Global tensor matter transitions in F-theory

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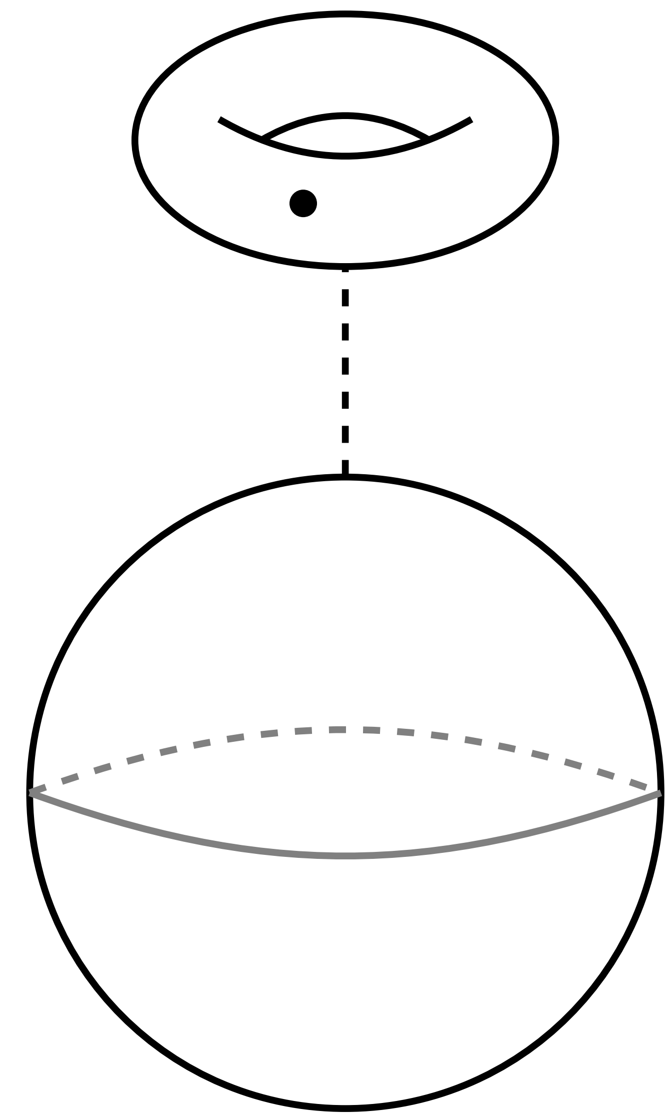
F-theory [Vafa '96]

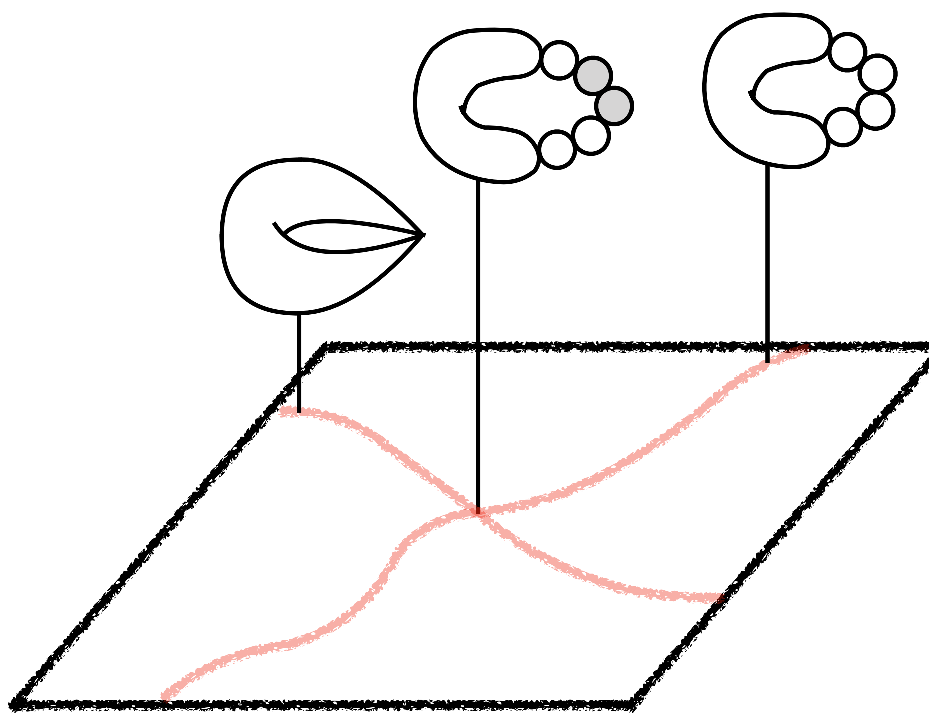
- **Non-perturbative** type IIB string theory
- **Varying axio-dilaton**
- Encoded in **additional auxiliary torus** over (compact complex) base manifold B via SL(2,Z) invariance
- **Geometrization** of physical properties
- Great framework for **phenomenology**

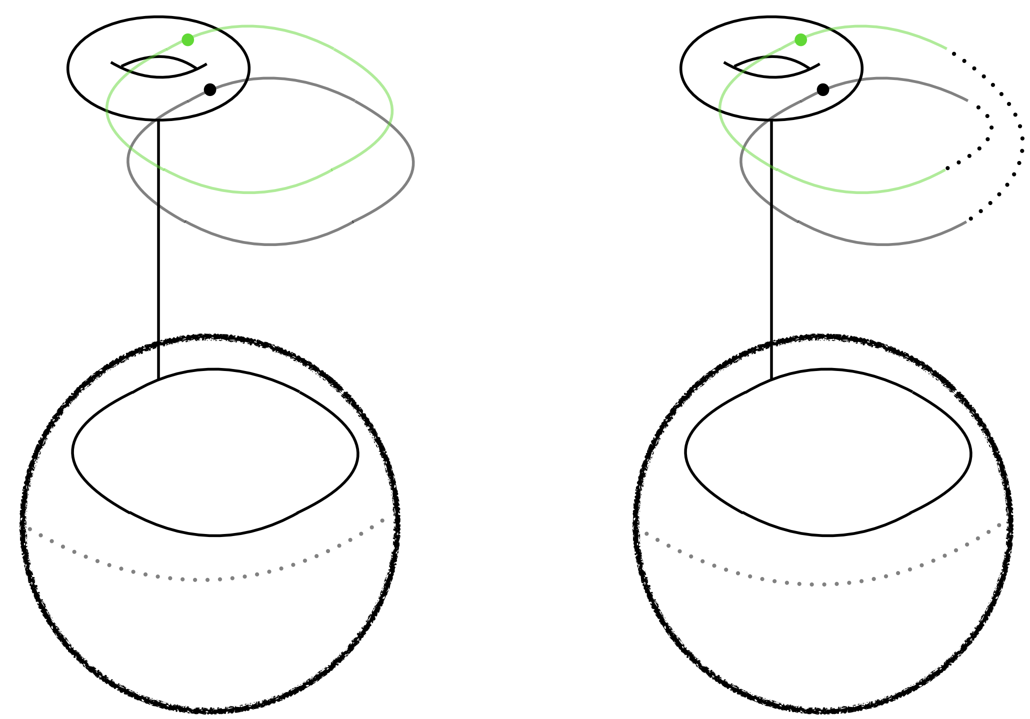


$$y^2 = x^3 + fx + g \quad \rightarrow \quad \mathrm{ord}(f, g, \Delta), \qquad \Delta = 27g^2 + 4f^3$$

Elliptically-fibered Calabi-Yau n-folds (here n = 3)

F-theory (local and global)





- **Codimension-one** singularity induces **non-Abelian gauge algebra**
- **Codimension-two** singularity encodes matter
- No gravitational anomalies

- Mordell-Weil group dictates **U(1) symmetries** and **non-Abelian gauge group**
- **Multi-sections** lead to **discrete gauge symmetries**
- Gravitational anomalies

Non-flat fibers

$$\text{ord}(f, g, \Delta) \geq (4, 6, 12)$$

- Resolution in fiber not enough for smooth geometry
- **Blow-up in the base**
- **Strongly coupled** dynamics (superconformal matter)

Why do we care?

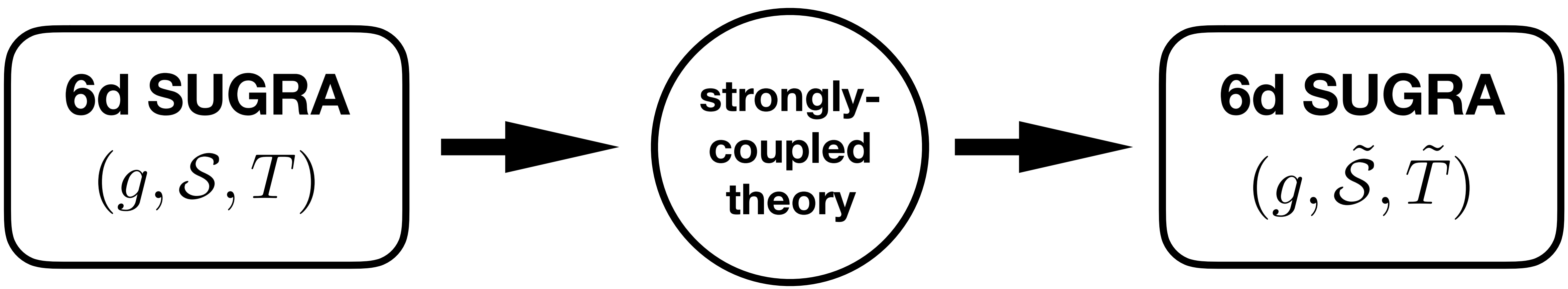
- Generically **appear for GUT** gauge groups, for SO(10) see e.g. [Buchmuller, MD, Oehlmann, Ruehle '17]
- Lead to **non-trivial 6d SCFTs in decoupling limit**, reviewed in [Heckman, Rudelius '18]

Transitions

Strong coupling dynamics **hard to study directly** (no supergravity limit)

Indirect investigation **via transitions**

see also [Anderson, Gray, Raghuram, Taylor '15]



- 6d SUGRA defined by **gauge algebra, matter spectrum and tensors**
- Complex structure deformation to **(4,6,12)** point
- Deformation to **tensor branch**

6d anomalies

- Strict **anomaly constraints** in **six dimensions**
- **Irreducible** anomalies have to **vanish**

$$H - V + 29T - 273 = 0$$

$$B_{\mathrm{adj}} - \sum_{\mathbf{R}} n[\mathbf{R}] B_{\mathbf{R}} = 0$$

$$\mathrm{tr}_{\mathbf{R}} F^4 = B_{\mathbf{R}}\, \mathrm{tr} F^4 + C_{\mathbf{R}}\, (\mathrm{tr} F^2)^2$$

$$\mathrm{tr}_{\mathbf{R}} F^2 = A_{\mathbf{R}}\, \mathrm{tr} F^2$$

- **Reducible** can be **cancelled by** generalized **Green-Schwarz mechanism,** [Green, Schwarz '84], [Sagnotti '92], [Sadov '96]

$$\mathcal{I}_8^{\mathrm{red}} = -\tfrac{1}{32} \Omega_{\alpha\beta} X^\alpha X^\beta$$

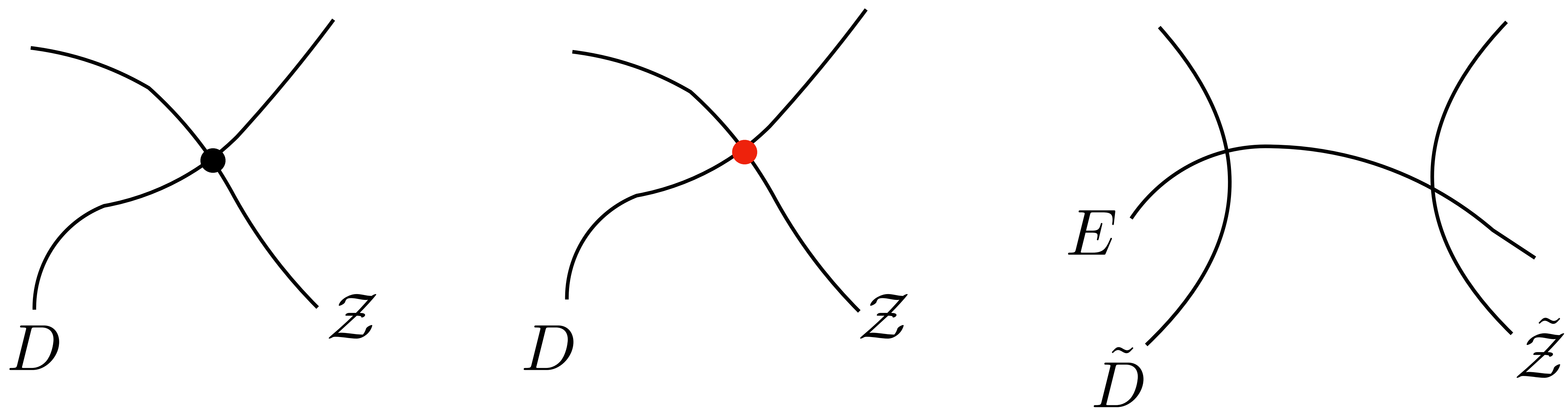
Green-Schwarz and geometry

e.g. [Park '11], [Park, Taylor '11], [Morrison, Park '12]

$$\mathcal{I}_8^{\mathrm{red}} = -\tfrac{1}{32}\Omega_{\alpha\beta}X^\alpha X^\beta\,, \quad X^\alpha = \tfrac{1}{2}a^\alpha \mathrm{tr}R^2 - \tfrac{2}{\lambda}b^\alpha \mathrm{tr}F^2 - \sum_{i,j} 2b^\alpha_{ij}F_iF_j$$

- $\{H_\alpha\}$ a **basis** for $H_2(B,\mathbb{Z})$ then: $\Omega_{\alpha\beta} = H_\alpha \cdot H_\beta$ describes the participating tensor fields
- a^α is the vector describing the **anticanonical class** K_B^{-1}
- b^α is the vector describing the **locus** on which **gauge algebra** g is localized
- b^α_{ij} describes the coefficient of the **height pairing** $-\pi(\sigma_i \cdot \sigma_j)$ accounting for **Abelian symmetries**

Transitions geometrically [Bershadsky, Johansen '96]



- Divisor $\mathcal{Z}$ with gauge algebra g
- Intersection with divisor D (possible matter locus)
- Complex structure **deformation to (4,6,12)** point
- Blow-up in base with self-intersection -1 curve E

Anomalies after blow-up

Additional tensor multiplet:

- Get rid of 29 degrees of freedom in hypermultiplet sector
- More possibilities for Green-Schwarz mechanism

Involved matter representations constrained?

- **Non-Abelian representations fixed**
- **Abelian charges can be constrained**

We use:

- **Smoothness of fiber** over E (no new gauge group)

- Properties of the **blow-down map** β :

$$\beta_* : \quad H_2(\tilde{B}, \mathbb{Z}) \to H_2(B, \mathbb{Z})\,, \qquad \beta_*^{-1}(D) \cdot \beta_*^{-1}(D')|_{\tilde{B}} = D \cdot D'|_B$$

- **Fixed transformation** of anticanonical class:

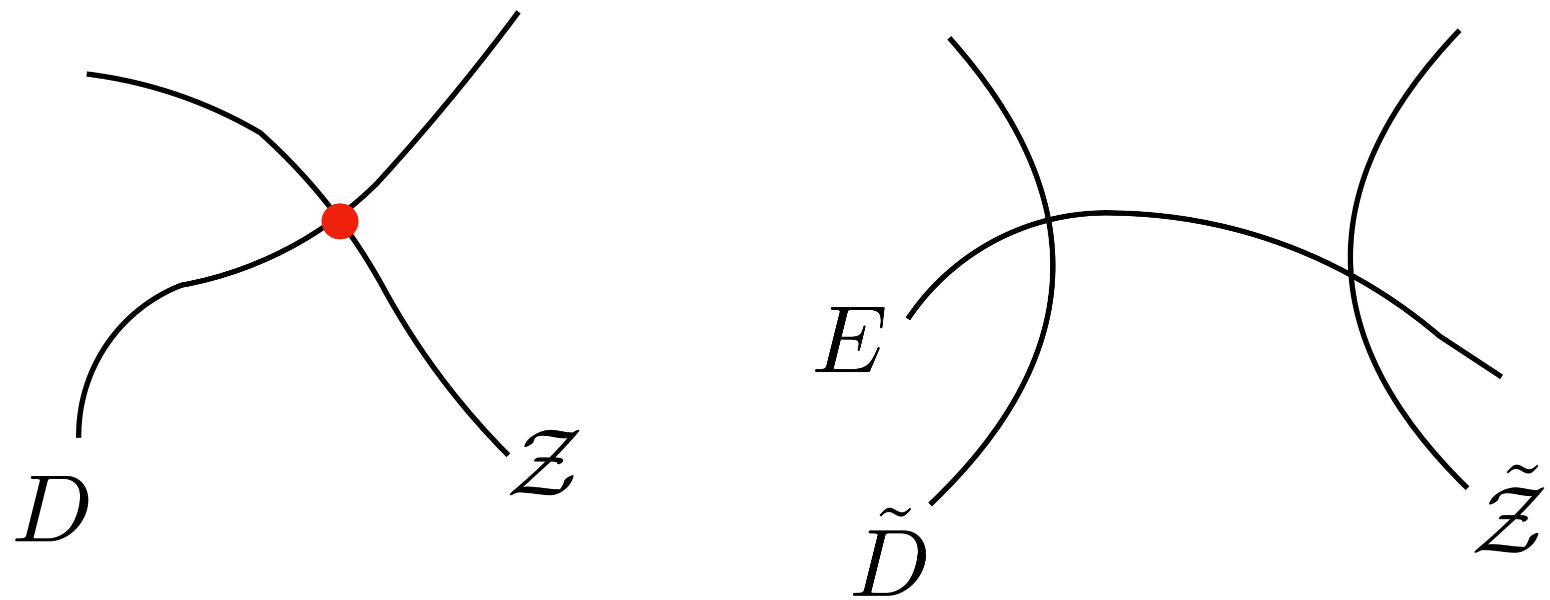
$$K_{\tilde{B}}^{-1} = \beta_*^{-1}(K_B^{-1}) - E$$

- **Orthogonality** properties of Shioda map

- **New basis** of second homology:

$$\beta_*^{-1}(H_\alpha) = \tilde{H}_\alpha + h_\alpha E\,,$$
$$\{H_\alpha\}_B \to \{\tilde{H}_\alpha, E\}_{\tilde{B}}$$

Anomalies in transition



From connection between geometry and anomalies:

$$\Omega_{\alpha\beta} \to \tilde{\Omega}_{AB} = \begin{pmatrix} \Omega_{\alpha\beta} - h_\alpha h_\beta & h_\alpha \\ h_\beta & -1 \end{pmatrix}$$

$$a^\alpha \to \tilde{a}^A = \begin{pmatrix} a^\alpha \\ a^\alpha h_\alpha - 1 \end{pmatrix}$$

$$b^\alpha \to \tilde{b}^A = \begin{pmatrix} b^\alpha \\ 0 \end{pmatrix}$$

Non-Abelian matter fixed

Back to matter states:

$$\sum_{\mathbf{R}} \Delta n[\mathbf{R}]\dim(\mathbf{R}) < 29\,,$$
$$\sum_{\mathbf{R}} \Delta n[\mathbf{R}] B_{\mathbf{R}} = 0\,,$$
$$\frac{\lambda^2}{3}\sum_{\mathbf{R}} \Delta n[\mathbf{R}] C_{\mathbf{R}} = -1\,,$$
$$\frac{\lambda}{6}\sum_{\mathbf{R}} \Delta n[\mathbf{R}] A_{\mathbf{R}} = -1$$

- SU(5): $\Delta\mathcal{S} \supset -(\mathbf{10} \oplus \mathbf{5} \oplus \mathbf{5} \oplus \mathbf{5})$
- SO(10): $\Delta\mathcal{S} \supset -(\mathbf{16} \oplus \mathbf{10})$

Abelian matter

With assumption:

$$\Omega_{\alpha\beta}\Delta b^{\alpha}_{ij}(b^{\beta}-a^{\beta})=0$$

Curiously always satisfied for toric hypersurfaces with genus-one $\mathcal{Z}$

$$\boxed{\lambda\sum_{\mathbf{R},i,j}\Delta n[\mathbf{R},q_i,q_j]q_iq_jA_{\mathbf{R}}-\tfrac{1}{6}\sum_{i,j}\Delta n[q_i,q_j]q_iq_j=0}$$

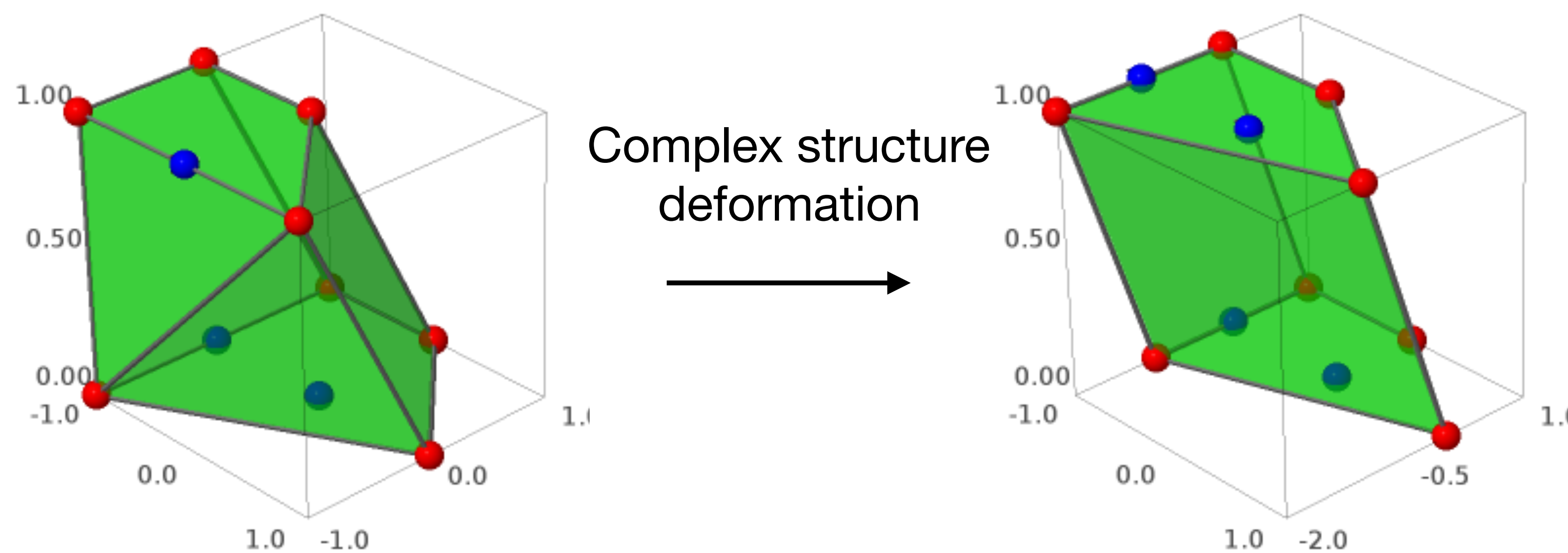
Satisfied in **all** examples:

- SU(5): $\Delta\mathcal{S}=-(\mathbf{10}_{-3/5}\oplus 3\times\mathbf{5}_{1/5}\oplus 3\times\mathbf{1}_1\oplus\mathbf{1}_0)$
- SO(10): $\Delta\mathcal{S}=-(\mathbf{16}_{1/4}\oplus\mathbf{10}_{1/2}\oplus\mathbf{1}_1\oplus 2\times\mathbf{1}_0)$

Toric geometry

e.g. [Klevers, Mayorga Pena, Oehlmann, Piragua, Reuter ‘14]

Using the **top construction** and connected **toric geometry:**
[Batyrev ’94], [Candelas, Font ’98],[Bouchard, Skarke ’03]



Point in face corresponds to **non-flat fiber** see also [Braun, Grimm, Keitel ’13]

$$h^{1,1}(Y_3) = \mathrm{rank}(G) + h^{1,1}(B) - 1 + \sum_i \delta_i n_{\mathrm{SCP}_i}$$

Modification of number of **neutral singlets** in presence of **non-flat fibers**

Conclusions

- **Consistency conditions** for **coupling strongly-coupled sectors** to **gravity** see also [del Zotto, Heckman, Morrison, Park ‘15]
- Inclusion of **global data** controlling **Abelian symmetries**
- **Toric realization** in terms of hypersurfaces and tops

Outlook

- Extension to **higher-order singularities**
- **Direct investigation** of **non-flat fibers** [Achmed-Zade, Garcia-Etxebarria, Mayrhofer ’18]
- Comparison with **Abelian flavor symmetries** in decoupling limit [Lee, Regalado, Weigand ’18]