2} + Q^{2})}{\mathrm{if}} e^{3} \right)$$

$$p = q = 0: \quad B_{1}^{0,0} = v_{0,0} \, e^{1} \, , \, B_{2}^{0,0} = v_{0,0} \, e^{2} \quad (Y_{-}^{0,0} = 0)$$

$$B_{3}^{0,0} = v_{0,0} \, e^{3} \quad (Y_{+}^{0,0} = \mathbf{f}^{2})$$

$$(P = p \frac{2\pi}{r^{1}} \, , \, Q = q \frac{2\pi}{r^{2}})$$

$$\mathbf{Y}_{\pm}^{\mathbf{p},\mathbf{q}} = P^{2} + Q^{2} + \frac{\mathbf{f}^{2}}{2} \pm \sqrt{\left(P^{2} + Q^{2} + \frac{\mathbf{f}^{2}}{2}\right)^{2} - (P^{2} + Q^{2})^{2}}$$

$$\mathbf{B}_{\pm}^{\mathbf{k},\mathbf{l},\mathbf{n}} = \sum_{a=1}^{6} \varphi_a^{k,l,n}(x^m) e^a$$

$$\varphi_1^{k,l,n} = \frac{1}{2}c\sqrt{2^n n!} \left(u_{k,l,n} - \frac{\sqrt{(n+1)(n+2)}\mathbf{f}^2}{\alpha_n + 2|\lambda| + (n+2)\mathbf{f}^2} u_{k,l,n+2}\right)$$

$$\varphi_2^{k,l,n} = -\frac{i}{2} \operatorname{sgn}(\lambda) c \sqrt{2^n n!} \left(u_{k,l,n} + \frac{\sqrt{(n+1)(n+2)} \mathbf{f}^2}{\alpha_n + 2|\lambda| + (n+2) \mathbf{f}^2} u_{k,l,n+2} \right)$$

$$\varphi_3^{k,l,n} = -\frac{1}{2|\lambda|^{\frac{1}{2}}} c \sqrt{2^n n!} \frac{\mathbf{f} \sqrt{2(n+1)} (\alpha_n + 2|\lambda|)}{\alpha_n + 2|\lambda| + (n+2) \mathbf{f}^2} u_{k,l,n+1} ,$$

$$=-\frac{\mathrm{i}}{2}\mathrm{sg}$$

$$\frac{i}{2}$$
sgn

$$\frac{i}{2}$$
sgn

$$\sqrt{2^n n!}$$

 $\mathbf{Y}_{+}^{\mathbf{k},\mathbf{l},\mathbf{n}} = k^2 \left(\frac{2\pi}{3}\right)^2 + (2n+3)\frac{2\pi}{3}|k\mathbf{f}| + \frac{1}{2}\mathbf{f}^2$

$$l_{l,n} - \frac{1}{\epsilon}$$

$$l,n-\frac{1}{2}$$

$$a$$
 $a = \frac{1}{\alpha_n}$

$$\sqrt{(n)}$$

$$\left(\frac{\mathbf{f}^2}{2}\right)^2 - \left(\frac{\mathbf{f}^2}{2}\right)^2$$

$$-(P^2 +$$

$$\overline{{ t f}|{ t f}^2}$$

$$\pm\sqrt{\left(\frac{2\pi}{r^3}|k\mathbf{f}| + \frac{1}{2}\mathbf{f}^2\right)^2 + 2(n+1)\frac{2\pi}{r^3}|k\mathbf{f}|\mathbf{f}^2}$$

Truncations and the swampland

Consistent truncations and gauged supergravity

Standard truncation on group manifolds:

Scherk-Schwarz truncation:

keep the left-invariant/Maurer-Cartan forms: constant $\times e^a$. (compare to previous spectrum: $v_{p,q} \times e^{1,2,3}$, $v_{0,0} \times e^{1,2,3}$) \hookrightarrow 4d theory: gauged supergravity, gaugings: f^a_{bc} .

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Example with an SU(3) structure: keep finite set of forms on \mathcal{M} : $1, \omega_I, \alpha_K, \beta^K, \tilde{\omega}^I, \text{vol}_6$, on which fields are developed

+ build SU(3) structure forms J, Ω , analogous to CY

 $\hookrightarrow \mathcal{N} = 2$ gauged supergravity.

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Example with an SU(3) structure:

keep finite set of forms on \mathcal{M} : $1, \omega_I, \alpha_K, \beta^K, \tilde{\omega}^I, \text{vol}_6$, on which fields are developed

- + build SU(3) structure forms J, Ω , analogous to CY
- $\hookrightarrow \mathcal{N} = 2$ gauged supergravity.

Scherk—Schwarz truncations are **consistent truncations**: select set of "independent" or decoupled modes \rightarrow 4d theory. However: **not low energy** truncation/not low energy theory: light modes a truncated, heavy modes are kept.

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Low energy truncation on the nilmanifold

$$\begin{array}{l} -f^3{}_{12} = \mathbf{f} = \frac{r^3N}{r^1r^2} \\ \hookrightarrow \text{ consider approximation: } r^3 \leqslant |N|r^3 \ll r^1, \ r^2 \\ \text{ (small fiber/large base)} \end{array}$$

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Low energy truncation on the nilmanifold

$$-f^3{}_{12} = \mathtt{f} = \tfrac{r^3N}{r^1r^2}$$

- \hookrightarrow consider approximation: $r^3 \leq |N|r^3 \ll r^1$, r^2 (small fiber/large base)
- \Rightarrow hierarchy of scales: $|\mathbf{f}| \ll \frac{1}{r^1}, \frac{1}{r^2} \ll \frac{1}{r^3}$ Hierarchy between curvature vs radii scales, geometric flux vs Kaluza–Klein scales.

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Low energy approximation: truncate masses $\geqslant \frac{1}{r^1}, \frac{1}{r^2}, \frac{1}{\sqrt{r^1 r^2}}$

 \rightarrow apply on the complete spectrum of Laplacian

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Low energy approximation: truncate masses $\geq \frac{1}{r^1}, \frac{1}{r^2}, \frac{1}{\sqrt{r^1 r^2}}$

 \rightarrow apply on the complete spectrum of Laplacian, left with light modes (mass = 0 or |f|):

$$1, \quad e^1, \ e^2, \ e^3, \quad e^2 \wedge e^3, \ e^3 \wedge e^1, \ e^1 \wedge e^2, \quad e^1 \wedge e^2 \wedge e^3$$

up to normalisation constant $v_{0,0} = \frac{1}{\sqrt{V}}$.

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Low energy truncation on the nilmanifold

$$-f^3{}_{12}=\mathtt{f}=\tfrac{r^3N}{r^1r^2}$$

 \hookrightarrow consider approximation: $r^3 \le |N|r^3 \ll r^1$, r^2 (small fiber/large base)

 \Rightarrow **hierarchy** of scales: $|\mathbf{f}| \ll \frac{1}{r^1}$, $\frac{1}{r^2} \ll \frac{1}{r^3}$ Hierarchy between curvature vs radii scales, geometric flux vs Kaluza–Klein scales.

Low energy approximation: truncate masses $\geqslant \frac{1}{r^1}, \frac{1}{r^2}, \frac{1}{\sqrt{r^1 r^2}}$

 \rightarrow apply on the complete spectrum of Laplacian, left with light modes (mass = 0 or |f|):

$$1, \quad e^1, \ e^2, \ e^3, \quad e^2 \wedge e^3, \ e^3 \wedge e^1, \ e^1 \wedge e^2, \quad e^1 \wedge e^2 \wedge e^3$$

up to normalisation constant $v_{0,0} = \frac{1}{\sqrt{V}}$.

Surprise: low energy truncation matches Scherk–Schwarz trunc. (on the Laplacian spectrum, in this regime)
Specific to nilmanifold w.r.t. to other group manifolds.

Testing the (refined) swampland distance conjecture

In field space, move from ϕ_0 to $\phi_0 + \Delta \phi$: a tower of modes of mass $m(\phi)$ becomes light:

$$m(\phi_0 + \Delta\phi) = m(\phi_0) f(\phi_0, \Delta\phi) e^{-\alpha \frac{\Delta\phi}{M_p}}$$

If at ϕ_0 : \checkmark effective theory \Rightarrow ruined at $\phi_0 + \Delta \phi$.

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If at ϕ_0 : \checkmark effective theory \Rightarrow ruined at $\phi_0 + \Delta \phi$. Test this here on towers of Laplacian eigenmodes:

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points in field space:
$$r^3 \ll r^{1,2} \ll \text{dist.} \longrightarrow \text{dist.}$$

number of light modes: finite

masses of light modes: 0, |f|

Testing the (refined) swampland distance conjecture

In field space, move from ϕ_0 to $\phi_0 + \Delta \phi$: a tower of modes of mass $m(\phi)$ becomes light:

$$m(\phi_0 + \Delta\phi) = m(\phi_0) f(\phi_0, \Delta\phi) e^{-\alpha \frac{\Delta\phi}{M_p}}$$

If at ϕ_0 : \checkmark effective theory \Rightarrow ruined at $\phi_0 + \Delta \phi$. Test this here on towers of Laplacian eigenmodes:

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points in field space: $r^3 \ll r^{1,2} \iff \text{dist.} \implies r^3 \gg r^{1,2}$ number of light modes: finite infinite

masses of light modes: 0, $|\mathbf{f}|$ 0, $\sqrt{Y^{p,i}}$

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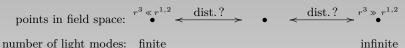
spectrum

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Distance? Compute kinetic terms...

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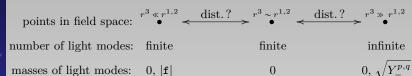
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points in field space:
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Distance? Compute kinetic terms...

Counterexample of the conjecture? String effects? Quantum effects?

Due to the nilmanifold geometry, away from the lamppost...

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De Sitter swampland criterion

Consider gravity minimally coupled to scalar fields with potential $V(\phi_i)$:

$$S = \int d^4x \sqrt{|g_4|} \left(\mathcal{R}_4 + \text{kin. terms} - V \right)$$

Solutions with constant scalars:

$$\mathcal{R}_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{R}_4 = -\frac{g_{\mu\nu}}{2} V \Rightarrow \mathcal{R}_4 = 2V , \quad \hat{\sigma}_{\phi_i} V = 0$$

Extrema of potential, value $V|_0$: maximally symmetric 4d space-time, cosmological constant $\Lambda = \frac{1}{2}V|_0$, de Sitter $V|_0 > 0$.

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$$cV \le |\nabla V|$$

where
$$c > 0$$
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Extreme of potential value VI: maximally symmetric

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 \Rightarrow extremum: $V|_0 \le 0 \Rightarrow$ no de Sitter solution (+ cosmological consequences)

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Inspired by situation of de Sitter vacua in string theory: difficult/unnatural.

Searches have provided similar conditions/criteria.

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Difficulty is to have V > 0, $\partial_{\phi_i} V = 0$, $\partial_{\phi_i}^2 V > 0$ together

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Difficulty is to have V > 0, $\partial_{\phi_i} V = 0$, $\partial_{\phi_i}^2 V > 0$ together \hookrightarrow a natural criterion:

$$\exists b_i \in \mathbb{R}, c_i \in \mathbb{R}_+ \text{ such that}$$

$$V + \sum_i b_i \ \phi_i \partial_{\phi_i} V + \sum_i c_i \ \phi_i^2 \partial_{\phi_i}^2 V \leqslant 0$$

Solution:
$$V|_0 + \sum_i c_i \ (\phi_i^2 \partial_{\phi_i}^2 V)|_0 \le 0$$

- \Rightarrow no stable de Sitter solution, tachyonic de Sitter sol. \checkmark .
- ⇒ checks? Cosmological implications?

There exist (unstable) 10d classical de Sitter solutions: in type IIA/B with intersecting D_p/O_p , on group manifolds

 $\Rightarrow |\nabla V| \geqslant c V \text{ wrong ?}$

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Distinguish: 10d solution and 4d low energy effective theory, especially if not a consistent truncation.

Argument saying that **two points can be compatible**: Known 10d de Sitter solutions + 4d low energy effective theory without de Sitter solutions: \checkmark .

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 \hookrightarrow which de Sitter swampland criterion is valid? (if any)

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Summary

Summary

- Obtained Laplacian spectrum on a nilmanifold
- Found a low energy truncation that matches Scherk–Schwarz truncation; probably only for nilmanifolds
- Used the spectrum to "test" (refined) swampland distance conjecture, maybe a counterexample
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- But an argument why 10d classical de Sitter solutions and no 4d de Sitter sol. in low energy effective theory can be compatible

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Thank you for your attention!