Global tensor matter transitions in F-theory

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based on: 1804.07386 with P.-K. Oehlmann and F. Ruehle
F-theory [Vafa '96]

- **Non-perturbative** type IIB string theory
- **Varying axio-dilaton**
- Encoded in **additional auxiliary torus** over (compact complex) base manifold B via SL(2,Z) invariance
- **Geometrization** of physical properties
- Great framework for **phenomenology**

\[ y^2 = x^3 + fx + g \rightarrow \text{ord}(f, g, \Delta), \quad \Delta = 27g^2 + 4f^3 \]

Elliptically-fibered Calabi-Yau n-folds (here n = 3)
F-theory (local and global)

- **Codimension-one** singularity induces non-Abelian gauge algebra

- **Codimension-two** singularity encodes matter

- No gravitational anomalies

- Mordell-Weil group dictates U(1) symmetries and non-Abelian gauge group

- **Multi-sections** lead to discrete gauge symmetries

- Gravitational anomalies
Non-flat fibers

$$\text{ord}(f, g, \Delta) \geq (4, 6, 12)$$

- Resolution in fiber not enough for smooth geometry
- Blow-up in the base
- Strongly coupled dynamics (superconformal matter)

**Why do we care?**

- Generically appear for GUT gauge groups, for SO(10) see e.g. [Buchmuller, MD, Oehlmann, Ruehle ’17]

- Lead to non-trivial 6d SCFTs in decoupling limit, reviewed in [Heckman, Rudelius ’18]
Transitions

**Strong coupling** dynamics **hard to study directly** (no supergravity limit)

**Indirect** investigation **via transitions**
see also [Anderson, Gray, Raghuram, Taylor ’15]

- 6d SUGRA defined by **gauge algebra, matter spectrum and tensors**
- Complex structure deformation to **(4,6,12)** point
- Deformation to **tensor branch**
6d anomalies

- **Strict anomaly constraints** in six dimensions

- **Irreducible** anomalies have to vanish

\[
H - V + 29T - 273 = 0
\]

\[
B_{\text{adj}} - \sum_R n[R] B_R = 0
\]

- **Reducible** can be cancelled by generalized Green-Schwarz mechanism, [Green, Schwarz '84], [Sagnotti '92], [Sadov '96]

\[
\mathcal{I}_8^{\text{red}} = - \frac{1}{32} \Omega_{\alpha\beta} X^\alpha X^\beta
\]
Green-Schwarz and geometry

e.g. [Park ’11], [Park, Taylor ’11], [Morrison, Park ‘12]

\[ \mathcal{I}_8^{\text{red}} = -\frac{1}{32} \Omega_{\alpha\beta} X^\alpha X^\beta, \quad X^\alpha = \frac{1}{2} a^\alpha \text{tr} R^2 - \frac{2}{\lambda} b^\alpha \text{tr} F^2 - \sum_{i,j} 2b^\alpha_{ij} F_i F_j \]

• \{H_\alpha\} a basis for \( H_2(B, \mathbb{Z}) \) then: \( \Omega_{\alpha\beta} = H_\alpha \cdot H_\beta \) describes the participating tensor fields

• \( a^\alpha \) is the vector describing the anticanonical class \( K_B^{-1} \)

• \( b^\alpha \) is the vector describing the locus on which gauge algebra \( g \) is localized

• \( b^\alpha_{ij} \) describes the coefficient of the height pairing \(-\pi(\sigma_i \cdot \sigma_j)\) accounting for Abelian symmetries
Transitions geometrically \[\text{[Bershadsky, Johansen '96]}\]

- Divisor $\mathcal{Z}$ with gauge algebra $g$
- Intersection with divisor $D$ (possible matter locus)
- Complex structure \textbf{deformation to (4,6,12) point}
- Blow-up in base with self-intersection -1 curve $E$

\[
\begin{align*}
\mathcal{Z} & \quad D \\
\mathcal{Z} & \quad D \\
\tilde{D} & \quad \tilde{\mathcal{Z}}
\end{align*}
\]
Anomalies after blow-up

Additional tensor multiplet:

• Get rid of 29 degrees of freedom in hypermultiplet sector

• More possibilities for Green-Schwarz mechanism

Involved matter representations constrained?

• Non-Abelian representations fixed

• Abelian charges can be constrained
We use:

- **Smoothness of fiber** over $E$ (no new gauge group)

- **Properties of the blow-down map** $\beta$:

  $\beta_* : H_2(\tilde{B}, \mathbb{Z}) \rightarrow H_2(B, \mathbb{Z})$, \hspace{1cm} $\beta_*^{-1}(D) \cdot \beta_*^{-1}(D')|_{\tilde{B}} = D \cdot D'|_B$

- **Fixed transformation** of anticanonical class:

  $K^{-1}_{\tilde{B}} = \beta_*^{-1}(K^{-1}_B) - E$

- **Orthogonality** properties of Shioda map

- **New basis** of second homology:

  $\beta_*^{-1}(H_\alpha) = \tilde{H}_\alpha + h_\alpha E$, \hspace{1cm} $\{H_\alpha\}_B \rightarrow \{\tilde{H}_\alpha, E\}_{\tilde{B}}$
Anomalies in transition

From connection between geometry and anomalies:

\[
\Omega_{\alpha \beta} \rightarrow \tilde{\Omega}_{AB} = \begin{pmatrix}
\Omega_{\alpha \beta} - h_\alpha h_\beta & h_\alpha \\
h_\beta & -1
\end{pmatrix}
\]

\[
a^\alpha \rightarrow \tilde{a}^A = \begin{pmatrix}
a^\alpha \\
a^\alpha h_\alpha - 1
\end{pmatrix}
\]

\[
b^\alpha \rightarrow \tilde{b}^A = \begin{pmatrix}
b^\alpha \\
0
\end{pmatrix}
\]
Non-Abelian matter fixed

Back to matter states:

\[
\sum_{\mathcal{R}} \Delta n[\mathcal{R}] \dim(\mathcal{R}) < 29 ,
\]

\[
\sum_{\mathcal{R}} \Delta n[\mathcal{R}] B_{\mathcal{R}} = 0 ,
\]

\[
\frac{\lambda^2}{3} \sum_{\mathcal{R}} \Delta n[\mathcal{R}] C_{\mathcal{R}} = -1 ,
\]

\[
\frac{\lambda}{6} \sum_{\mathcal{R}} \Delta n[\mathcal{R}] A_{\mathcal{R}} = -1
\]

- SU(5): \( \Delta S \supset -(10 \oplus 5 \oplus 5 \oplus 5) \)
- SO(10): \( \Delta S \supset -(16 \oplus 10) \)
Abelian matter

With assumption:

$$\Omega_{\alpha\beta} \Delta b^\alpha_{ij} (b^\beta - a^\beta) = 0$$

Curiously always satisfied for toric hypersurfaces with genus-one $\mathcal{Z}$

Satisfied in all examples:

- **SU(5):** \( \Delta S = -(10_{-3/5} \oplus 3 \times 5_{1/5} \oplus 3 \times 1 \oplus 1_0) \)
- **SO(10):** \( \Delta S = -(16_{1/4} \oplus 10_{1/2} \oplus 1 \oplus 2 \times 1_0) \)
Toric geometry

e.g. [Klevers, Mayorga Pena, Oehlmann, Piragua, Reuter ‘14]

Using the **top construction** and connected **toric geometry**: [Batyrev ’94], [Candelas, Font ’98],[Bouchard, Skarke ’03]

Point in face corresponds to **non-flat fiber** see also [Braun, Grimm, Keitel ’13]

\[ h^{1,1}(Y_3) = \text{rank}(G) + h^{1,1}(B) - 1 + \sum_i \delta_i n_{\text{SCP}_i} \]

**Modification** of number of neutral singlets in presence of **non-flat fibers**
Conclusions

• Consistency conditions for coupling strongly-coupled sectors to gravity see also [del Zotto, Heckman, Morrison, Park '15]

• Inclusion of global data controlling Abelian symmetries

• Toric realization in terms of hypersurfaces and tops

Outlook

• Extension to higher-order singularities

• Direct investigation of non-flat fibers [Achmed-Zade, Garcia-Etxebarria, Mayrhofer ’18]

• Comparison with Abelian flavor symmetries in decoupling limit [Lee, Regalado, Weigand ’18]