Circle compactification, Anomalies and Field Distances

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Utrecht University

1710.07626 with Diego Regalado and Thomas Grimm

and work in progress with Irene Valenzuela

Introduction

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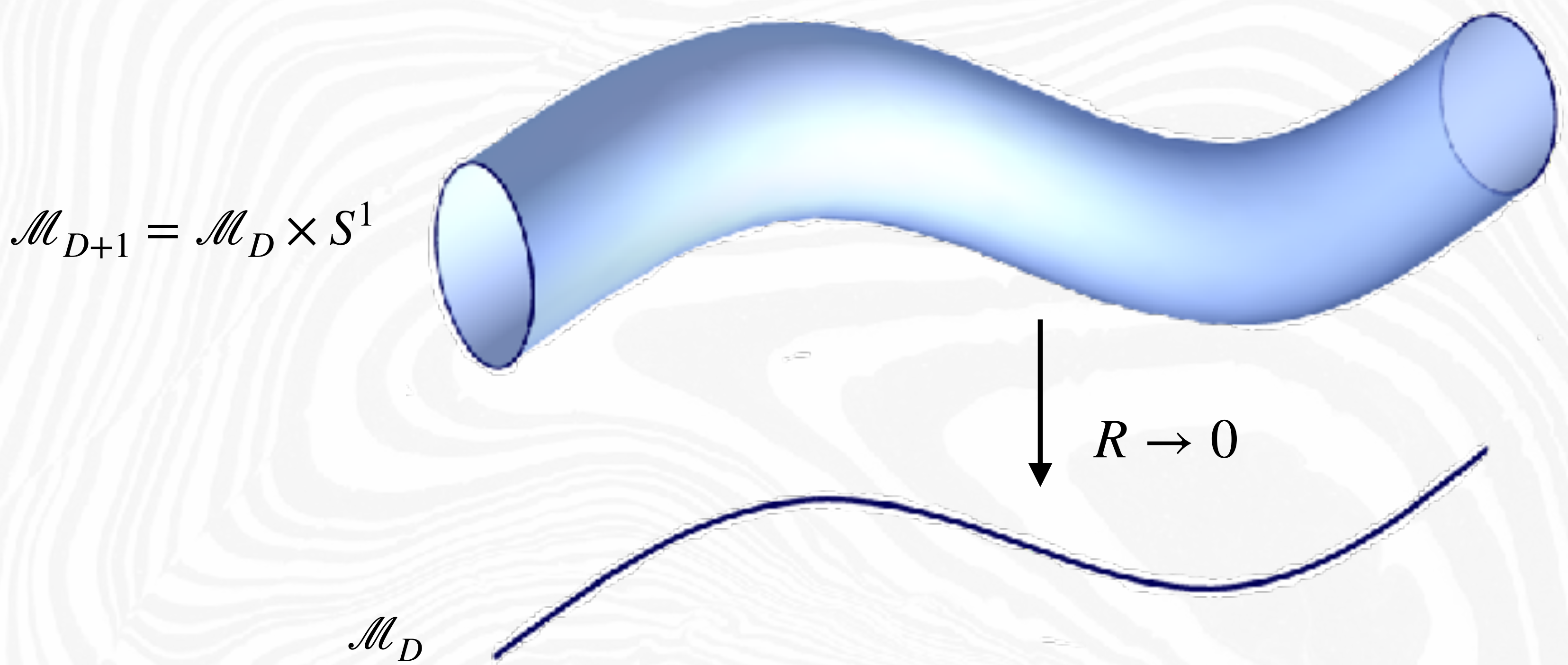

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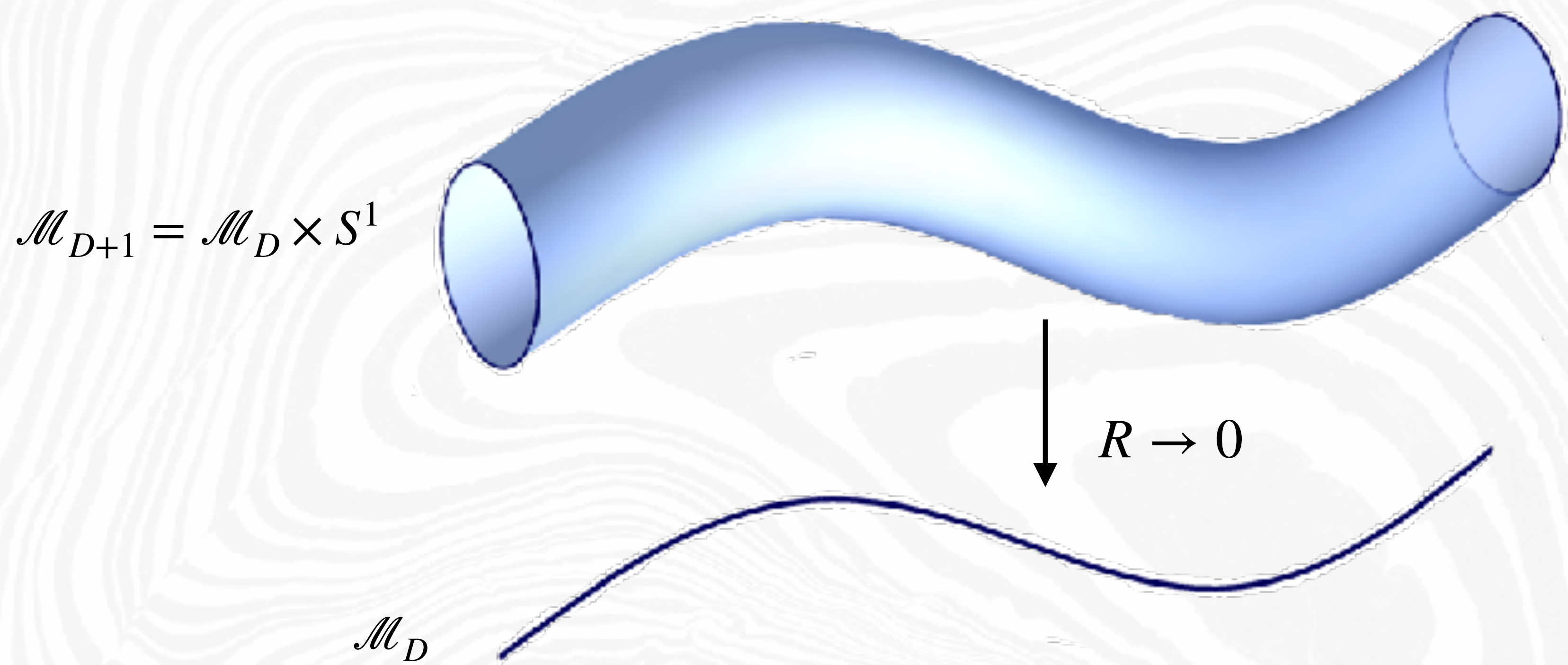

Décomposition in Fourier modes $\hat{\phi} = \sum_{n \in \mathbb{Z}} \phi_n e^{iny}$ $y \sim y + 2\pi$

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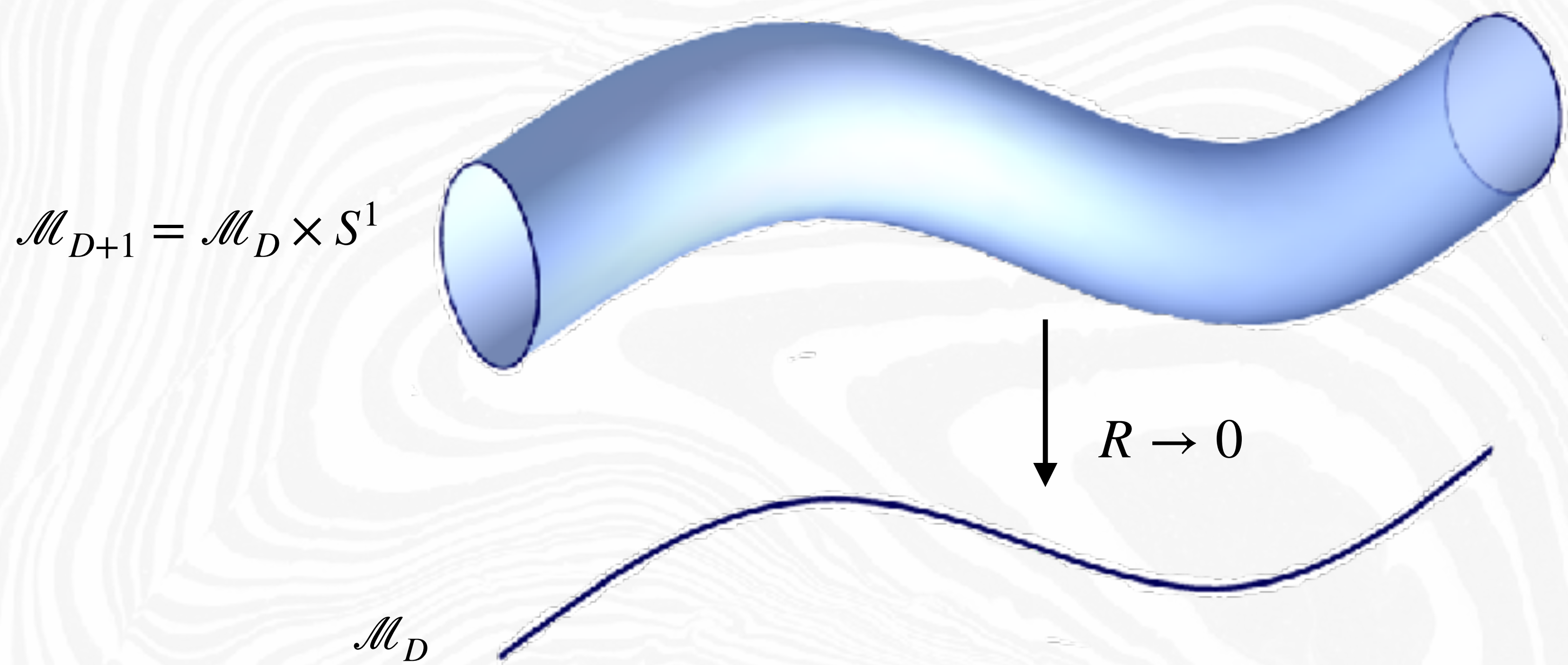

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mass $\qquad m_n = \dfrac{n}{R}$

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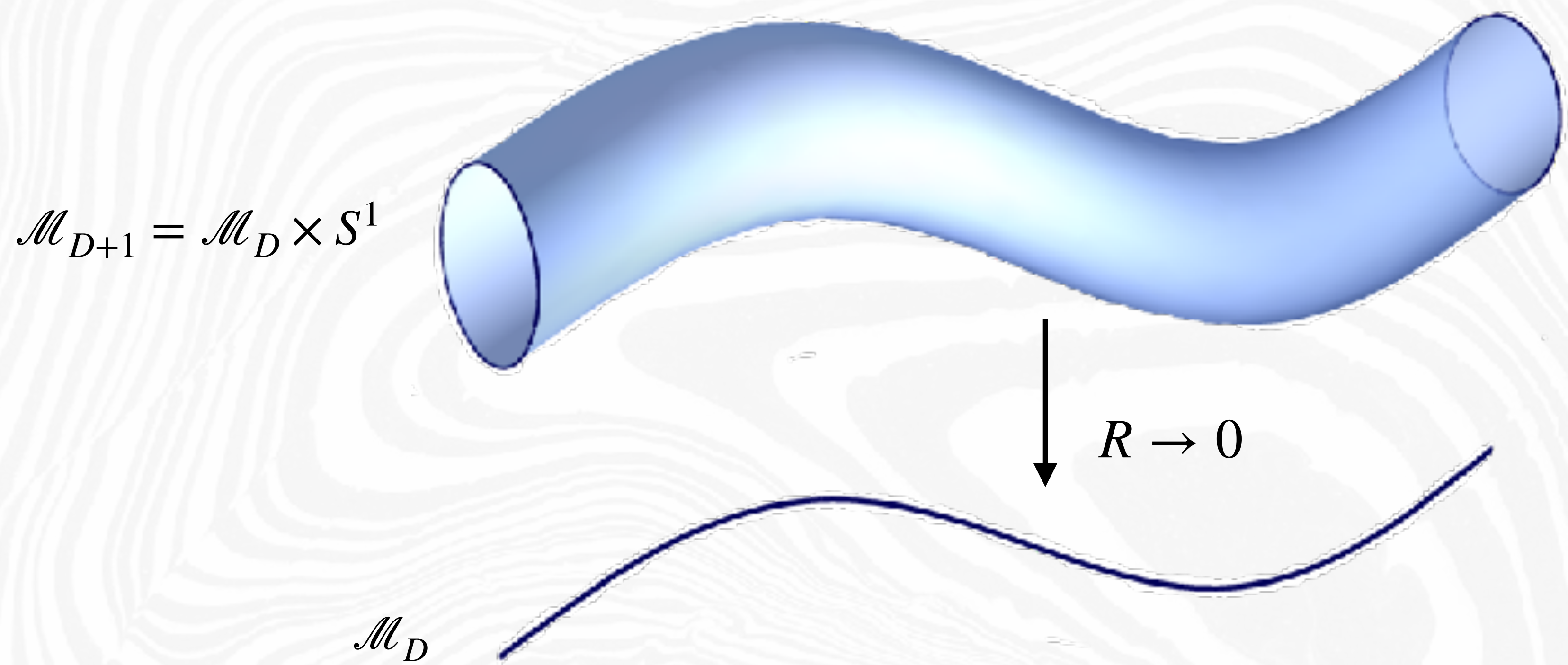

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mass $m_n = \frac{n}{R}$

- massive modes run in **loops**
- may be relevant (e.g. anomalies)

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- Play a central role in **effective actions of F-theory**

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[Grimm'10]

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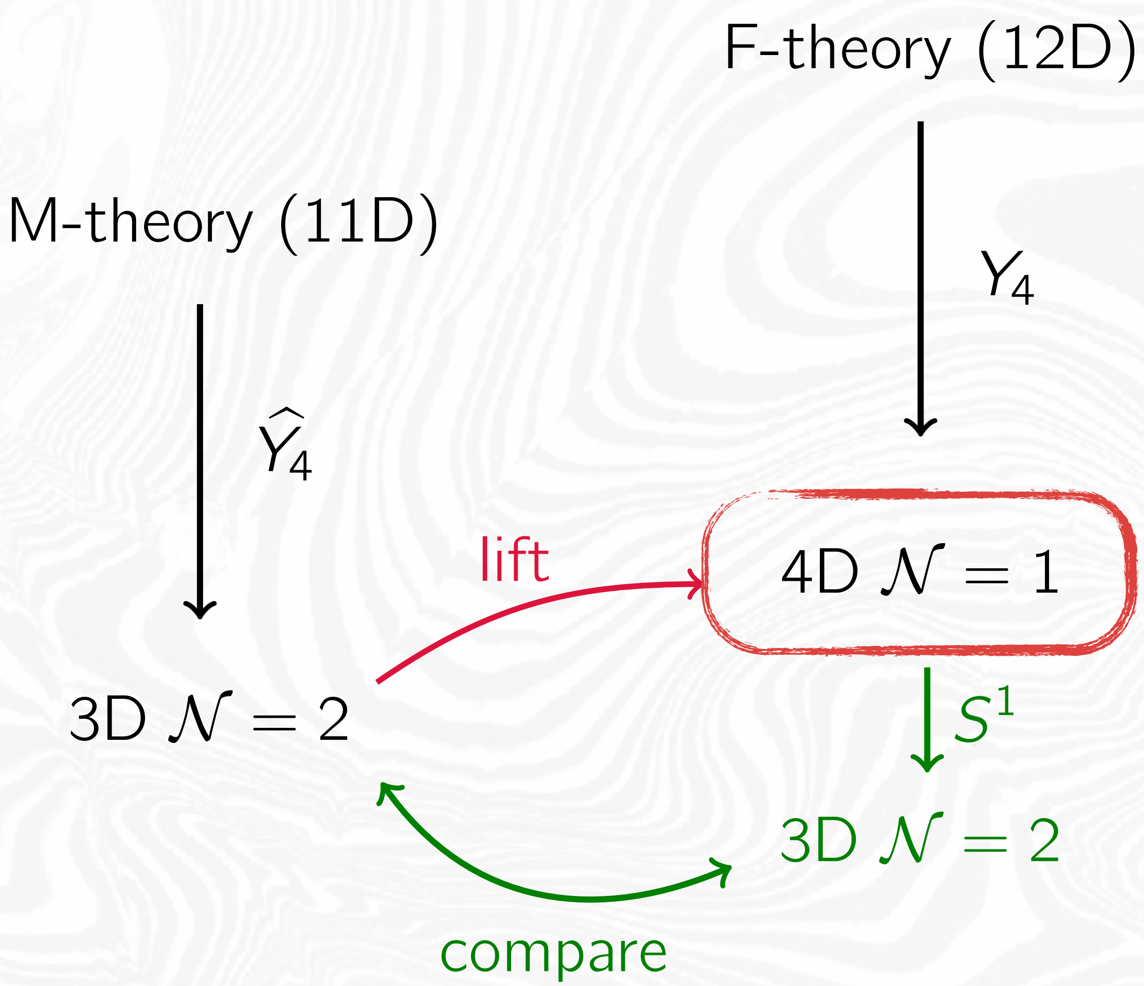

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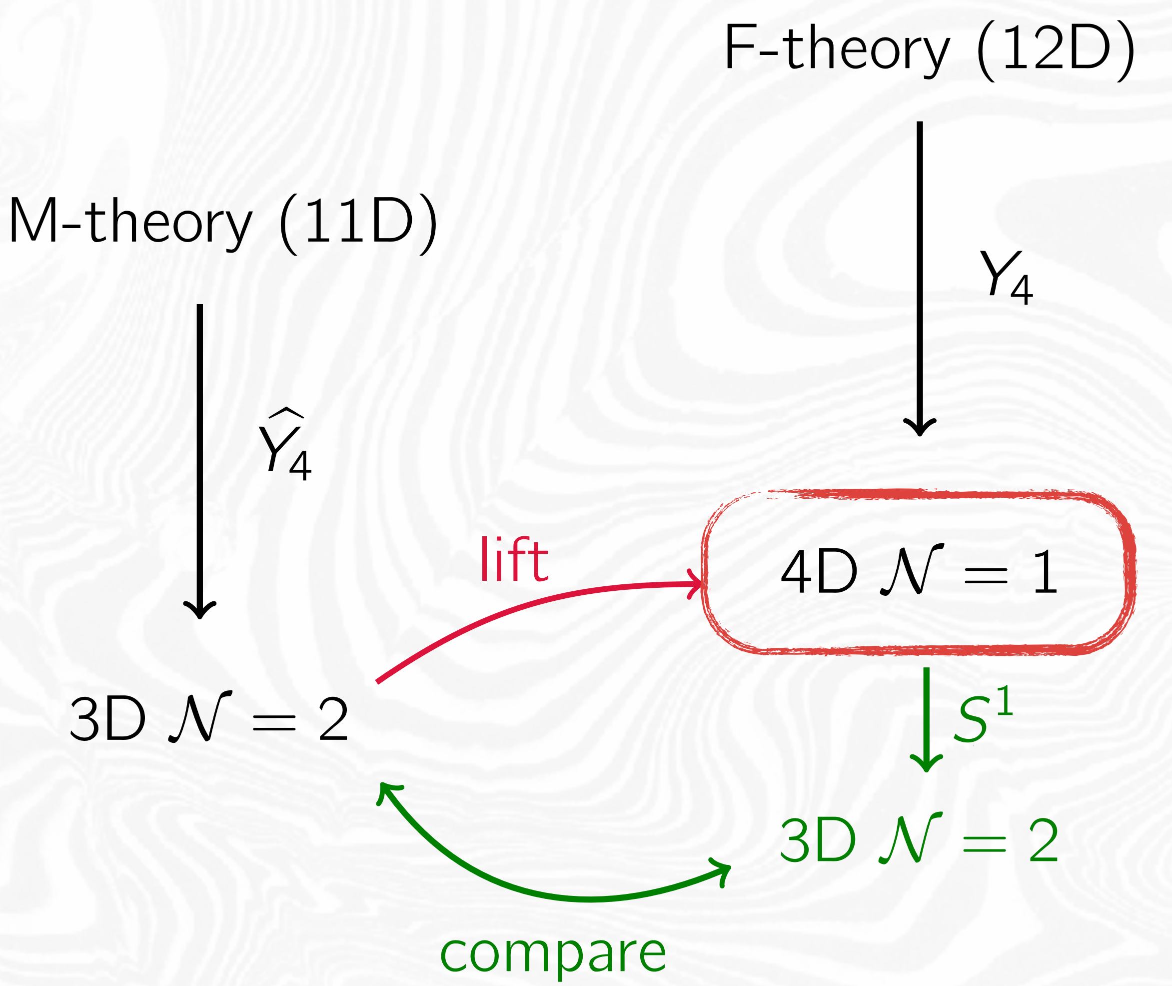

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- **Swampland Distance Conjecture** for KK reduction

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Anomalous?

Chiral Anomalies on a Circle

4D gauge theory with chiral fermions $\quad \hat{\bar\psi}\,\gamma^\mu\,(i\partial_\mu + q\hat{A}_\mu)\,P_L\,\hat{\psi}$

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$$\hat{A} \to \hat{A} + \mathrm{d}\hat{\lambda}$$

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Expect 3D theory **not** to be invariant under $\zeta \to \zeta + 1$

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One loop Chern-Simons $\Theta\, A \wedge F$

One mode $\Theta = \dfrac{1}{2}\,\mathrm{sign}\, m_n = \dfrac{1}{2}\mathrm{sign}\,(n + q\,\zeta)$

KK mass

CB mass

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where $$a_k = k^2 - n^2 - M^2$$
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Result $\quad \Theta = \frac{1}{2} + \lfloor \zeta \rfloor - \frac{2}{3} \zeta$

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shifts as expected!

[PC,Grimm,Regalado'10]

4D hard cutoff

Problems:

- Cumbersome
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3D theory is gauge invariant at one loop	$\Longleftrightarrow$	4D chiral anomalies are cancelled

M-theory reduction

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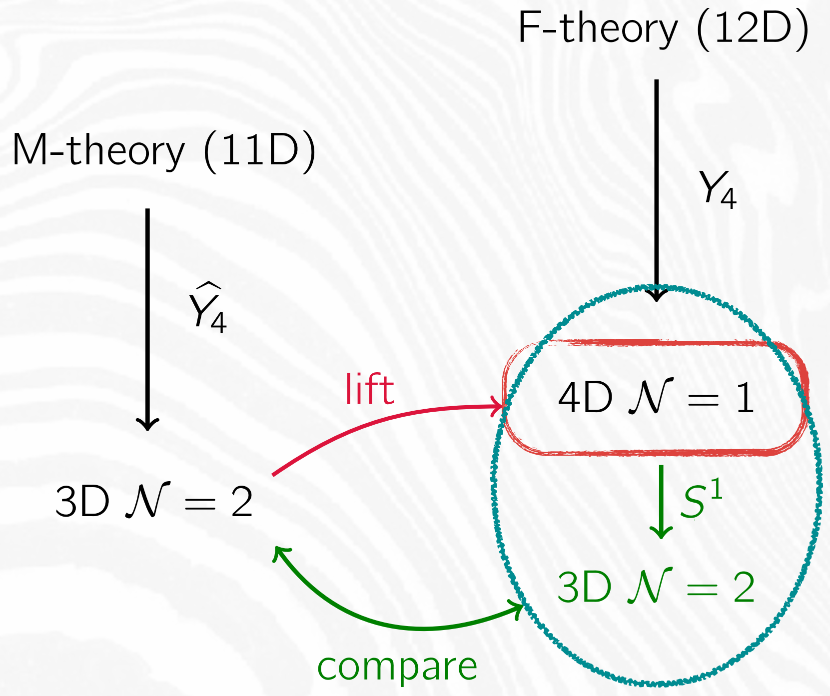

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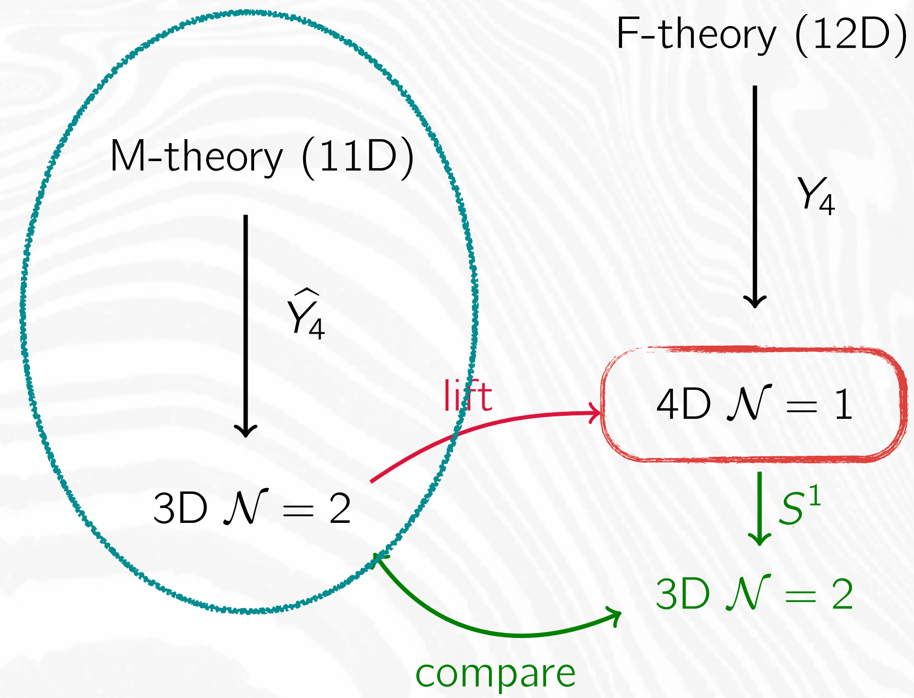

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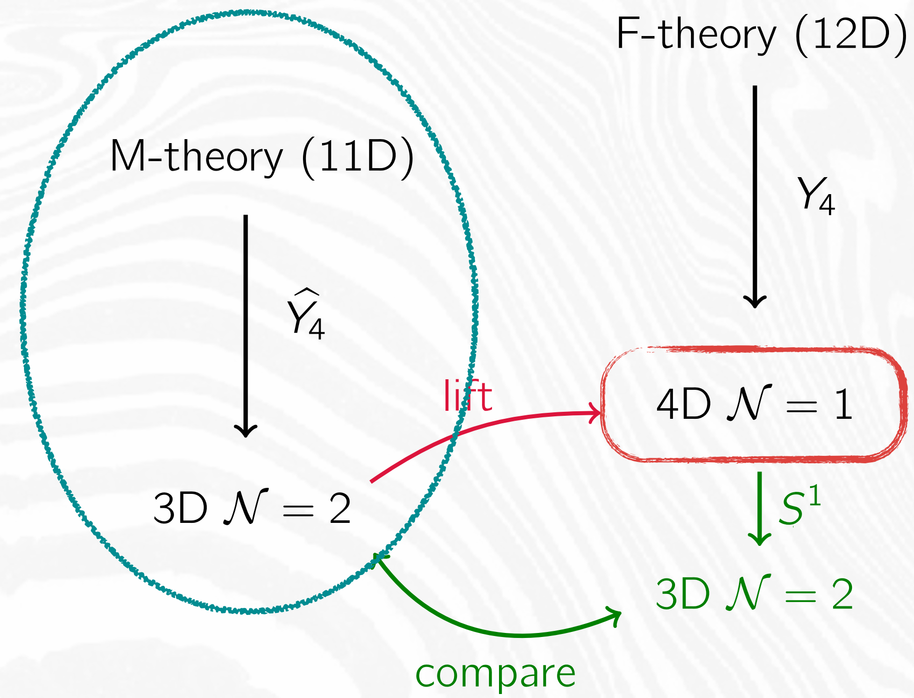

Classical reduction of M-theory with G_4 -flux:

- gauge invariant
- non zero Chern-Simons Θ

[Haack,Louis'01]

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$$C = A\wedge\omega^{1,1} \quad \underbrace{\int_{CY_4}\omega\wedge\omega\wedge G_4}_{\Theta}\int_{\mathcal{M}_3} A\wedge F$$

incorporates one-loop effects!

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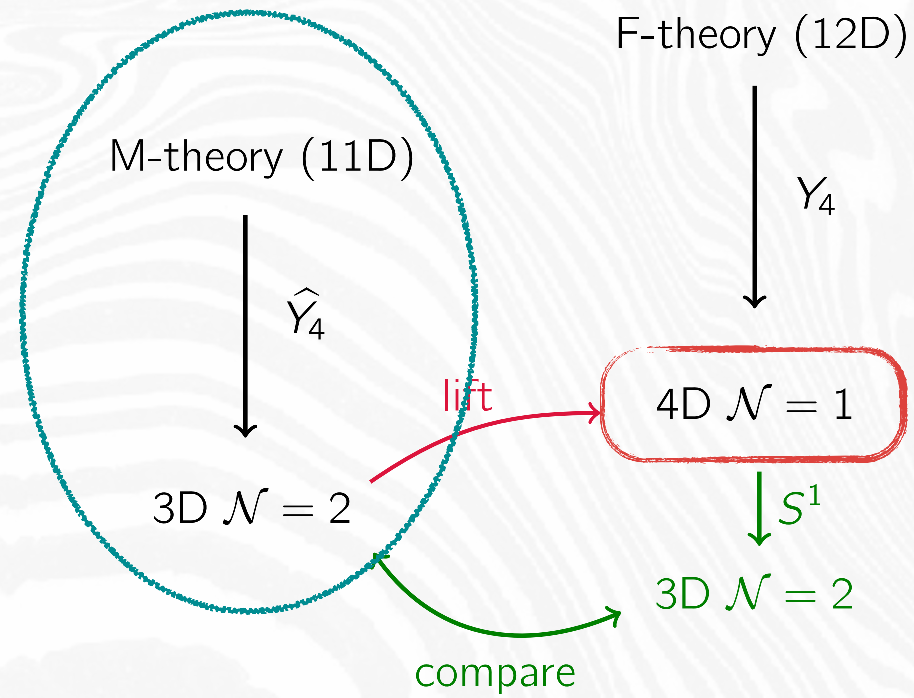

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[PC,Grimm,Regalado'10]

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3D effective action from M-theory:
- gauge invariant
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$\Downarrow$

4D effective actions of F-theory have no chiral anomaly

[PC,Grimm,Regalado'10]

Assumptions:
- Geometry can be smoothed
- Chirality induced by 'standard' G_4 -flux

Swampland Distance Conjecture

Infinite Distance in moduli space	$\xrightarrow{\text{SDC}}$	Infinitely many light states

[Ooguri, Vafa '06]

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In particular, **Infinite distance singularities**

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metric $\sim g_{ij}\, \mathrm{d}\phi^i \wedge \star\, \mathrm{d}\phi^j$

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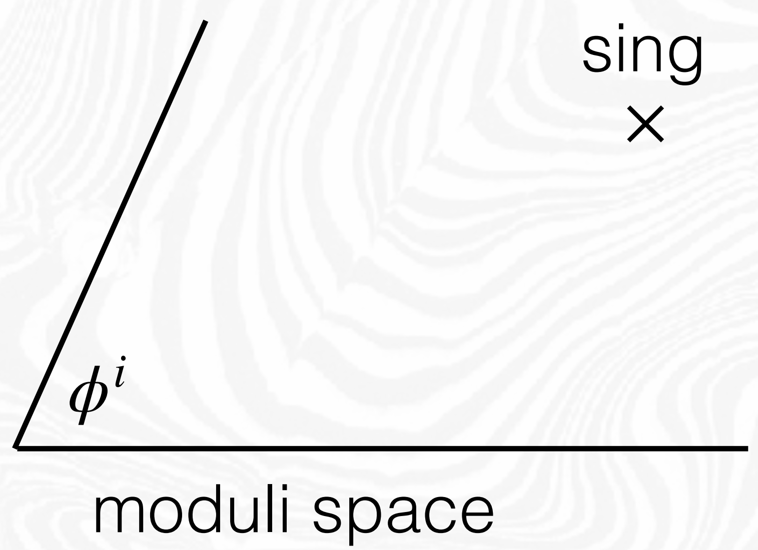

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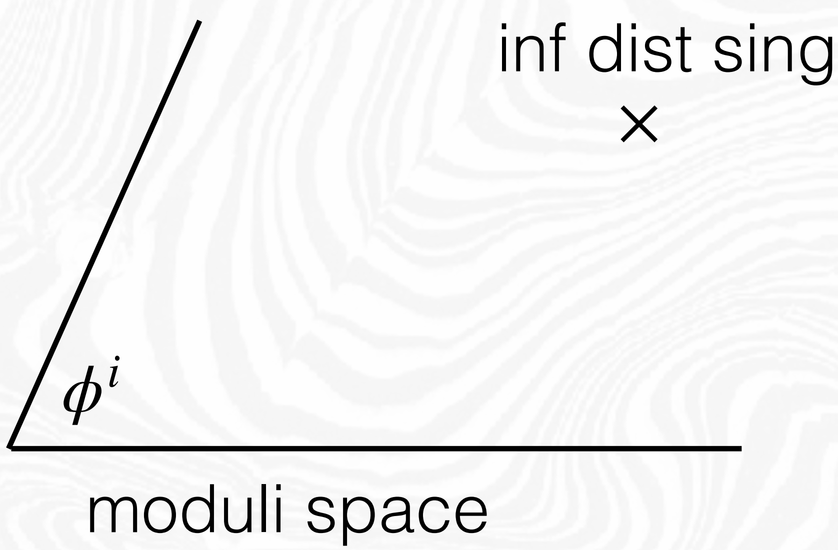

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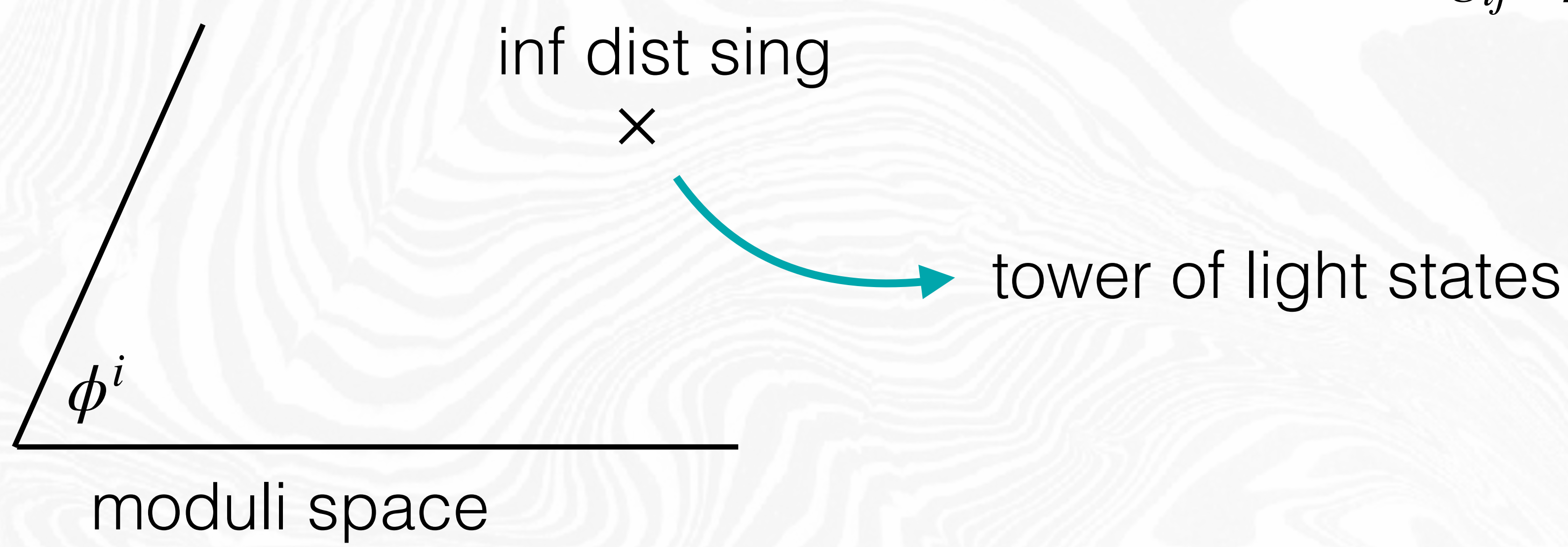

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Strong evidence for this conjecture in the complex structure moduli space of IIB on a CY

[Grimm, Palti, Valenzuela '18] [Irene's talk]

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Circle compactification: tower of KK modes $m_n = \dfrac{n}{R}$

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moduli space:

$R = 0$ $R = \infty$

metric $\sim \frac{1}{R^2}\,\mathrm{d}R \wedge \star\,\mathrm{d}R$

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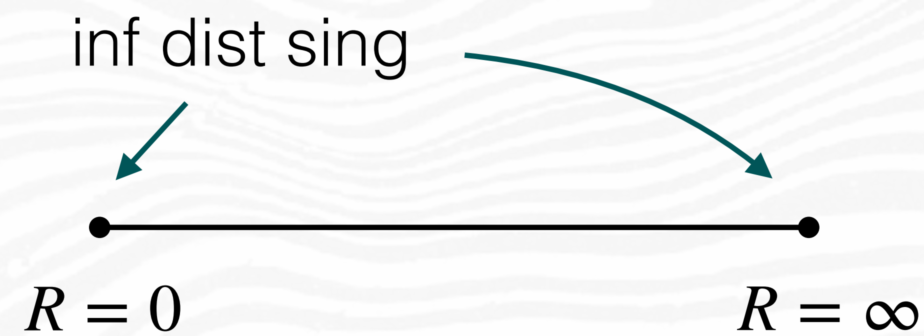

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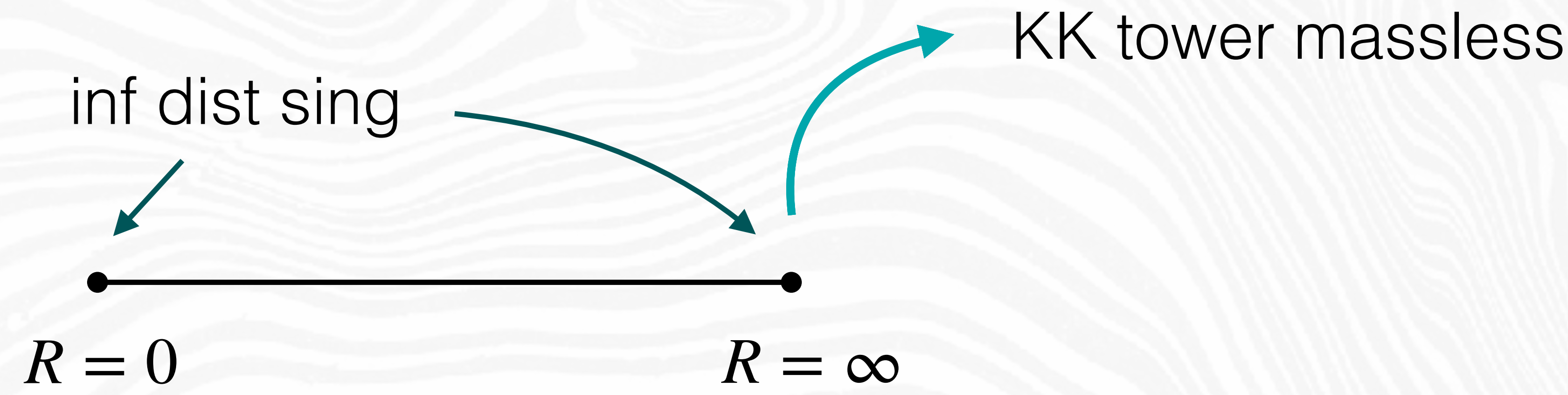

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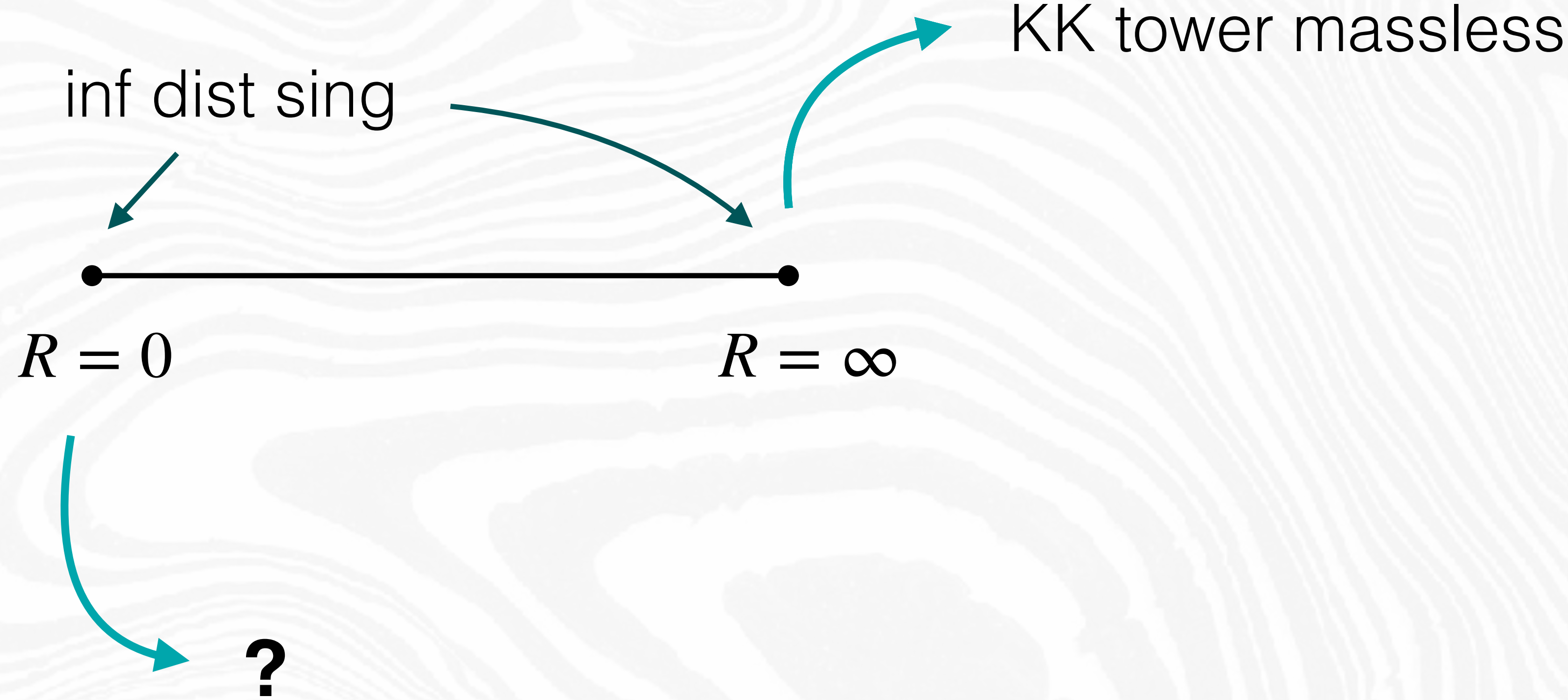

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Circle compactification: tower of KK modes $m_n = \frac{n}{R}$

KK tower massless

moduli space: inf dist sing

$R = 0$ $R = \infty$

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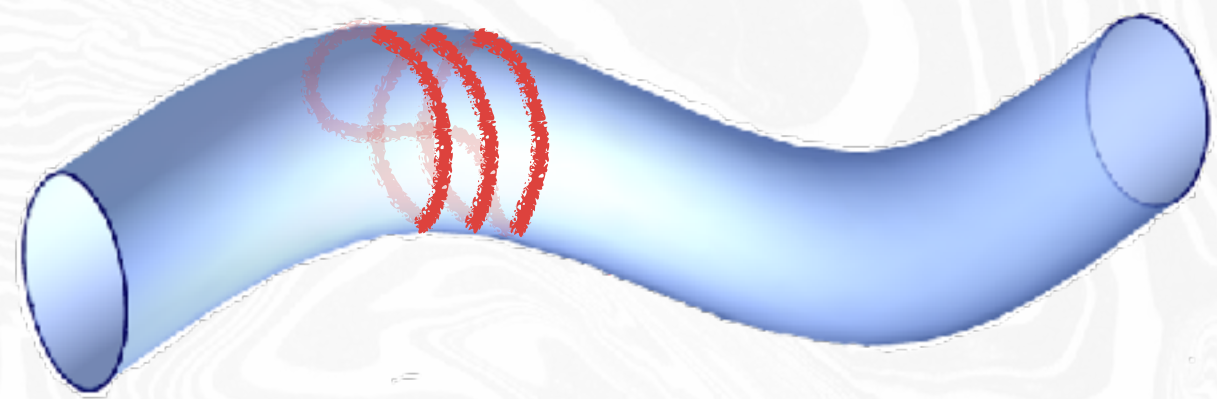
? winding modes



become massless as $R \to 0$

Emergence

Infinite Distance in moduli space $\xrightarrow{\text{SDC}}$ Infinitely many light states

Emergence

Infinite Distance in moduli space

SDC

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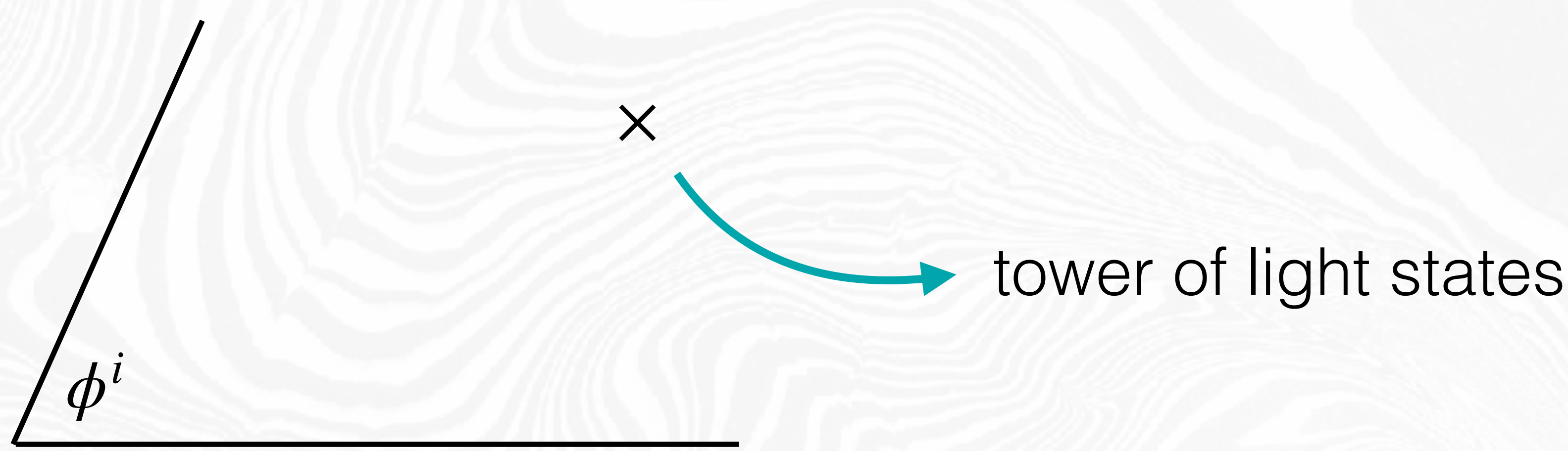
SDC $\rightleftarrows$ Emergence

Infinitely many light states



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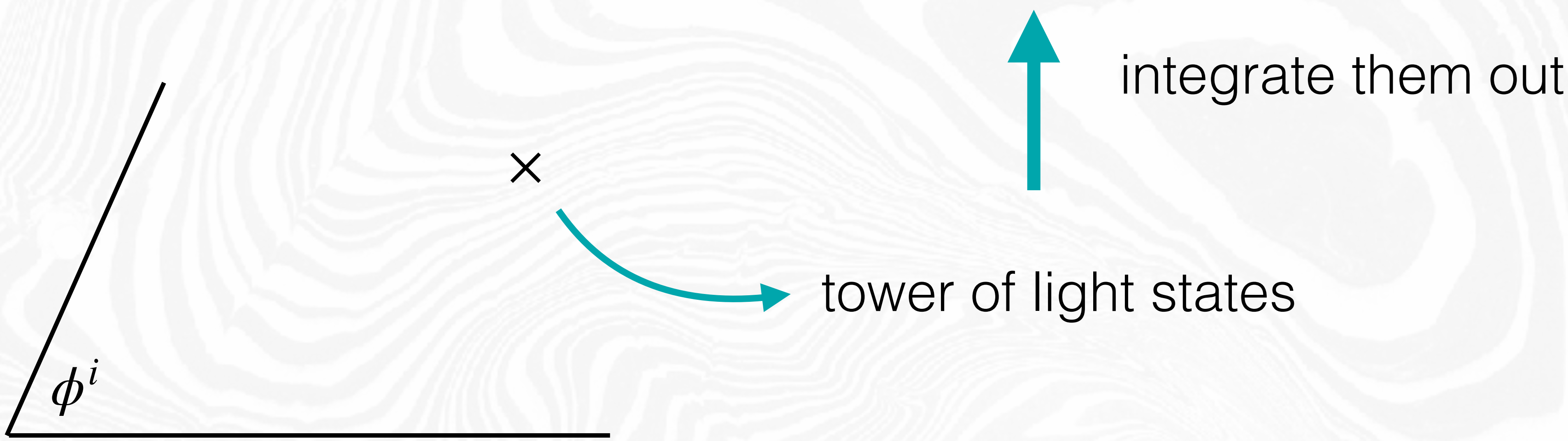
Infinitely many light states $\xrightarrow{\text{Emergence}}$ Infinite Distance in moduli space



Emergence

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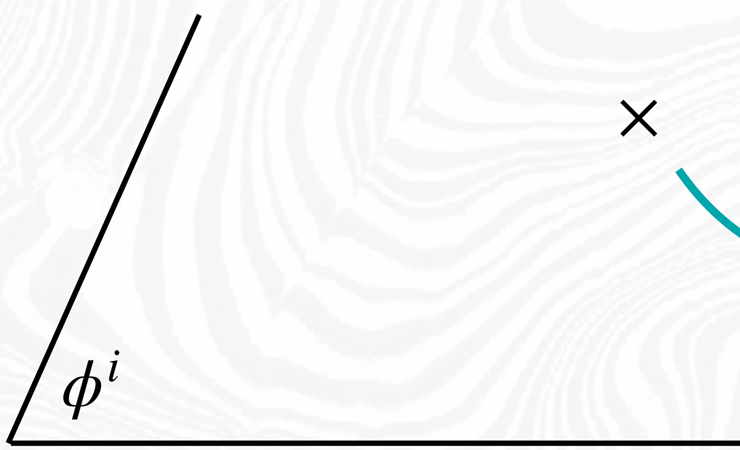
Infinitely many light states

metric $\sim g_{ij}\,\mathrm{d}\phi^i \wedge \star\, \mathrm{d}\phi^j$

integrate them out



tower of light states

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Strong evidence in the complex structure moduli space of IIB on a CY

[see Irene's talk]

Emergence for circle reduction?

Circle: tower of KK modes $m_n = \dfrac{n}{R}$

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sum up to the **species bound**:

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For a KK reduction?

Species bound and KK

Species bound in 3D $$\Lambda_{QG} \lesssim \frac{M_{\mathrm{pl},3}}{N}$$ [Dvali '07]

Number of species below Λ_{QG}

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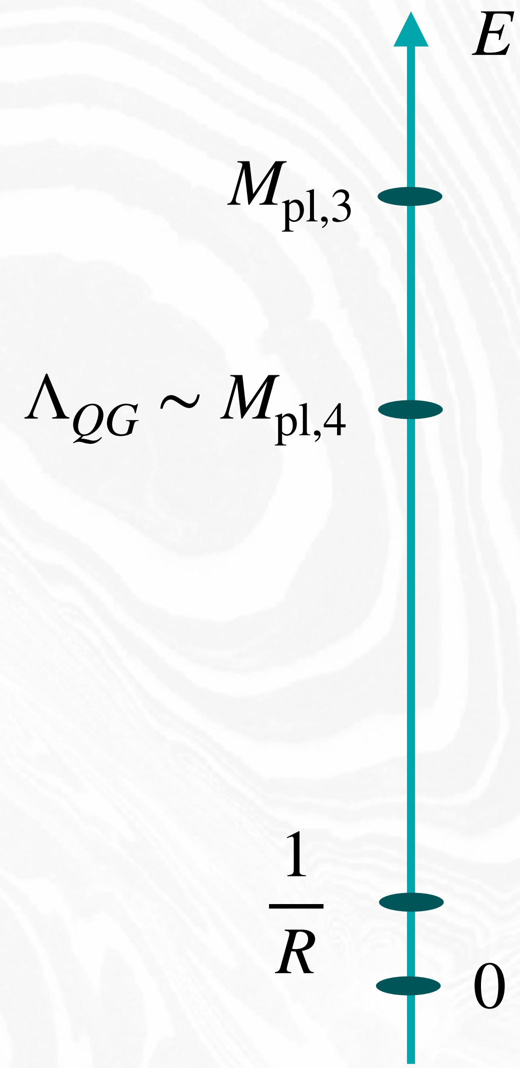

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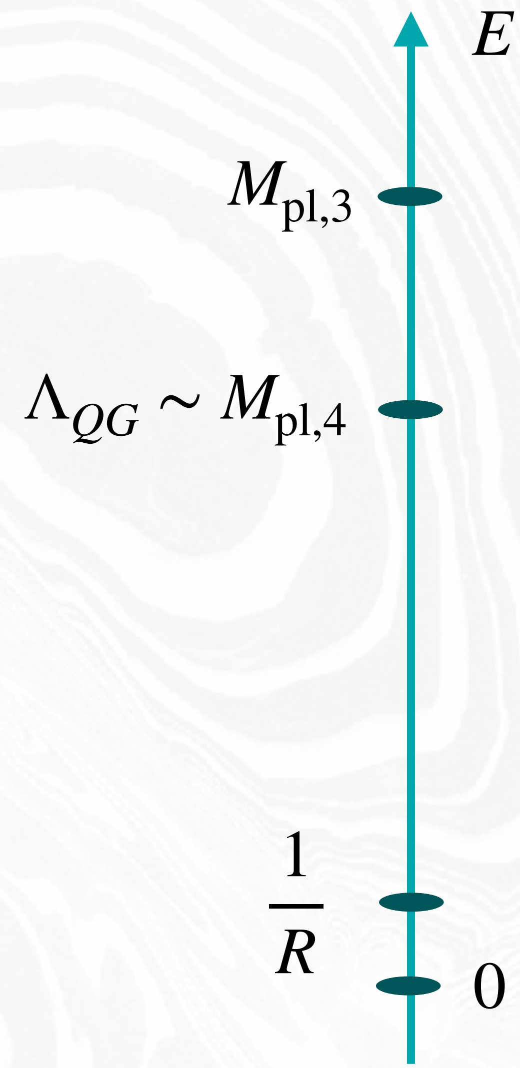

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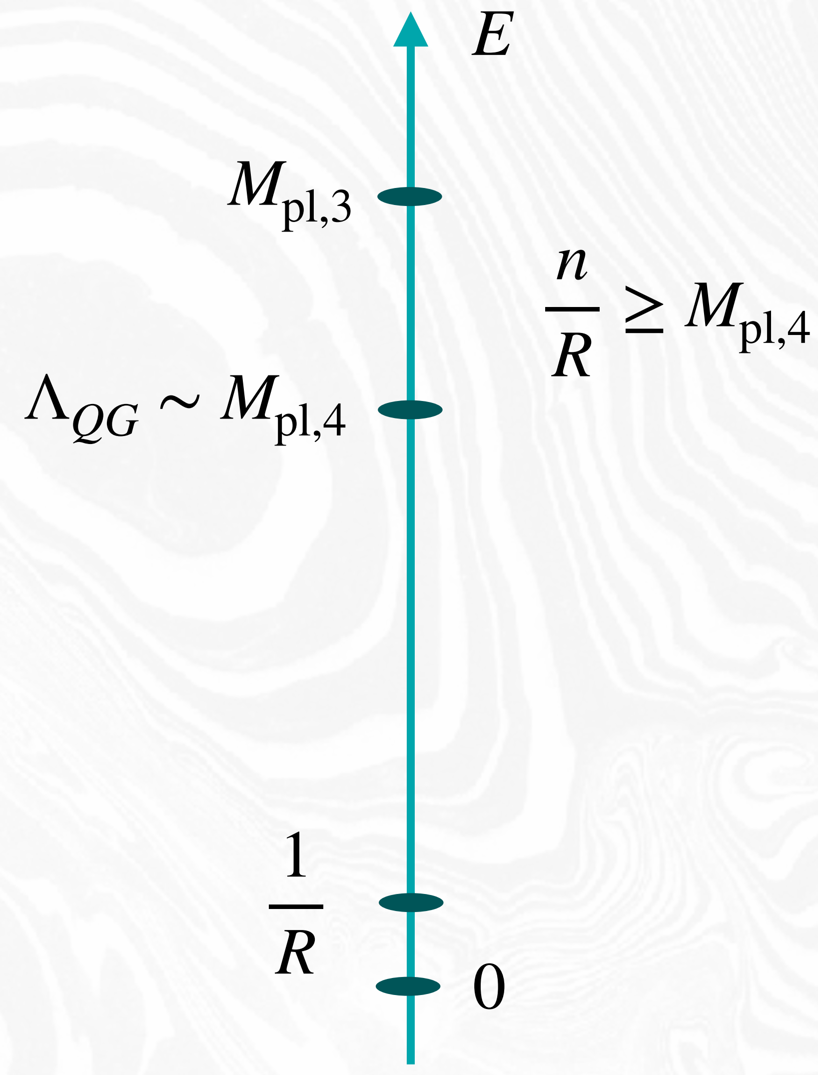

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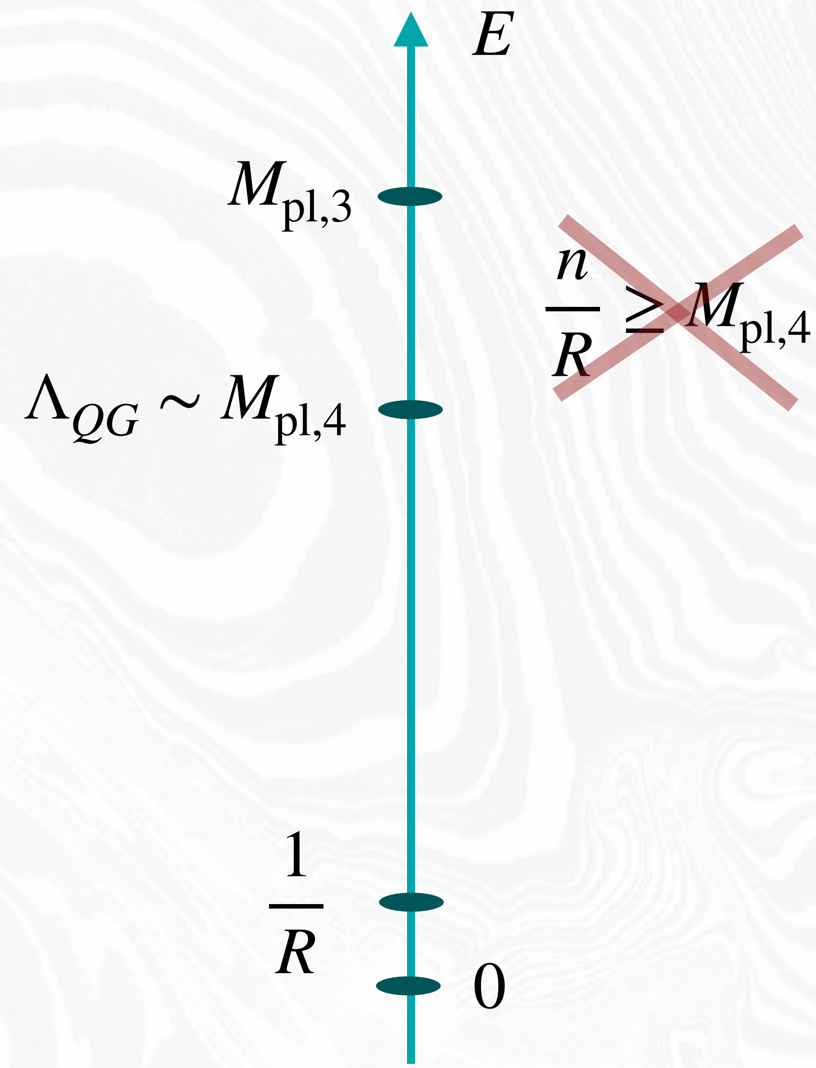

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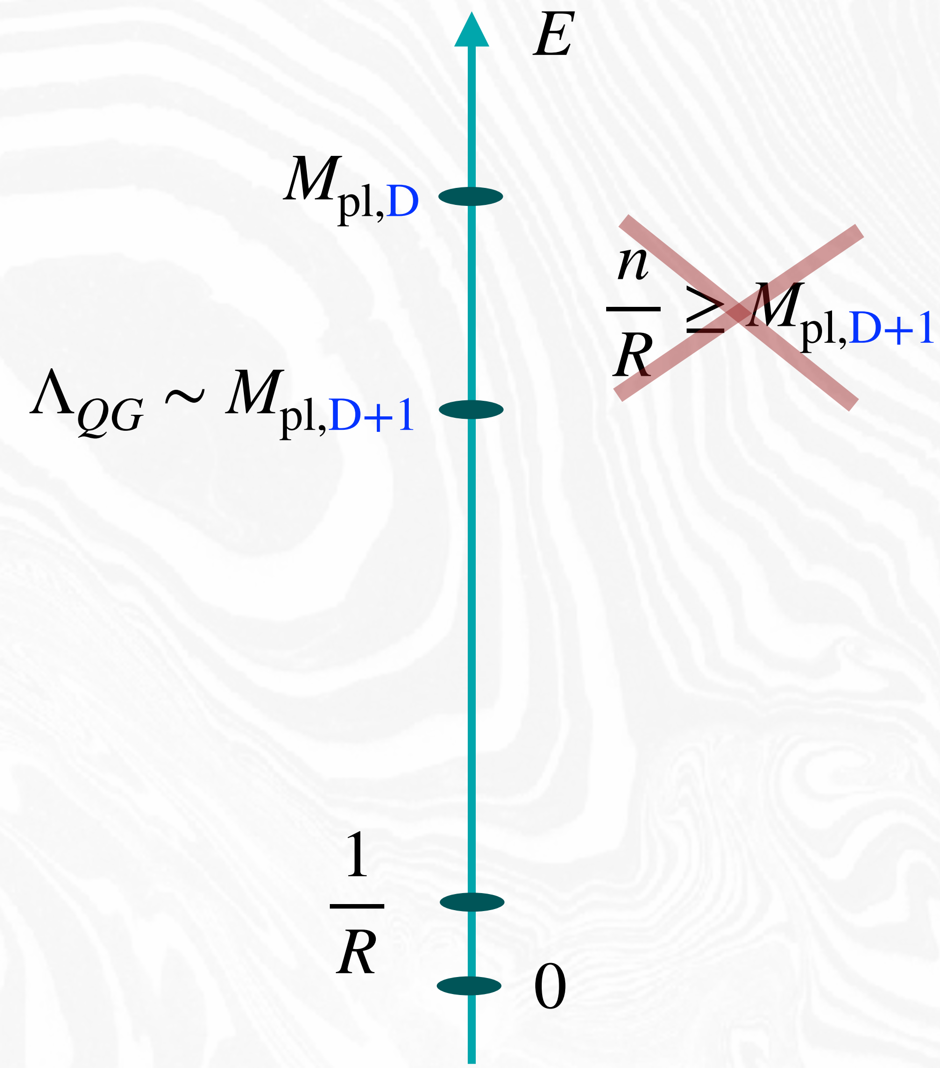

One-loop metric

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recall: 4D reg $\displaystyle\sum_{n=-\Lambda R}^{\Lambda R}$

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Other interpretation: unification

[Heidenreich,Reece,Rudelius'18]

Summary

- Explored in detail how to sum modes of a KK tower
  - ↳ preserving higher dimensional symmetries
- Showed that information about chiral anomaly preserved in 3D
  - ↳ Chiral anomaly canceled in 4D effectives action of F-theory
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  - ↳ $\Lambda_{QG} \sim M_{\mathrm{pl,D+1}}$
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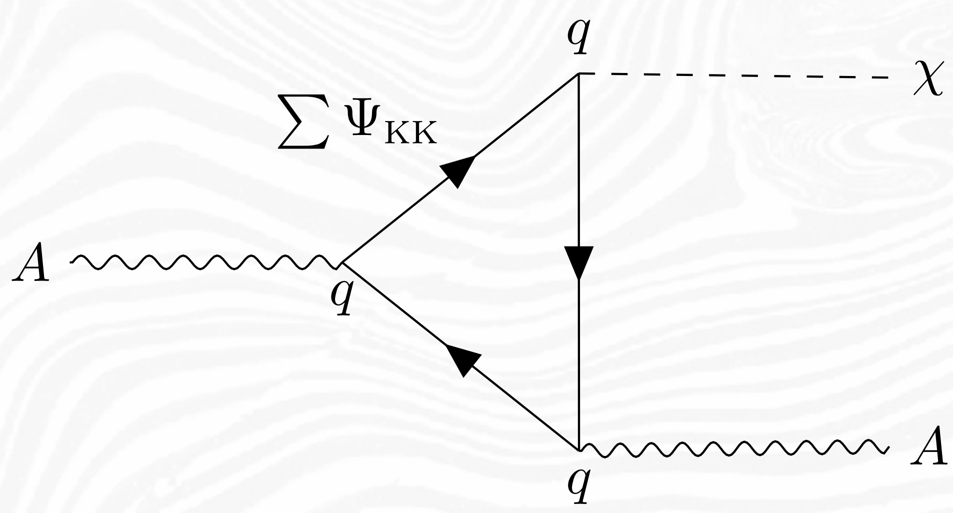
Thank you!

back-up slides

3d computation: triangle diagram



$$\zeta = \langle \zeta \rangle + \chi$$

With the whole KK tower running in the loop!

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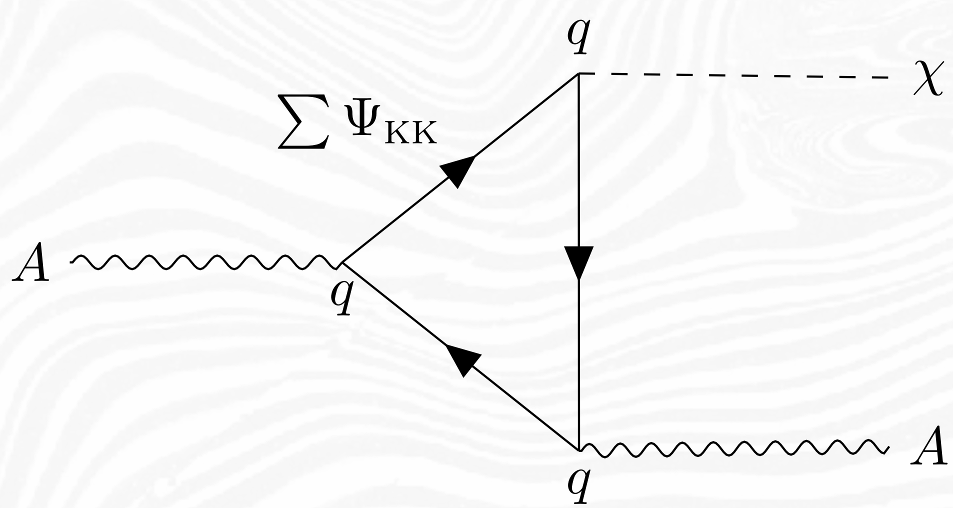

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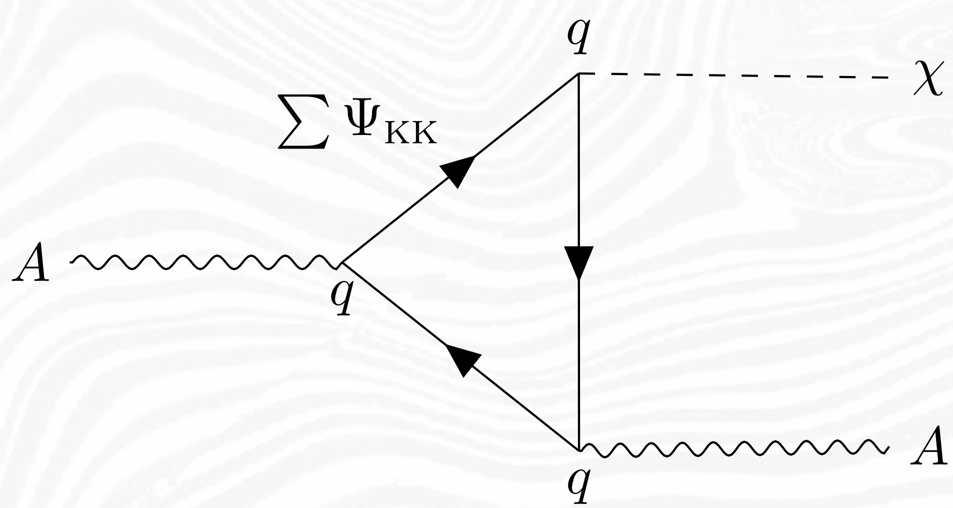

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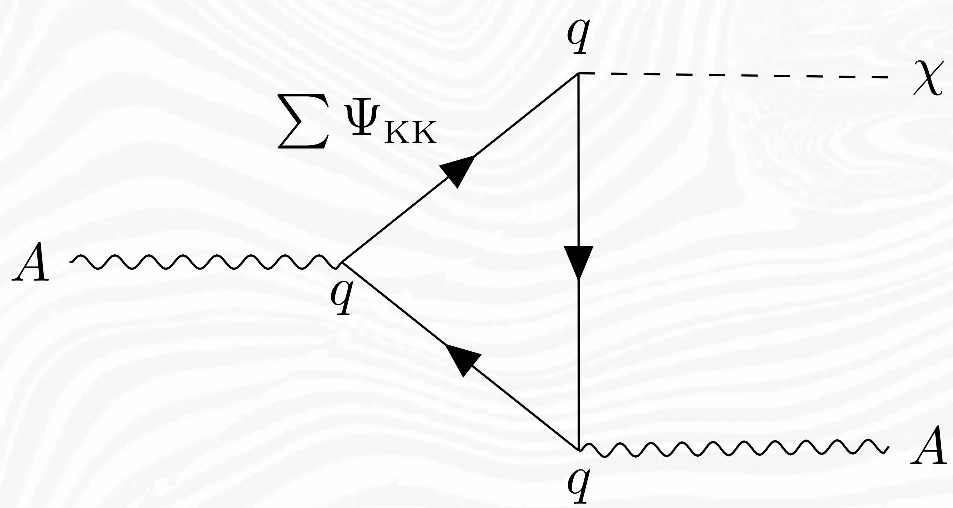

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NOT invariant under $A \to A + \mathrm{d}\lambda$ as expected

Unification

Assume $K(\phi)\,(\partial\phi)^2$

Suppose the scale strongly couple same as gravity ~ unification

Compute the quantum corrections at that scale Λ_{QG}

Ask that QC ~1 $\longrightarrow$ $K(\phi) \sim \dfrac{1}{\phi^2}$

[Heidenreich,Reece,Rudelius'18]

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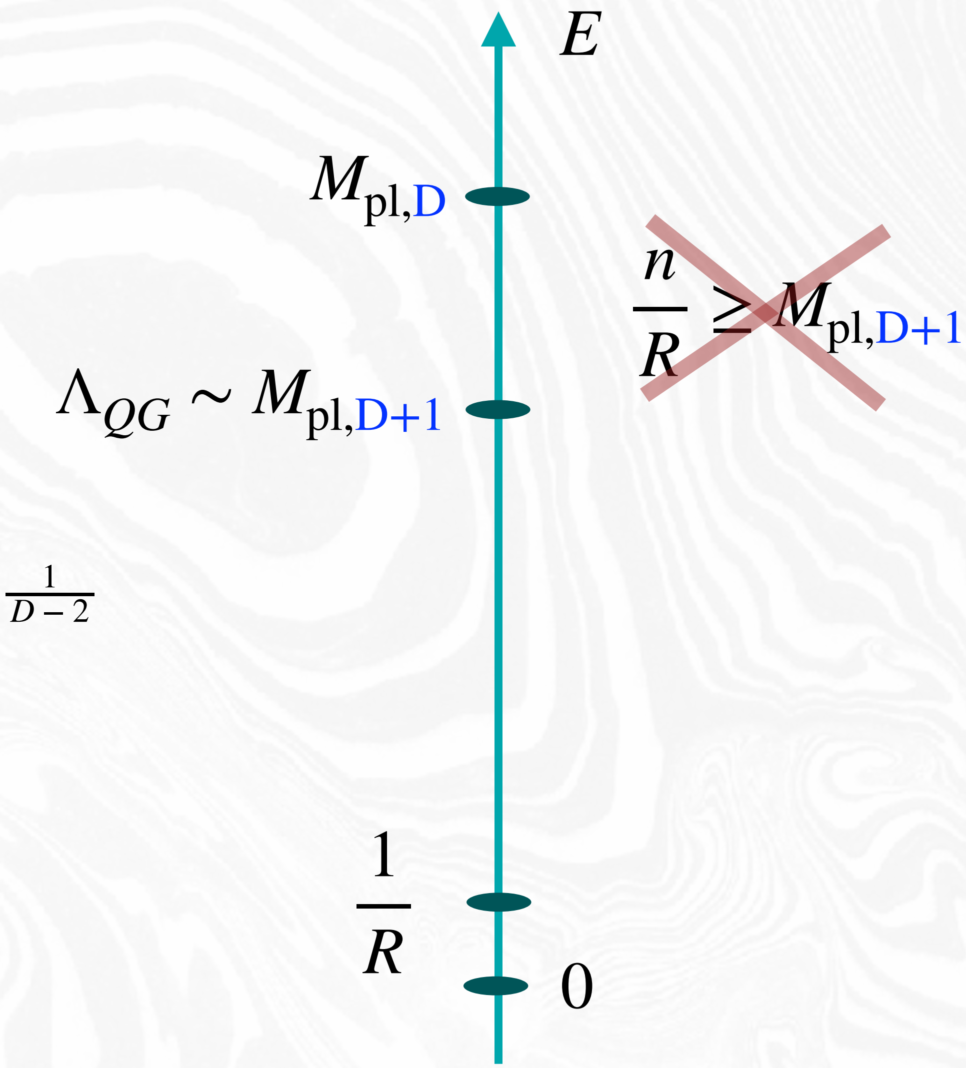

one loop metric

$$\lambda_n \sim m_n(R)\, m_n'(R)$$

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