Primordial Black Holes from String Inflation

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in collaboration with M. Cicoli & F. Pedro

Related Talk's : Cicoli, Zavala
Outline

• Motivation
  why PBHs?
  astrophysical constraints
  why string theory?

• PBH formation

• PBHs from fibre inflation

• Summary & conclusions
Motivation

LIGO Experiment
(Obs. of gravitational wave merge of two BH’s)

Why PBHs?
revived

Idea of BHs as part of/main constituent of DM

large population of BHs of 10’s $M_\odot$

fulled old idea

long-lived BHs produced during the early stage of the universe

What about smaller masses?

[Hawking’71]
[Carr, Hawking’74]
\( \Omega_{\text{PBH}} / \Omega_{\text{DM}} \)

\( M_{\text{PBH}} [M_{\odot}] \)

\[ 10^{-16} \leq M_{\odot} \leq 10^{-14} M_{\odot} \]

way far from the LIGO region but still an interesting idea to study due to the lack of astrophysical explanation

[Ballestero, Taoso'18]
How can we form PBH? amplification of the density perturbation during inflation is needed that form an important contribution to DM abundance $\sim (10\%-100\%)$.

Enhancement of the power spectrum → How to generate this enhancement? → Can it be done at the right scales?

Answer: framework of single field inflation reproducing the Planck data.

Enhancement achieved → Inflationary potential : slow-roll requiring near inflection point → slow-roll

[Ballesteros, Taoso'18] [Özsoy, Parameswaran, Tasinato, Zavala'18]
[Ivanov, Naselsky, Novikov'94] [Ezquiaga, Bellido, Ruiz'17]
NOTE:

- DM $\equiv$ PBHs $\Rightarrow$ Potential need to have enough tuning freedom

few examples: [Germani, Prokopec’17], [Ballestero, Taoso’17], [Ivanov, Naselsky, Novikov’94], [Ezquiaga, Bellido, Ruiz’17]

These models are a bottom-up approach lack/ignores the fund. issues of deriving the model from a UV perspective

Possible solution:

String Theory can search for concrete examples of infl. models to allow PBH formation from a top–down perspective
PBH formation

**PBH formation**

PBH → Large & rare δρ re-enter the Hubble horizon and collapse

\[ \beta_f(M) = \frac{\rho_{PBH}(M)}{\rho_{tot}} \]  
\[ \beta_f(M) : \text{fraction of the total energy density in PBH with mass } M \text{ at formation} \]

\[ \beta_f(M) = \int_{\delta_c}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_M} e^{-\frac{\delta^2}{2\sigma_M^2}} d\delta \]

\[ \delta_c : \text{critical value for collapse into PBH} \]

Probability of large distribution

gaussian

Gaussian distributions, we know \[ \sigma_M^2 \sim \langle \zeta \zeta \rangle \text{ at CMB scales is } \mathcal{O}(10^{-9}) \]

\[ \zeta = \frac{u}{z} ; \text{ for } \sigma_M \ll \delta_c \Rightarrow \beta_f(M) \sim \frac{\sigma_M}{\sqrt{2\pi}\delta_c} e^{-\frac{\delta_c^2}{2\sigma_M^2}} \]

If PBH = important fraction of DM  ⇒ few order of magnitude large than on CMB scales
Estimation of the enhancement

\[ M = \gamma_G \frac{4\pi}{3} \rho_{\text{tot}} \left| \frac{H}{f} \right| = 4\pi \gamma_G \frac{M_p^2}{H_f} \]

\[ \gamma_G : \text{correction factor} \]
\[ M : \text{Mass of PBH forming when large density pert. re-enter the horizon} \]

PBH behave like matter \( \Rightarrow \)

\[ \beta_f(M) = \left( \frac{H_0}{H_f} \right)^2 \frac{0.26}{a_f^3} f_{\text{PBH}}(M) \]

Assumption: PBH form before Matter–Radiation equality

\[ H_f^2 = 8 \times 10^{-5} \frac{H_0^2}{a_f^4} \left( \frac{g_{*f}}{g_{*0}} \right)^{-1/3} \]
\[ \Rightarrow \]
\[ \beta_f(M) \simeq \frac{4}{\sqrt{\gamma_G}} \times 10^{-9} \left( \frac{g_{*f}}{g_{*0}} \right)^{1/4} \sqrt{\frac{M}{M_\odot}} f_{\text{PBH}}(M) \]

\[ \beta_f(M) \simeq 10^{-8} \sqrt{\frac{M}{M_\odot}} \]
\[ \Rightarrow \]
\[ \delta_c \sim 1 \quad \sigma_M \sim 0.12 \]

Power spectrum \( P_k \sim \sigma^2 \sim \mathcal{O}(10^{-2}) \)

We need an enhancement of 7 orders of magnitude respect to CMB
Observations:

1. Estimation

Broadly distributed \( f_{PBH}(M) = \int df_{PBH}(M) = \int \frac{df_{PBH}(M)}{d \log(M)} d \log(M) \)

\( df_{PBH}(M) \) is the fraction between \( M \) and \( M + d \log(M) \)

2. \( H \sim \) is constant during inflation

\[
\Delta N_{CMB}^{PBH} = \ln \left( \frac{a_{PBH} H_{inf}}{a_{CMB} H_{inf}} \right) = \ln \left( \frac{a_f H_f}{0.05 \text{ Mpc}^{-1}} \right) = 18.4 - \frac{1}{12} \ln \left( \frac{g_*}{g^*_0} \right) + \frac{1}{2} \ln \gamma_G - \frac{1}{2} \ln \left( \frac{M}{M_\odot} \right)
\]

\( M \in [10^{-16}, 10^{-14}] \, M_\odot \quad \rightarrow \quad \Delta N_{CMB}^{PBH} \sim 34.2 - 36.5 \) efoldings
**PBH from fibre inflation**  
\[ \mathcal{V} = t_{\mathbb{P}^1} \tau_{K3} - \tau_{d\mathbb{P}}^{3/2} \]

\( \tau_{K3} \): Volume of K3–surface fibre over a \( \mathbb{P}^1 \)

\( \tau_{d\mathbb{P}} \): Volume of a diagonal del Pezzo

\( t_{\mathbb{P}^1} \): Volume of \( \mathbb{P}^1 \)

\( \mathcal{V}, \tau_{d\mathbb{P}} \) Lifted by non-pert. effects to W & \( \alpha' \) correct. to K at leading order in \( 1/\mathcal{V} \)

\( \tau_{K3} \): remain as a flat direction \( \Rightarrow \) our inflaton candidate

**Way of lifting:**

- 1–loop string corrections: KK–corrections & Winding corrections
  
  \[ \text{[Berg, Haack, Kors'05]} \]
  \[ \text{[Berg, Haack, Pajer'07]} \]

- Higher derivative \( \alpha' \) effects
  
  \[ \text{[Ciupke, Louis, Westphal'15]} \]
Scalar potential

\[ V_{\text{inf}} = \frac{W_0}{\sqrt[3]{V}} \left[ \frac{C_{\text{up}}}{\sqrt[3]{V}} - \frac{C_W}{\sqrt{\tau_{K3}}} + \frac{A_W}{\sqrt{\tau_{K3}} - B_W} + \frac{\tau_{K3}}{V} \left( D_W - \frac{G_W}{1 + R_W \frac{\tau_{K3}^{3/2}}{V}} \right) \right] \]

\[ \tau_{K3} = \langle \tau_{K3} \rangle e^{\frac{2}{3} \hat{\phi}} \]

- **minimum**: \( \langle \tau_{K3} \rangle \sim \frac{C_W B_W^2}{(\sqrt{C_W} - \sqrt{A_W})^2} \)

- **5th term in the potential has a critical point**: \( \frac{2^{2/3}}{(R_W/V)^{2/3}} \)

\[ G_W, R_W \quad \text{crucial for the generation of the near inflection point} \]
**Power spectrum**

Mukhanov-Sasaki

\[ P_k = \frac{k^3}{2\pi^2} \left| \frac{u_k}{z} \right|^2 \]

slow-roll

\[ P_k = \frac{H^2}{8\pi^2\epsilon} \left. \right|_{k=aH} \]

\[ u''_k(\eta) + \left( k^2 - z''/z \right) u_k(\eta) = 0 \]

\[ \frac{z''}{z} = (aH)^2 \left[ 2 - \epsilon + \frac{3}{2} \eta - \frac{1}{2} \epsilon \eta + \frac{1}{4} \eta^2 + \frac{1}{2} \eta \kappa \right] \]

\[ \epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \frac{\dot{\epsilon}}{\epsilon H}, \quad \kappa = \frac{\dot{\eta}}{\eta H} \]

\[ u_k : \text{re-scaled curvature pert.} \quad \zeta = u/z \]

\[ z \equiv \sqrt{2\epsilon} a \]

Cosmological observables:

<table>
<thead>
<tr>
<th>( P )</th>
<th>( n_s )</th>
<th>( r )</th>
<th>( \frac{dn_s}{d\ln k} )</th>
<th>( \Delta N_{\text{CMB}} )</th>
<th>( P_k )</th>
<th>peak</th>
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</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>0.9457</td>
<td>0.015</td>
<td>-0.0017</td>
<td>34.5</td>
<td>0.01365</td>
<td></td>
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<tr>
<td>( P_2 )</td>
<td>0.9437</td>
<td>0.015</td>
<td>-0.0017</td>
<td>34.5</td>
<td>0.03998</td>
<td></td>
</tr>
<tr>
<td>( P_3 )</td>
<td>0.9457</td>
<td>0.015</td>
<td>-0.0019</td>
<td>34.5</td>
<td>0.013341</td>
<td></td>
</tr>
</tbody>
</table>
Universe evolve $\rightarrow$ slow-roll $\rightarrow$ constant roll $\rightarrow$ slow-roll (with big $\eta$)

\[ \ddot{\phi} \equiv -(3 + \alpha)H\dot{\phi} \]

slow-roll $\alpha = -3 \Rightarrow \ddot{\phi} = 0$

Ultra slow-roll $\alpha = 0 \Rightarrow \ddot{\phi} = -3H\dot{\phi}$

constant roll appear due to the presence of extreme flat region causing the inflaton to brake upon reaching it

\[ P_k \propto H^{2\alpha+3-1}a^{3+2\alpha+|3+2\alpha|} \]

constant mode $2\alpha + 3 < 0$

growing mode $2\alpha + 3 > 0$

CMB scales

PBH scales
Summary & Outlooks

• First string theory inflationary model consistent cosmological obs. at CMB scales generates PBH at small distances scales

• PBH formed in the low-mass region ⇒ significant fraction of DM

Relevant features of fibre models:

(i) Coeff. of potential depends on background fluxes & intersection numbers

(ii) Potential enjoys approx. abelian re-scaling symmetry suppressing quantum corrections

(iii) The contribution generating the near inflection point has been generated in

[Cicoli, Ciupke, VAD, Guidetti, Muia, Shukla’17]
To do:

- Re-do analysis with $\delta_c \sim 0.5$, quantum diffusion & non-Gaussianities

- More general fibre inflation potential

- Consider formation in a matter-dominated period

- Find curvaton-like mechanism for PBH production

- Study oscillon scope collapse into BHs at the end of inflation

[Ando, Kinomata, Kawasaki, Mukaida, Yanagida]
Gracias.