Gravity and U(1)s in F-theory

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Outline

Introduction and Summary

Decoupling Gravity

— *Supergravity* perspective

— *F-theory* perspective

Weakening U(1)

Conlusions and Outlook

Introduction

F-theory vacua as quantum theories of gravity

- **6d EFTs of String/F-theory**
  - Controlled

    more supersymmetries, geometries under better control, …
  - Interesting

    max dim for an SCFT, quantum gravity properties persist, …
- **When gravity decouples**
  - SCFTs associated with nonabelian G [Heckman-Morrison-Vafa '15]
  - What about U(1)s?
- **When U(1) becomes weak**
  - Can it be weaker than gravity? . . . . Weak Gravity Conjecture [Arkani-Hamed et al. '06]
  - Infinite light particles? . . . . Swampland Distance Conjecture [Ooguri-Vafa '06]

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F-theoretic answers to the two questions

- **What happens to U(1)s as gravity is decoupled?**
  - U(1) gauge fields lead to global U(1) symmetries [SJL-Regalado-Weigand '18]
  - Proven in two perspectives
    (a) ***Physics*** of supergravity
    (b) ***Geometry*** of string/F-theory

- **What happens to** (F-)**EFT as U(1) gets weaker than gravity?**
  - A tensionless string appears (lead to infinite light particles) [SJL-Lerche-Regalado-Weigand '18]
  - Amogst them are particles with $q \geq \gamma m$ for each charge, where $\gamma = \frac{c}{g M_{\mathrm{Pl}}^2}$
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Decouple Gravity

from Supergravity

[SJL–Regalado–Weigand '18]

6D N=(1,0) Supergravity

Basic Setup

- **Multiplets**

Multiplet	Field Contents
Gravity	$(g_{\mu\nu}, \psi^+_\mu, B^+_{\mu\nu})$
Tensor	$(B^-_{\mu\nu}, \chi^-, \phi)$
Vector	(A_μ, λ^+)
Hyper	$(\psi^-, 4\varphi)$

$G = \prod G_\kappa \times U(1)^r$

- **Action** (with $M_{\rm Pl} = 1$)

$$\int_{\mathbb{R}^{1,5}} \Big(\frac{1}{2}R * 1 - \frac{1}{4}g_{\alpha\beta}H^\alpha \wedge * H^\beta - \frac{1}{2}g_{\alpha\beta}dj^\alpha \wedge * dj^\beta - \sum \frac{2j\cdot b_\kappa}{\lambda_\kappa}{\rm tr}F_\kappa \wedge * F_\kappa - (2j\cdot b)F\wedge * F - \frac{1}{2}\Omega_{\alpha\beta}B^\alpha \wedge X^\beta\Big) + S_{\rm hyp}$$

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$$\int_{\mathbb{R}^{1,5}} \Big(\frac{1}{2} R * 1 - \frac{1}{4} g_{\alpha\beta} H^\alpha \wedge * H^\beta - \frac{1}{2} g_{\alpha\beta} dj^\alpha \wedge * dj^\beta$$
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6D N=(1,0) Supergravity

Basic Setup

- **Multiplets**

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Anomaly cancellation

- **Anomaly cancellation**
  - Coupling of tensors involves $X_4^\alpha = \frac{1}{2}a^\alpha \mathrm{tr}R^2 + \sum \frac{2b_\kappa^\alpha}{\lambda_\kappa}\mathrm{tr}F_\kappa^2 + 2b^\alpha F^2$
  - One-loop anomalies are cancelled

    $I_8^{1-\mathrm{loop}} = \frac{1}{32}\Omega_{\alpha\beta}X^\alpha \wedge X^\beta$

- **Anomaly equations**

$$
\begin{aligned}
\mathbf{b}_\kappa \cdot \mathbf{b}_\kappa &= \frac{1}{3}\lambda_\kappa^2\left(\sum \mathcal{M}_I^\kappa C_\kappa^I - C_{\mathrm{Adj}_\kappa}\right) && [G_\kappa^4] \\
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\mathbf{b} \cdot \mathbf{b} &= \frac{1}{3}\sum \mathcal{M}_I q_I^4 && [U(1)^4]
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where

M's are multiplicities;

A's, C's, and E's are contants.

Decoupling Gravity in Sugra

Nonabelian gauge interaction

- **Decomposition of $\mathbf{SO(1, n_T)}$ Vector Space**
  - (Anti-)Self-duality: $*H^\alpha = D^\alpha{}_\beta H^\beta$ where $D(j)^\alpha{}_\beta := 2j^\alpha j_\beta - \delta^\alpha{}_\beta$
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  - $\mathbb{R}^{1,n_T} = \mathcal{V}^+ \oplus \mathcal{V}^-$
  - $v = v^+(j) + v^-(j)$

$$dH^\pm(j) = \frac{1}{2}a^\pm(j)\mathrm{tr}R^2 + \sum_\kappa \frac{2b_\kappa^\pm(j)}{\lambda_\kappa}\mathrm{tr}F_\kappa^2 + 2b^\pm(j)F^2$$

- **$\mathbf{G}_\kappa$ Interactions in the Decoupling Limit**
  - $g_{\hat{\kappa}} \to \infty$ while $g_{\check{\kappa}}$ finite, for a choice j_0
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  - $1/g_{\check{\kappa}}^2 \sim j_0 \cdot b_{\check{\kappa}} \neq 0$ ⇢ $b_{\check{\kappa}}^+ \neq 0$ ⇢ non-zero $G_{\check{\kappa}}$-anomaly after decoupling
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6D N=(1,0) Supergravity

Basic Setup

- **Multiplets**

	Multiplet	Field Contents
$\mathbf{1}$	Gravity	$(g_{\mu\nu}, \psi^+_\mu, B^+_{\mu\nu})$
$\mathbf{n_T}$	Tensor	$(B^-_{\mu\nu}, \chi^-, \phi)$
	Vector	(A_μ, λ^+)
	Hyper	$(\psi^-, 4\varphi)$

$G = \prod \boxed{G_\kappa} \times \boxed{U(1)}$

- **Action (with $M_{\rm Pl} = 1$)**

$$\int_{\mathbb{R}^{1,5}} \Big(\frac{1}{2}R*1 - \frac{1}{4}g_{\alpha\beta}H^\alpha \wedge *H^\beta - \frac{1}{2}g_{\alpha\beta}dj^\alpha \wedge *dj^\beta$$
$$- \sum \frac{2j\cdot b_\kappa}{\lambda_\kappa}\mathrm{tr}F_\kappa \wedge *F_\kappa - (2j\cdot b)F\wedge *F$$
$$- \frac{1}{2}\Omega_{\alpha\beta}B^\alpha \wedge X^\beta\Big) + S_{\rm hyp}$$

- **Notations**

CS forms

- $\alpha, \beta = 0, \cdots, n_T$ ········> B^α $\quad H^\alpha = dB^\alpha + \frac{1}{2}a^\alpha\omega_L + \sum\frac{2b^\alpha_\kappa}{\lambda_\kappa}\omega^\kappa_Y + 2b^\alpha\omega$
- $a^\alpha, b^\alpha_\kappa, b^\alpha$ ························> $SO(1,n_T)$ $\quad \alpha, \beta, \cdots$ $\quad \Omega_{\alpha\beta}$
- j^α ································> $SO(1,n_T)$ $\quad j\cdot j = 1$ $\quad n_T$
- $g_{\alpha\beta} = 2j_\alpha j_\beta - \Omega_{\alpha\beta}$ ···> kinetic metric

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  - $b \cdot b = b^+ \cdot b^+ + b^- \cdot b^-$ $\ldots$ becomes non-positive upon decoupling

    $\ldots$ anomaly is no longer cancelled
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  - $b \cdot b = \underbrace{\cancel{\boxed{b^+ \cdot b^+}}}_{\geq 0} + \underbrace{\boxed{b^- \cdot b^-}}_{\leq 0}$ $\ldots$ becomes non-positive upon decoupling

    $\ldots$ anomaly is no longer cancelled
  - U(1) cannot remain dynamical ('t Hooft anomaly)
- **Worth Checking**
  - ABJ anomalies remain cancelled since $b^+_{\hat{\kappa}} = 0$ and GS contributions to:

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Decoupling Gravity in Sugra

Abelian gauge interaction

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Decouple Gravity

from String/F-theory EFT

[SJL—Regalado—Weigand '18]

F-theory EFT

Physics via Geometry

- **6d EFT of F-theory**
  - IIB on B_2 with varying axio-dilaton
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- **EFT via Geometry**
  - Tensor fields $\cdots\cdots\rightarrow$ $C_4 = B^\alpha \wedge w_\alpha$ with $w_\alpha \in H^{1,1}(B_2)$; $1 + n_T = h^{1,1}(B_2)$
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Revisiting the Sugra Results

Geometric interpretation via F-theory

- **Criterion for Being Dynamical in the Decoupling Limit**
  - g's $\rightarrow \infty$ with M_{Pl} fixed
  - $\mathrm{vol}_J(b\text{'s}) = 0$ with $\mathrm{vol}_J(B_2)$ fixed; b's need to be "***contractible***"

- **Geometric Intuition**
  - B_2 may have both contractable curves and noncontractable ones
  - $b_{\hat{\kappa}}$ can only be of the former type
  - b should never be contractible as U(1) is bound to become a global symmetry
  - U(1) anomaly equation gives a direct geometric clue

— Mumford's contractibility criterion:

$$\boxed{\{C_i\} \text{ contract to point(s)} \quad \Rightarrow \quad I_{ij} = C_i \cdot C_j \text{ negative (semi)definite}}$$

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Geometric Constraint in F-theory

“U(1) curves” are never contractible

- **Rudiments**
  - An elliptic Calabi-Yau 3-fold, $\pi : \hat{Y}_3 \to B_2$, as IIB/F-theory background
  - — G_κ: degenerate fibers along curves $b_\kappa \in H_2(B_2)$
  - — $U(1)$: an extra section $S \in H_4(\hat{Y}_3)$ (in addition to the zero-section S_0)
  - U(1) gauge coupling
  - — $C_3 = A_D\,[D] + \cdots$, where $[D] \in H^{1,1}(\hat{Y}_3)$
  - — S gives the A once shifted, $\sigma(s) := S - S_0 - \pi^{-1}\pi_*((S - S_0)\cdot S_0) \in H_4(\hat{Y}_3)$
  - — $\boxed{1/g^2 = \int_{\hat{Y}_3} [\sigma(s)] \wedge * [\sigma(s)] \xrightarrow{\text{F-theory limit}} \mathrm{vol}_J(-\pi_*(\sigma(s)\cdot\sigma(s)))}$

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  - Suppose the gauge group is U(1) $\Rightarrow\ b = 2\bar{K}_{B_2} + 2\pi_*(S\cdot S_0)$
  - **Claim**: $\bar{K}_{B_2}$ is non-contracible

— Any base curve with $C\cdot C \leq -3$ supports a nonabelian gauge field [Morrison-Taylor '12]

— If all base curves have self-intersection bigger than -3, then $\bar{K}_{B_2}\cdot\bar{K}_{B_2} > 0$

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Geometric Constraint in F-theory

"U(1) curves" are never contractible

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Weaken U(1)

with gravity coupling fixed

[SJL–Lerche–Regalado–Weigand '18]

Testing QG Conjectures in F-theory

WGC, SDC, …

- **(sL)WGC: in the limit where U(1) is weak**
  - Can prove, for a *general* F-theory model with U(1), that a curve in B_2 ***must*** srhink
  - D3-wrapped string is tensionless and leads to infinite light particles
  - Of the charged particles at each mass-level:
    — the maximal charge is proportional to mass
    — their counting is also given analytically
- **SDC**
  - The masses of those particles can be written in terms of the moduli space distance
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Details in the Talk by T.Weigand

See also:

[talk by I.Valenzuela], [talk by Klaewer]

Conclusions

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  - U(1) gauge fields lead to global U(1) symmetries

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