The Weak Gravity Conjecture in F-theory

- 1803.07998 w/ Seung-Joo Lee and Diego Regalado
- to appear w/ Seung-Joo Lee, Wolfgang Lerche, Diego Regalado

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Gravity and U(1)s

Quantum gravity effects deeply woven into fabric of string theory:
No arbitrary independent tuning of gravitational and gauge degrees of freedom possible

- Heterotic string: Gravity and gauge d.o.f. both in closed sector
- Open string/brane theories: Non-abelian gauge theory can be decoupled from gravitational bulk only when UV complete by itself - geometrically by shrinking brane cycles of non-abelian branes

$U(1)$ gauge theories special

- Field theory: by themselves not UV complete
- Geometry: Non-shrinkability of height pairing of rational sections in F-theory [Lee,Regalado,TW'18] see talk by S.-J. Lee

Deep interrelations between quantum gravity and string geometry

Gravity and U(1)s

Lesson:

For $U(1)$ gauge theory in F-theory, $M_{Pl} \to \infty$ only possible for $g_{\rm YM} \to 0$

This talk considers the converse question:

What happens if take $g_{\rm YM} \to 0$ at M_{Pl} finite?

Field theory intuition:

1. Weak Gravity Conjecture [Arkani-Hamed,Motl,Nicolis,Vafa'06], ...
   Gravity is weakest force.
2. In presence of gravity, no global symmetries. [Banks,Dixon'88], ...
   All symmetries must be gauged in the UV.

General expectation: [Ooguri,Vafa'06], ...

✓ Offensive limit should be at infinite distance (beyond reach)

✓ Effective theory must break down (quantum gravity censorship):

As $g_{\rm YM} \to 0$ expect infinitely many charged massless states.

Gravity and U(1)s

In particular: [Grimm,Palti,Valenzuela'18] analyses this (see talk by I. Valenzuela)

- in context of 4d $N = 2$ Type IIB compactifications
- for $U(1)$ in question from Type IIB Ramond Ramond sector

[Palti'17] [Kläwer,Palti'16] [Heidenreich,Reece,Rudelius'16/'18] [Andriolo,Junghans,Noumi,Shiu'18]

[Blumenhagen,Kläwer,Schlechter,Wolf'18], . . . cf. talks by Corvilain, Kläwer, Wolf, . . .

Hardly studied quantitatively so far:
Quantum Gravity Conjectures and $U(1)$/gauge symmetries
in 'open string sector'

This talk:

- Quantitative study of various QG conjectures for 'open string' $U(1)$s in F-theory
- For better control of quantum corrections to geometry: F-theory in 6d
- Results apply to abelian or non-abelian gauge groups alike

Summary of results

Main results: [Lee,Lerche,Regalado,TW to appear]

1. For fixed M_{Pl}, limit $g_{\mathrm{YM}} \to 0$ lies at infinite distance in Kähler moduli space of base B_2.
2. As $g_{\mathrm{YM}} \to 0$, necessarily tensionless strings arise in the 6d compactification.
   Math: **Mori's cone theorem**
3. These give rise to infinitely many massless BPS particles which are charged under the gauge group.
   Math: **Theory of weak Jacobi forms** and **elliptic genera**
4. The Sublattice Weak Gravity Conjecture is satisfied (at least asymptotically) if one takes into account the (massive) states from strings wrapped on a D3-brane.

Formal new results:

Computation of elliptic genus for general Mordell-Weil flavour group

F-theory in 6d

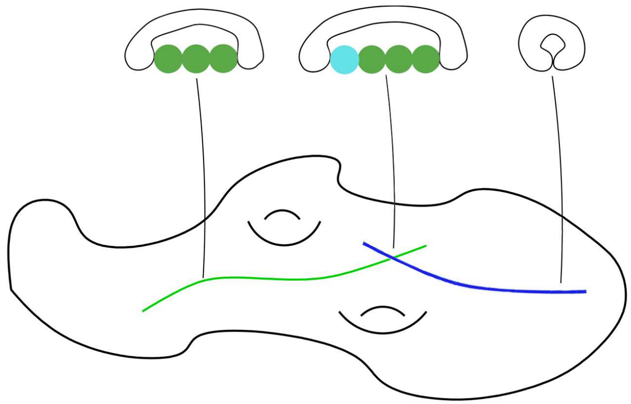
Context: F-theory compactified on elliptic CY_3 $Y_3 \to B_2$

Effective theory in $\mathbb{R}^{1,5}$:

$N = (1,0)$ supergravity (8 SUSYs)

base B_2: complex surface

7-branes on complex curve $C \subset B_2$



Couplings:

$\mathbf{M_{\mathrm{Pl}}^4 = 4\pi\mathrm{vol}_J(B_2)}$ $\qquad$ $\mathbf{\frac{1}{g_{\mathrm{YM}}^2} = 4\pi\mathrm{vol}_J(C)}$

- non-abelian gauge algebra $\mathfrak{g}$:
  C contained in discriminant of fibration (wrapped by brane stack)
- abelian gauge algebra $\mathfrak{g} = \mathfrak{u}(1)_A$: [cf talks by Cvetič, Mayorga, Dierigl]
  $C = -\pi_*(\sigma(S_A) \cdot \sigma(S_A))$ (height pairing of rational section S_A)

Global limit in Kähler geometry

[Lee,Lerche,Regalado,TW to appear]

Aim: $$\frac{1}{g^2_{\rm YM}} \sim {\rm vol}_J(C) \to \infty \quad \text{while} \quad M_{\rm Pl} \sim {\rm vol}_J(B_2) \equiv 1 \quad (*)$$

Result: There must exist another curve C_0 with

$$C_0 \cdot C \neq 0 \qquad \text{and} \qquad {\rm vol}_J(C_0) \to 0$$

General intuition

"On finite volume surface, if one direction gets big, normal direction must get very small".

Proof:

Step 1) Limit $(*)$ requires asymptotically - (in simplest case!)

$$J = a(t)\left(tJ_0 + \frac{1}{2t}N\right) \qquad \text{with} \qquad t \to \infty, \quad a(t) = (1+\frac{\gamma}{4t^2})^{-1/2}$$

- $\int_{B_2} J_0 \cdot J_0 = 0$, $\quad \int_{B_2} J_0 \cdot N = 1$, $\quad \gamma = \int_{B_2} N \wedge N$
- $\int_C J_0 = 1$

Global limit in Kähler geometry

$$\frac{1}{g_{\mathrm{YM}}^2} \sim \mathrm{vol}_J(C) \to \infty \qquad \text{while} \qquad M_{\mathrm{Pl}} \sim \mathrm{vol}_J(B_2) \equiv 1$$

$$J \sim \left(tJ_0 + \frac{1}{2t}N\right) \qquad \text{with} \quad \int_{B_2} J_0 \cdot J_0 = 0, \quad \int_{B_2} J_0 \cdot N = 1$$

1. W.l.o.g. write $J = A_i J_i + \tilde{A}_i \tilde{J}_i + a'_k J'_k$
   - J_i, $\tilde{J}_i$, J'_k Kähler cone generators
   - $A_i, \tilde{A}_i \to \infty$, a'_k finite ≥ 0
   - $\int_C J_i \neq 0$, $\int_C \tilde{J}_i = 0$
2. Simplest case: Fastest rate of divergence for some A_i else - see paper!

$$\Rightarrow J = t(a_i J_i + \tilde{a}_i \tilde{J}_i) + a'_k J'_k =: tJ_0 + J_2 \qquad \int_C J_0 \neq 0$$

3. $\mathrm{vol}_J(B_2) = \int_{B_2} J^2 = \int_{B_2} (t^2 J_0^2 + J_2^2 + 2tJ_0 J_2)$ finite as $t \to \infty$

   $\Rightarrow J_0^2 = 0$ and $J_0 \cdot J_2 \sim \frac{1}{2t}$

Global limit in Kähler geometry

Step 1) $J \sim tJ_0 + \frac{1}{2t}N$ with $\int_{B_2} J_0^2 = 0$, $\int_{B_2} J_0 \cdot N = 1$

Step 2)

Idea: If J_0 is the class of a holomorphic curve C_0, i.e. if

$$J_0 \sim [C_0]$$

✓ $\mathrm{vol}_J(C_0) \sim \frac{1}{2t}\int_{C_0} N \to 0$ as $t \to \infty$

✓ $C_0 \cdot C \neq 0$ because $\int_C J_0 \neq 0$

Indeed this is the case, and in addition C_0 is a $\mathbb{P}^1$.

- $J_0 \in \overline{\mathcal{K}}$ (Kähler cone closure) $\subset \overline{NE(B_2)}$ (Mori cone closure)
- Mori's cone theorem: $\overline{NE(B_2)} = \overline{NE(B_2)}_{\bar{K}\leq 0} + \sum_i \mathbb{R}_{\geq 0}[C_i]$ $\qquad C_i$ are $\mathbb{P}^1$s
- We can rule out $J_0 \cdot \bar{K} \leq 0$ for F-theory base:
  $\bar{K} \geq 0$ and exists $n > 0$: $n\bar{K} \geq C$ with $J_0 \cdot C > 0$ for C a discriminant component or height pairing

  $\Rightarrow$ J_0 is effective curve class C_0. Uses that B_2 is F-theory base!

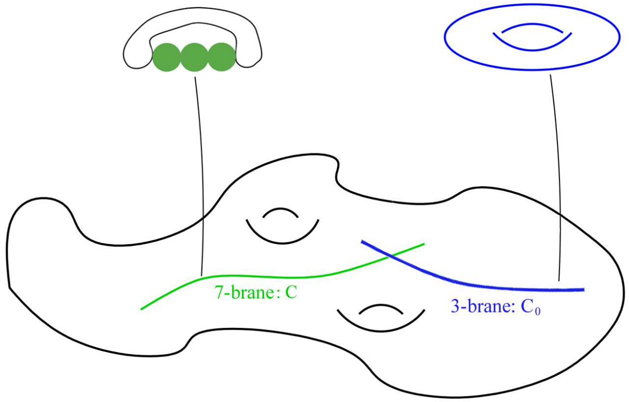
Example

$B_2 = dP_1$: $\mathbb{P}^2$ (class: H) with one blowup curve E

$$H^2 = 1\,, \qquad E^2 = -1\,, \qquad H \cdot E = 0\,.$$

- Kähler cone generators: $H - E$, H
  $\Rightarrow J = a(H-E) + bH$ for $a \geq 0$, $b \geq 0$
- Mori cone generators: $H - E$, E
- $\mathrm{vol}(H) = a + b$, $\mathrm{vol}(E) = a \Rightarrow H$ large and E large for a large
- $\mathrm{vol}(B_2) = J^2 = b^2 + 2ab$ finite as $a \sim t \to \infty$ only for $b \sim \frac{1}{t}$
  $\Rightarrow C_0 = H - E$ with $\mathrm{vol}(C_0) \sim 1/t$

Tensionless Strings



- **7-brane on C**
  $\mathrm{vol}_J(C) \sim t \sim \frac{1}{g^2_{\mathrm{YM}}} \to \infty$
- **curve C_0 with $C_0 \cdot C \neq 0$**
  $\mathrm{vol}_J(C_0) \sim \frac{1}{2t} \to 0$

D3-brane wrapped on C_0 gives rise to string in $\mathbb{R}^{1,5}$
$=$ non-critical string theory in 6d with tension $T = (2\pi)\mathrm{vol}_J(C_0)$

Tower of BPS excitations: [Witten'97]

$$M_n^2 \sim n\,T \qquad n = 1, 2, 3, \ldots \quad \text{excitation level}$$

Maximal charge q_n per excitation level n: - derived later

$$q_n^2 = \beta\, n \qquad \beta = \mathcal{O}(1)$$

As $g_{\mathrm{YM}} \to 0$: the string becomes **tensionless** and
this **tower of charged states becomes massless**.

Quantum Gravity Conjectures

1) Since global symmetries are not possible, the limit must be at infinite distance in moduli space, i.e. it cannot be reached.

Indeed this is the case here.

Proof: For general surface B_2

- Basis ω_α $\alpha = 1, \ldots, h^{1,1}(B_2) = d, \quad \omega_\alpha \cdot \omega_\beta = \mathrm{diag}(+1, -1, \ldots, -1)$
- Kähler form $J = j^\alpha \omega_\alpha \qquad J^2 = 1$
  $\Rightarrow j^0 = \cosh x, \qquad j^i = (\sinh x) \underbrace{u^i(\phi^1, \ldots, \phi^{d-2})}_{S^{d-2}}$ with $\sum_i (u^i)^2 = 1$
- Metric on Kähler moduli space:
  $ds^2 = dx^2 + (\sinh x)^2 d\Omega_{S^{d-2}}$
- Distance between points P and Q in moduli space:

$$d(P,Q) = \int \sqrt{1 + (\sinh x)^2 h_{AB} \phi'^A \phi'^B} dx \ \geq x_P - x_Q$$

Present case: $t \sim \frac{1}{2} e^x \quad \Rightarrow \quad$ **Limit $t \to \infty$ at distance $\Delta \sim \log(t)$**

Quantum Gravity Conjectures

1) No global symmetries.

$\Rightarrow$ The limit must be at infinite distance in moduli space.

Indeed this is the case here.

Conclusion: Limit $t \to \infty$ at distance $\Delta \sim \log(t) \to \infty$

2) Swampland Distance Conjecture: [Ooguri,Vafa] [Klaewer,Palti'16], [Grimm,Palti,Valenzuela'18] [Heidenreich,Reece,Rudelius'17,'18]

[Andriolo,Junghans,Noumi,Shiu'18]

Infinitely many - charged! - states become massless at exponential rate.

Present case:

$$\Delta \sim \log(t), \qquad m_n^2 \sim n\,T \sim n\,\mathrm{vol}_J(C_0) \sim \frac{n}{2}\frac{1}{t} \sim e^{-\Delta}$$

Quantum Gravity Conjectures

3) Sublattice Weak Gravity Conjecture [Heidenreich,Reece,Rudelius'16-'18]

[Shiu et al. 16-18], ...

= for each Q in a charge sublattice $\exists$ state with

$$Q^2 g_{\mathrm{YM}}^2 \geq k M^2 G_{\mathrm{N}} = \frac{k}{8\pi} \frac{M^2}{M_{\mathrm{Pl}}^4} \,,$$

Present case: [Lee,Lerche,Regalado,TW to appear]

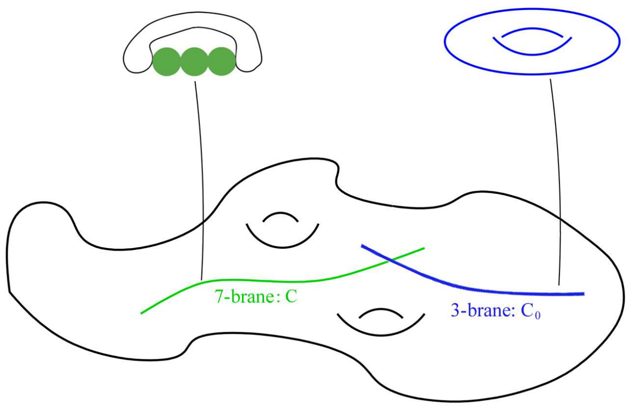
- Maximal charge per excitation level: $q_n^2 = \beta n$
- $m_n^2 = \alpha\, n\, T = 2\pi\alpha\, n\, \mathrm{vol}(C_0) = 2\pi\alpha\, n\, a(t)\, \frac{1}{2t}$
- $\frac{1}{g_{\mathrm{YM}}^2} = 4\pi \mathrm{vol} C = 4\pi\, a(t) t$

$$q_n^2\, g_{\mathrm{YM}}^2 = \frac{\beta}{\pi\alpha} A(t) \frac{m_n^2}{M_{\mathrm{Pl}}^4} \qquad A(t) = 1 + \frac{\gamma}{4t^2} \to 1$$

Note: All charges are present and satisfy the quantum completeness conjecture of [Polchinski'03] .

Non-critical strings

- 7-brane on C
- D3-brane on curve C_0 with $C_0 \cdot C \neq 0$
  $\Rightarrow$ non-critical string on curve C_0 with $C_0^2 = 0$, $C_0 \cdot \bar{K} = 2$



Twisted reduction of $N = 4$ SYM with varying gauge coupling along C_0

[Martucci'14][Haghighat,Murthy,Vafa,Vandoren'15][Lawrie,Schafer-Nameki,TW'16] talk by Mayer

- 2d $N = (0, 4)$ effective theory describes worldsheet theory
- At intersection $C_0 \cap C$:
  isolated 3-7 string modes **charged** under 7-brane gauge group

$\Rightarrow$ gauge group on C becomes global/flavour symmetry group of non-critical string

$\Rightarrow$ excitations of string will be charged under 7-brane group

The elliptic genus

A certain index of resulting BPS states is counted by elliptic genus:
Consider string on a torus T^2 with periodic (R) boundary conditions:

$$Z_{C_\beta}(\tau, \lambda_s, z_a) = \mathrm{Tr}_R (-1)^F q^{H_L} \bar{q}^{H_R} u^{2J_-} \prod_a (y_a)^{J_a}$$

$q = e^{2\pi i \tau}$: τ complex structure of T^2

$u = e^{2\pi i \lambda_s}$: fugacity w.r.t. $SU(2)_- \supset SO(4)_T$ (6d spin)

$y_a = e^{2\pi i z_a}$: fugacity w.r.t. flavour symmetry Cartan $U(1)_a$

Z_{C_β} is a Weyl invariant Jacobi form of weight $w = 0$
fugacity index m_u and m_{y_a} determined by the geometry

$$\begin{aligned} \varphi_{w,\mathbf{m}}\left(\frac{a\tau+b}{c\tau+d}, \frac{\zeta}{c\tau+d}\right) &= (c\tau+d)^w e^{2\pi i \frac{\mathbf{m}\, c}{c\tau+d} \frac{(\zeta,\zeta)}{2}} \varphi_{w,\mathbf{m}}(\tau,\zeta) \\ \varphi_{w,\mathbf{m}}(\tau, \zeta+\lambda\tau+\mu) &= e^{-2\pi i \mathbf{m}\left(\frac{(\zeta,\zeta)}{2}\tau + 2\frac{(\lambda,\zeta)}{2}\right)} \varphi_{w,\mathbf{m}}(\tau,\zeta) \end{aligned}$$

The elliptic genus

$$\begin{aligned} Z_{C_\beta}(\tau, \lambda_s, z_a) &= \mathrm{Tr}_R (-1)^F q^{H_L} \bar{q}^{H_R} u^{2J_-} \prod_a (y_a)^{J_a} \\ &= \sum c(n, r_u, r_a)\, q^n\, u^{r_u} \prod_a y_a^{r_a} \end{aligned}$$

$\Rightarrow$ $c(n, r_u, r_a)$ index of BPS states at excitation level n
and $U(1)_a$ flavour charge r_a

Z_{C_β} studied intensively for shrinkable curves $C_\beta \cdot C_\beta = -1, -2, \ldots, -12$

For extensive 6d literature - see review [Heckman,Rudelius'18]

[Klemm,Mayr,Vafa'96] [Klemm,Manschot,Wotschke'12] [Alim,Scheidegger'12]
[Benini,Eager,Hori,Tachikawa'13] [Haghighat,Lockhart,Vafa'14] [Cai,Huang,Sun'14]
[Haghighat,Klemm,Lockhart,Vafa'14] [Huang,Katz,Klemm'15] [Haghighat,Murthy,Vafa,Vandroen'15]
[Kim,Kim,Lee'15] [DelZotto,Lockhart'16/'18] [Gu,Huang,Kashani-Poor,Klemm'17]
[Kim,Lee,Park'18],...

The elliptic genus

Novelty: $C_0 \cdot C_0 = 0$ and with general $U(1)_A$ flavour symmetries

- Ansatz from analysis of pole structure: [Klemm et al. -'12-'18], [Vafa et al.]

$$Z_{C_0}(\tau, \lambda_s, \mathbf{z}) = \left(\frac{1}{\eta^2(\tau)}\right)^{6C_\beta \cdot \bar{K}} \frac{\Phi_{W,L,\mathbf{m}}(\tau, \lambda_s, \mathbf{z})}{\varphi_{-2,1}(\tau, \lambda_s)} .$$

$$W = 6C_0 \cdot \bar{K} - 2 = 10, \qquad L = \tfrac{1}{2}\, C_0 \cdot (C_0 + K) + 1 = g(C_0) = 0$$

- Fix fugacity index w.r.t. $U(1)_A$ [Lee,Lerche,Regalado,TW to appear]

$$m_A = \frac{1}{2} C_0 \cdot h_A \qquad h_A = -\pi_*(\sigma(S_A) \cdot \sigma(S_A)) \quad \text{(height pairing)}$$

- General ansatz for $\Phi_{W,L,\mathbf{m}}(\tau, \lambda_s, \mathbf{z})$ in terms of products of $E_4(\tau)$, $E_6(\tau)$, $\varphi_{-2,1}(\tau, z)$, $\varphi_{0,1}(\tau, z)$

- General properties of Eisenstein-Jacobi forms imply for maximal $U(1)_A$ charge per excitation level:

$$(\mathbf{q_A})^{\mathbf{2}}_{\mathbf{n}} = \beta\, \mathbf{n} \quad \text{for} \quad \beta \geq m_A$$

The elliptic genus

Compare with known BPS numbers via mirror symmetry computation of BPS invariants using duality with M-theory to fix ansatz completely

6d F-theory on S^1	5d M-theory
string wrapped on S^1	BPS particle in 5d
wrapping number m and	M2 brane on
KK momentum k	$mC_\beta + k\mathbb{E}_\tau$

Relation: $m\,k = n$ [Klemm,Mayr,Vafa'96]

$\Rightarrow$ Counting of excitation level n in 6d equivalent to counting M2-brane states in 5d via Gopakumar-Vafa BPS invariants

Encoded in free energy of topological string on same CY Y_3:

$\mathcal{F}(\lambda_s, \tau, \mathbf{t}, \mathbf{z}) = \sum_{g\geq 0} \mathcal{F}^{(g)}(\tau, \mathbf{t}, \mathbf{z})\lambda_s^{2g-2}$

$$Z_{\text{top}} = \exp\left(\mathcal{F}(\lambda_s, \tau, \mathbf{t}, \mathbf{z})\right) = Z_0(\tau, \lambda_s)\left(1 + \sum_{C_\beta} Z_{C_\beta}(\tau, \lambda_s, \mathbf{z})\, e^{2\pi i t_\beta + \frac{1}{2}(C_\beta\cdot\bar{K})\tau}\right)$$

[Haghighat et al.'13] [Haghighat,Klemm,Lockhart,Vafa'14],...

Non-critical strings

Explicit computation of elliptic genus and hence BPS spectrum for general $U(1)$ models of Morrison-Park type on Hirzebruch and dP_2 surfaces

Example: $B_2 = \mathbb{F}_{a=1}$

topological types of MP model specified by integer

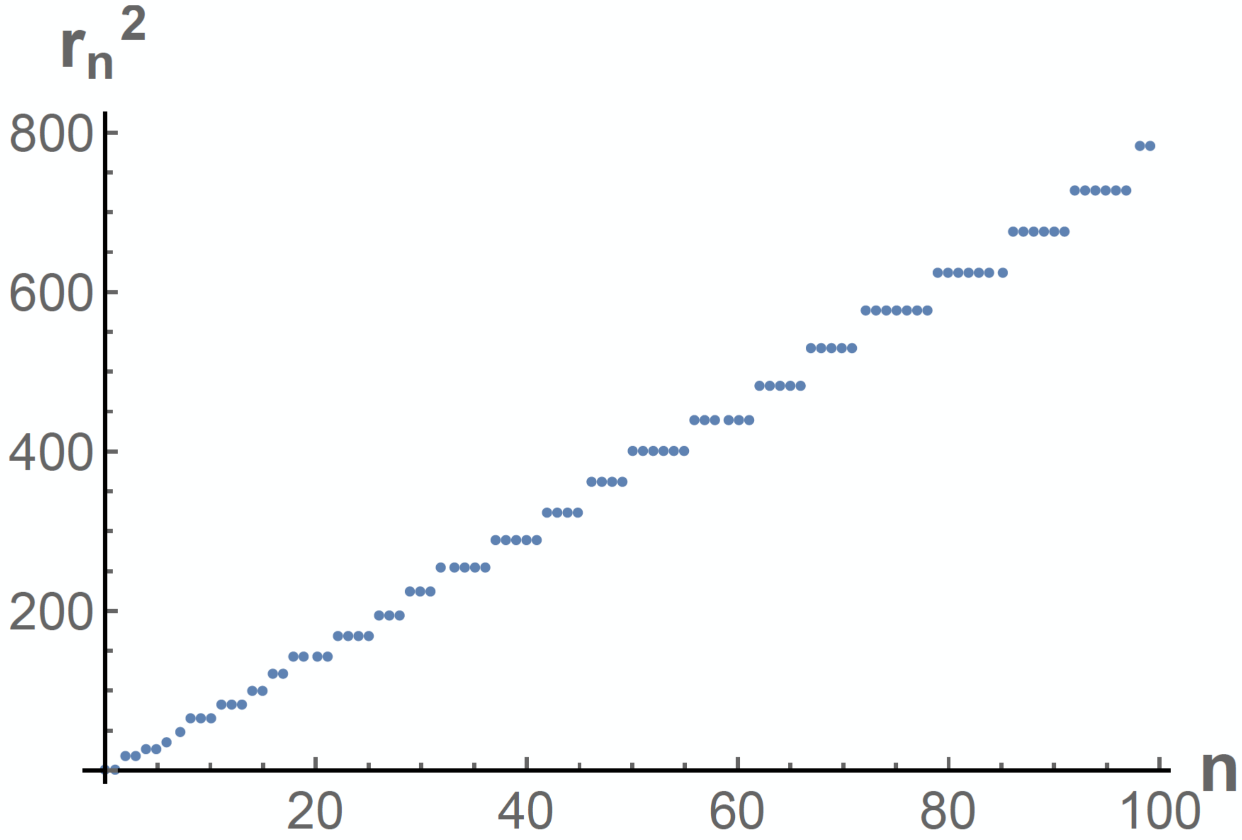
$0 \leq x \leq 4,\ 0 \leq y \leq 4 + 2a = 6$

$(x, y) = (4, 6) : U(1)_A$ fugacity $m_A = 2$ $\qquad$ $Z_{C_0} = \dfrac{\mathcal{F}^{(0)}_{C_0}(\tau, \zeta)}{\varphi_{-2,1}(\tau, \lambda_s)}$

with

$$\begin{aligned}
\mathcal{F}^{(0)}_{C_0}(\tau, \zeta) &= \frac{q}{\eta^{24}}\left(-\frac{1}{72}E_4^2 E_6 \varphi_{-2,1}^2 + \frac{1}{54}E_4^3 \varphi_{-2,1}\varphi_{0,1} + \frac{1}{108}E_6^2 \varphi_{-2,1}\varphi_{0,1} - \frac{1}{72}E_4 E_6 \varphi_{0,1}^2\right) \\
&= -2 + \left(\frac{96}{\zeta} + 288 + 96\zeta\right) q \\
&\quad + \left(-\frac{2}{\zeta^4} + \frac{96}{\zeta^3} + \frac{10192}{\zeta^2} + \frac{69280}{\zeta} + 123756 + 69280\zeta + 10192\zeta^2 + 96\zeta^3 - 2\zeta^4\right) \\
&\quad + \mathcal{O}(q^3)\,.
\end{aligned}$$

Mass-Charge relation



r_n: maximal charge per excitation level: $r_n^2 = \beta\, n$

Proven lower bound: $\beta \geq m_A = 2$ (fugacity index)

In fact: closer to $\beta \sim 4m_A$ in all studied examples

Summary

Systematic study of Quantum Gravity constraints for open string U(1)s

1. Gauge group become global at infinite distance.
2. $g_{\mathrm{YM}} \to 0$ implies tensionless non-critical strings.
3. Infinite tower of charged BPS states becomes massless, in agreement with
   - Swampland Distance Conjecture
   - Sublattice Weak Gravity Conjecture
   - Completeness Hypothesis
4. Explicit and quantitative analysis of elliptic genus in presence of general U(1)s

Weak Jacobi forms imply the weak gravity conjecture.

String Pheno 2019

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