Supersymmetry enhancement via T-branes

Federico Carta.

IFT-UAM/CSIC

4th of July 2018

Based on...

- F.C, S.Giacomelli, R.Savelli. [To appear]

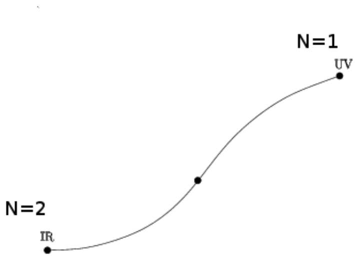
Related work:

- K.Maruyoshi, J.Song. 2016
- P.Agarwal, K.Maruyoshi, J.Song. 2016
- P.Agarwal, A.Sciarappa, J.Song. 2017
- S.Benvenuti, S.Giacomelli. 2017
- J.Heckman, Y.Tachikawa, C.Vafa, B.Wecht. 2010

Supersymmetry enhancement.

- A UV QFT follows an RG flow to a IR QFT with more explicit supersymmetry.
- Intrinsically interesting phenomenon in QFT.
- Can use the lagrangian to compute RG-protected quantities.

A schematic picture.



Different approaches.

1. MS flows. $4d\ \mathcal{N}=1 \to \mathcal{N}=2$
2. BG flows. $3d\ \mathcal{N}=2 \to \mathcal{N}=4$
3. Ungauging quiver tails. (Gadde-Rastelli-Razamat).
4. $3d\ \mathcal{N}=1 \to \mathcal{N}=2$. (Komargodski).
5. $3d$ CS-Maxwell. (Yamazaki).

Maruyoshi-Song flows.

- Start in UV with a $4d\ \mathcal{N}=2$ SCFT $\mathcal{T}$ with flavor symmetry F.
- Add by hand a $\mathcal{N}=1$ chiral M.
- M is gauge singlet and in the adjoint of the F.
- Turn on a superpotential term $W_{def} = TrM\mu$ where μ is the moment map operator. ($\mu \simeq q\tilde{q}$ if $\mathcal{T}$ is lagrangian).
- Give a *special kind of vev* to M. This triggers a RG flow.
- Depending on the choice of $\mathcal{T}$ and $\langle M \rangle$ sometimes we find that $\mathcal{T}[\langle M \rangle]$ flows in the IR a *new* $\mathcal{N}=2$ SCFT: call it $\mathcal{T}[\langle M \rangle]_{IR}$.
- “New" means $\mathcal{T}[\langle M \rangle]_{IR} \neq \mathcal{T}$

Nilpotent orbit vev.

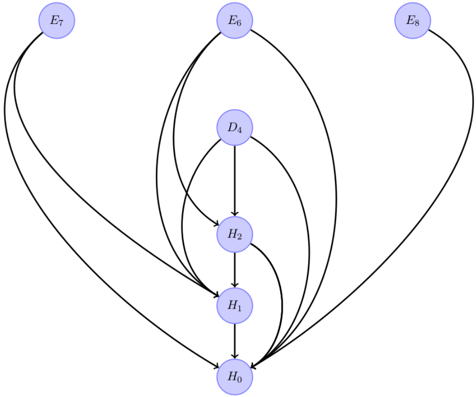
- M is a matrix in the adjoint of F.
- We choose $\langle M \rangle$ to be along a nilpotent orbit of $\mathfrak{f}_{\mathbb{C}}$
- Nilpotent orbit is defined as ghg^{-1} for $g \in F$ and $h \in \mathfrak{f}_{\mathcal{C}}$ nilpotent.
- Nilpotent orbits are *completely classified* by mathematicians.
- Classification by partitions of numbers for classical algebras.
- Classification by Bala-Carter labels for exceptional algebras.
- For a fixed $\mathcal{T}$, finite number of possible MS deformations.

Example:

$$\langle M \rangle = \begin{pmatrix} 0 & 1 \\ \text{x} & 0 \end{pmatrix} \tag{1}$$

is nilpotent vev in the maximal nilpotent orbit of $\mathfrak{sl}_2$.

Enhancing flows connecting rank 1 theories



A figure summarizing all the existent MS connecting rank one theories.
Multiple flows happen for different nilpotent orbit deformations.

Evidence for the enhancement.

- a-maximization.
  1. $\mathcal{N}=2\ U(1)_r \times SU(2)_R$ R-symmetry is broken to $U(1)^{UV}_{\mathcal{N}=1} \times U(1)_{\mathcal{F}}$
  2. $U(1)_{\mathcal{N}=1} = U(1)^{UV}_{\mathcal{N}=1} + \epsilon\, U(1)_{\mathcal{F}}$ mixing of R-symmetry and flavor.
  3. $a(\epsilon)$ and $c(\epsilon)$ central charges depend on ϵ.
  4. $a(\epsilon)$ is extremal at the IR fixed point. $\implies$ find ϵ^*
  5. If a and c are rational, good evidence for enhancement.
  6. Often find a and c of previously known AD theories.

- The full Superconformal Index.
  1. Compute the UV $\mathcal{N}=1$ Index by SUSY localization.
  2. Upon finding the extremizing ϵ^*, shift the index fugacities
  3. Find the $\mathcal{N}=2$ index
  4. Check limits of the index against previously computed cases. (Shur, McDonald, Coulomb, HL, HS of 3d mirror CB)
  5. Huge evidence for enhancement.

The easiest example. H_1 AD theory.

1. Let $\mathcal{T} = AD_{N_f=2}[SU(2)] \simeq H_1 \simeq (A_1, A_3)$. The flavor symmetry is $SU(2)$. Add M and give vev along the maximal nilpotent orbit of $\mathfrak{sl}_2$. Namely
$$\langle M \rangle = \begin{pmatrix} 0 & 1 \\ x & 0 \end{pmatrix} \quad (2)$$
2. The trial central charge is
$$a(\epsilon) = \frac{3}{32}\left(3\mathrm{tr}\mathcal{R}^3_{\mathcal{N}=1} - \mathrm{tr}\mathcal{R}_{\mathcal{N}=1}\right) \quad (3)$$
3. By extremizing $\frac{da(\epsilon)}{d\epsilon} = 0 \implies \epsilon^* = \frac{9}{15}$
$$a(\epsilon^*) = \frac{43}{120} \quad c(\epsilon^*) = \frac{11}{30} \implies H_0 \text{ Theory!} \quad (4)$$

Open questions.

1. Why only some specific nilpotent vevs for M give Susy enhancement?
2. Can one find more examples of SCTs showing these features?
3. Can one understand in a more direct way why the enhancement happens, from a physical point of view?
4. Is there a way to see this phenomenon from a String Theory engineering of the QFT?

The BIG scan

1. MS flows starting form lagrangian SCFTs are constrained.
   1. Superconformal quivers with bifund hypers have always ADE shape.
   2. The number of nilpotent orbit is finite, and not too huge.
   3. Even allowing non-bifund hypers, small list of exception (Tachikawa-Bhardwaj)
2. Write a program to run the a-max test for a very large calss of $(\mathcal{T}, \langle M \rangle)$.
3. Find one new case in rank 1: E_7 MN with orbit with BC label E_6 flows to H_1.
4. $\sim$ 1000 cases checked among superconformal quivers of D and E shape, or also non-bifund matter.

The BIG scan

1. MS flows starting form lagrangian SCFTs are constrained.
   1. Superconformal quivers with bifund hypers have always ADE shape.
   2. The number of nilpotent orbit is finite, and not too huge.
   3. Even allowing non-bifund hypers, small list of exception (Tachikawa-Bhardwaj)
2. Write a program to run the a-max test for a very large calss of $(\mathcal{T}, \langle M \rangle)$.
3. Find one new case in rank 1: E_7 MN with orbit with BC label E_6 flows to H_1.
4. $\sim$ 1000 cases checked among superconformal quivers of D and E shape, or also non-bifund matter. ZERO enhancements.

The BIG scan

1. MS flows starting form lagrangian SCFTs are constrained.
   1. Superconformal quivers with bifund hypers have always ADE shape.
   2. The number of nilpotent orbit is finite, and not too huge.
   3. Even allowing non-bifund hypers, small list of exception (Tachikawa-Bhardwaj)
2. Write a program to run the a-max test for a very large calss of $(\mathcal{T}, \langle M \rangle)$.
3. Find one new case in rank 1: E_7 MN with orbit with BC label E_6 flows to H_1.
4. $\sim$ 1000 cases checked among superconformal quivers of D and E shape, or also non-bifund matter. ZERO enhancements. We conjecture that this class never give S.E.

The geometrical picture. Part 1

- Consider a F-theory setup, with a D3 probing the elliptic fibration.
- Write the Weierstrass model for the elliptic fibration. The theory on the D3 will be the theory $\mathcal{T}$ in the UV. In particular the Weiestrass model fixes the flavor group F.
- Elliptic fiber $\simeq$ Seiberg-Witten curve of the QFT.
- The chiral M is geometrized by a T-brane deformation of the 7-brane stack.
- There is a one-to-one correspondence between nilpotent orbits and versal deformation of the Weierstrass model.

The geometrical picture. Part 2

Singularity	Curve	Flavor group
II^*	$u^2 = v^3 + v(M_2 z^3 + M_8 z^2 + M_{14} z + M_{20}) + (z^5 + M_{12} z^3 + M_{18} z^2 + M_{24} z + M_{30})$	E_8
III^*	$u^2 = v^3 + v(z^3 + M_8 z + M_{12}) + (M_2 z^4 + M_6 z^3 + M_{10} z^2 + M_{14} z + M_{18})$	E_7
IV^*	$u^2 = v^3 + v(M_2 z^2 + M_5 z + M_8) + (z^4 + M_6 z^2 + M_9 z + M_{12})$	E_6
I_0^*	$u^2 = v^3 + v(\tau z^2 + M_2 z + M_4) + (z^3 + \tilde{M}_4 z + M_6)$	$SO(8)$
IV	$u^2 = v^3 + v(M_{1/2} z + M_2) + (z^2 + M_3)$	$SU(3)$
III	$u^2 = v^3 + vz + (M_{2/3} v + M_2)$	$SU(2)$
II	$u^2 = v^3 + v M_{4/5} + z$	no

Table: Maximally deformed Weierstrass models

- The parameters M_i are the versal deformations of the model
- They correspond to casimir operators of the Higgs field in the F-theory picture, which we take to be $\langle M \rangle$
- $\langle M \rangle$ is in turn fixed by the chosen nilpotent orbit.

The geometrical picture. Part 3

- When the D3 probes the deformed Weiestrass model, the theory is $\mathcal{N} = 1$, due to the T-brane presence.
- RG flow is a local zoom at the singularity. In the IR the probe does not have enough energy to resolve global aspects of the singularity.
- In the IR, some terms in the Weiestrass become subleading. We throw them away and recover the Weierstrass for $\mathcal{T}^{IR}$
- It is often esay to see which terms are subleading, just by rescaling $\langle M \rangle \to m \langle M \rangle$ and looking how m enters in the casimirs M_i.

Conclusions

- We perform an a-max scan on $(\mathcal{T}, \langle M \rangle)$ larger than the one presented in the literature. Among rank 1 theories we find a new enhancement case, for the MN E_7 theory flowing to H_1.

Conclusions

- We perform an a-max scan on $(\mathcal{T}, \langle M \rangle)$ larger than the one presented in the literature. Among rank 1 theories we find a new enhancement case, for the MN E_7 theory flowing to H_1.
- For $\mathcal{T}$ given by a superconformal quiver of rank > 1 we find only the examples already discovered in the literature (Flows with $\mathcal{T}$ =linear quiver ending to (A_n, A_m) and (A_n, D_m)). It is an open question why superconformal quivers of type D and E generically don't give enhancement.

Conclusions

- We perform an a-max scan on $(\mathcal{T}, \langle M \rangle)$ larger than the one presented in the literature. Among rank 1 theories we find a new enhancement case, for the MN E_7 theory flowing to H_1.
- For $\mathcal{T}$ given by a superconformal quiver of rank > 1 we find only the examples already discovered in the literature (Flows with $\mathcal{T}$ =linear quiver ending to (A_n, A_m) and (A_n, D_m)). It is an open question why superconformal quivers of type D and E generically don't give enhancement.
- We have a geometrical understanding of S.E. for the rank 1 case. Extra chiral $M =$ a T-brane deformation of a 7-brane stack. RG flow = local zoom in the singularity locus of the elliptic fibration. $MS = T$.

Conclusions

- We perform an a-max scan on $(\mathcal{T}, \langle M \rangle)$ larger than the one presented in the literature. Among rank 1 theories we find a new enhancement case, for the MN E_7 theory flowing to H_1.
- For $\mathcal{T}$ given by a superconformal quiver of rank > 1 we find only the examples already discovered in the literature (Flows with $\mathcal{T}$ =linear quiver ending to (A_n, A_m) and (A_n, D_m)). It is an open question why superconformal quivers of type D and E generically don't give enhancement.
- We have a geometrical understanding of S.E. for the rank 1 case. Extra chiral $M =$ a T-brane deformation of a 7-brane stack. RG flow = local zoom in the singularity locus of the elliptic fibration. $MS = T$.
- Geometrizing rank > 1 is Work in Progress.

Conclusions

- We perform an a-max scan on $(\mathcal{T}, \langle M \rangle)$ larger than the one presented in the literature. Among rank 1 theories we find a new enhancement case, for the MN E_7 theory flowing to H_1.
- For $\mathcal{T}$ given by a superconformal quiver of rank > 1 we find only the examples already discovered in the literature (Flows with $\mathcal{T}$ =linear quiver ending to (A_n, A_m) and (A_n, D_m)). It is an open question why superconformal quivers of type D and E generically don't give enhancement.
- We have a geometrical understanding of S.E. for the rank 1 case. Extra chiral $M =$ a T-brane deformation of a 7-brane stack. RG flow = local zoom in the singularity locus of the elliptic fibration. $MS = T$.
- Geometrizing rank > 1 is Work in Progress.

Thank you for your attention.