Stability Walls for T-branes on del-Pezzo Surfaces

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Based on: *Marchesano, Savelli, S.S. [1707.03797] & to appear*
See also talk by R. Savelli

BPS-stability of 7-branes

- Worldvolume theory of stack of 7-branes on 4-cycle S is 8d $\mathcal{N} = 1$ SYM
- Bosonic field content:
  $\Phi \in H^0(S, \mathrm{End}(V) \otimes K_S) \cong H^{2,0}(S, \mathrm{End}(V)^*)$
  $A \in H^1(S, \mathrm{End}(V))$
- Vacuum is stable if BPS-conditions are satisfied

BPS-stability of 7-branes

- They split into F-term conditions $F^{2,0} = 0$ and $\overline{\partial}_A \Phi = 0$ and D-term conditions $J \wedge F + \frac{1}{2}[\Phi, \Phi^\dagger] = 0$
- F-term conditions are holomorphic data, i.e. they are invariant under complexified gauge transformations of the vector bundle
- D-terms depend on Kähler moduli and are subjected to α'-corrections *Marchesano and S.S. 2016*

Intersecting branes on the same cycle class

- Cartan of Φ parametrises transversal deformations of brane stack
- consider 2 coinciding branes, i.e. gauge group $U(2)$
- Switching on a vev $\Phi = \begin{pmatrix} v & \\ & -v \end{pmatrix}$ breaks the gauge group to $U(1)^2$
- geometrically we are taking apart the branes
- if v is non-constant the symmetry is restored along its vanishing loci — intuitively this describes a pair of branes intersecting along the curve $v = 0$

T-branes

- What happens if we switch on a non-Cartan vev? *Cecotti, Cordova, Heckman, and Vafa 2011*
- $\Phi = \begin{pmatrix} 0 & m \\ 0 & 0 \end{pmatrix}$ breaks the gauge group but does not encode geometric deformation
  (different context *Vafa and Witten 1994*)
- information is lost completely in standard M-/F-theory uplift but well-defined states from world-volume perspective
- Some interest in the last years *Anderson, Heckman, and Katz 2014; Collinucci and Savelli 2014; Collinucci and Savelli 2015; Collinucci, Giacomelli, Savelli, and Valandro 2016; Anderson, Heckman, Katz, and Schaposnik 2017; Collinucci, Giacomelli, and Valandro 2017*

T-branes on compact 4-cycles

- Consider T-brane on compact, simply connected 4-cycle S
- no stable T-branes on positive curvature 4-cycles or K3 b/c of tension between F- and D-terms *Marchesano, Savelli, S.S. 2017*
- No-go can be evaded if we other stacks of 7-branes intersect the T-brane stack
- Requires vevs for defect fields σ, σ^c localised on intersection curve Σ
- BPS-conditions change
  F-terms: $\overline{\partial}_A \Phi = \delta_\Sigma \wedge << \sigma^c, \sigma >>_{\mathrm{ad}(P)}$
  D-terms: $\omega \wedge \mathbb{F}_S + \frac{1}{2}[\Phi, \Phi^\dagger] = \frac{1}{2}\omega \wedge \delta_\Sigma \left[\mu(\bar{\sigma}, \sigma) - \mu(\bar{\sigma}^c, \sigma^c)\right]$

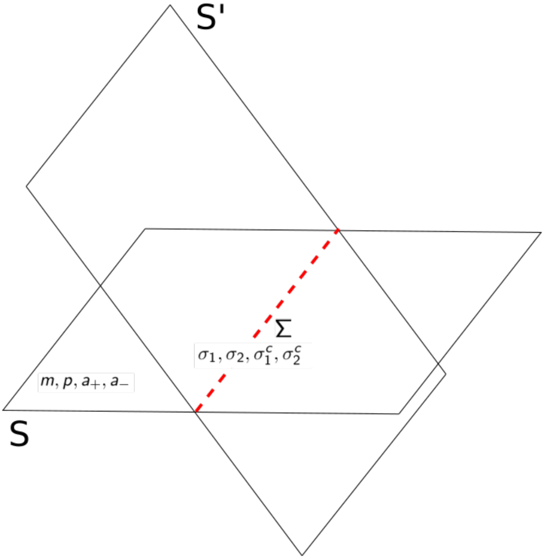
Stable T-branes on Positive Curvature 4-cycles

- Idea: Vevs for defect fields may relax tension between F- and D-terms stabilizing T-brane on positive curvature 4-cycles
  $\longrightarrow$ See talk by R. Savelli
- 1st way: Allow for both signs in FI-terms leaving Φ holom.
  $\longrightarrow$ *holomorphic scenario*
- 2nd way: Introduce poles in Φ, such that only $\mathscr{M} \otimes [\Sigma]$ is effective, but not $\mathscr{M}$
  $\longrightarrow$ *meromorphic scenario*
- Both scenarios require vevs for different defect fields

Walls of Stability

- How do the open string d.o.f. behave when we move in closed string moduli space?
  $\Rightarrow$ How does stability depend on Kähler moduli?
- Can we learn something about holomorphic quantities (such as Yukawa couplings)?
- Consider a topology that allows for a stable T-brane of this kind
- Put ourselves in a point in moduli space sufficiently close to vanishing FI-terms where it is stable
- Move in moduli space and check if T-brane system decays

Walls of Stability



Walls of Stability

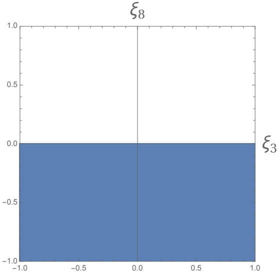
- We can read off their charges under the two line-bundles and consider the 4d
  $\frac{\xi_3-\xi_8}{2} = |m|^2 + |a_+|^2 + |\sigma_2|^2 - |p|^2 - |a_-|^2 - |\sigma_2^c|^2$
  $\xi_8 = |\sigma_1^c|^2 + |\sigma_2^c|^2 - |\sigma_1|^2 - |\sigma_2|^2$
- Stable T-brane requires existence of some of those fields
- If topology forbids existence of some combination of remaining fields, regions of moduli space may become inaccessible
  $\Rightarrow$ T-brane decays if we move into this regions.
- Minimal conditions for such regions to exist?

Holomorphic Case

- Stable T-branes on del-Pezzo with holomorphic Φ require holomorphic sections m and σ_1
- So by assumption topology satisfies $0 \leq \int_S J \wedge c_1(\mathscr{M})$ on 4-cycle in order to have m modes
- and to have σ_1 modes it satisfies $0 \leq \deg_{\Sigma_0}\left(\mathscr{L}_3^{-1} \otimes \mathscr{L}_8^{-1} \otimes K_\Sigma^{1/2}\right)$ on at least one component Σ_0
- Read off from FI-terms that T-brane stable in $\xi_8 \leq 0$ and $\xi_8 \leq \xi_3$

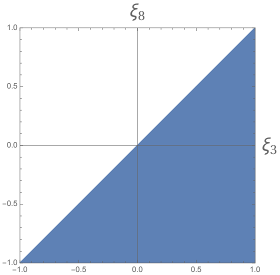
Holomorphic Case

- What happens if we move to other chambers of moduli space?
- if topology forbids e.g. zero-modes in σ_1^c and σ_2^c we cannot reach $0 < \xi_8$
- Combining the existence conditions for zero-sections of the relevant bundles gives sufficient topological conditions $\deg \mathscr{L}_8\big|_{\Sigma_i} < 0$ and $\deg \mathscr{L}_8\big|_{\Sigma_i} < -\deg \mathscr{L}_3\big|_{\Sigma_i}$ for all components Σ_i



Holomorphic Case

- Can the second region $\xi_3 < \xi_8$ become inaccessible?
- Require absence of zero modes in p, a_-, σ_2^c
  $\Rightarrow$ similiar topological conditions
- If only p and σ_2^c are absent moving into this region in moduli space requires a_- to acquire a vev
- Different kind of bound-state of 7-branes $\rightarrow$ "T-bundle"
- Sufficient condition $0 \leq \int_S J \wedge c_1(K_S) < 2 \int_S J \wedge c_1(\mathscr{L})$
  $\deg \mathscr{L}_8|_{\Sigma_0} < 0$ and $\deg \mathscr{L}_8|_{\Sigma_0} < 1 - g_{\Sigma_0} + \deg \mathscr{L}_3 < |_{\Sigma_0}$



Conclusions

- T-branes may be stable on positive or vanishing curvature 4-cycles if intersected by other 4-cycles carrying 7-branes
- Mechanism requires vevs for defect fields on intersection curve
- Stability depends on topology and position in Kähler mod. space
  $\Rightarrow$ under certain topol. conditions, T-branes decay if we move into different chambers or they form bound states of different kinds
- Analysis hints at different mechanisms for realistic Yukawa-couplings
  $\rightarrow$ Moving in moduli space cannot change holom. quantities; if T-brane becomes unstable in some region in mod. space, Yukawa's need to be realised differently