The type IIA flux potential, 4-forms and Freed-Witten anomalies

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Based on:

Herraez, Ibañez, F.M., Zoccarato
[1802.05771]

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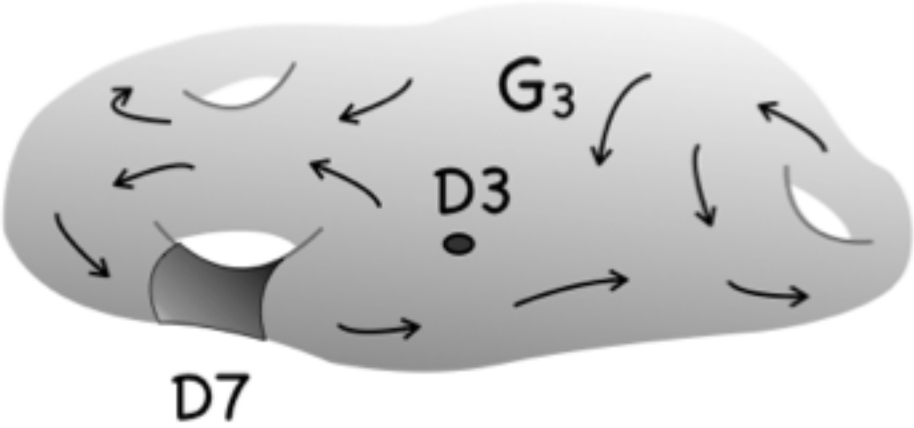
Motivation: moduli stabilisation program

- Typical example: type IIB compactified on a Calabi-Yau orientifold
- Background fluxes for F_3 and H_3 generate
  - Superpotential for moduli $W = \int_{X_6} (F_3 - \tau H_3) \wedge \Omega$ Gukov, Vafa, Witten '99
  - Warped Calabi-Yau geometry → neglected in the large volume limit

Giddings, Kachru, Polchinski '01



taken from Ibañez & Uranga '12

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$$V_F = \frac{1}{\kappa_4^2} e^K \left(K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \overline{W} - 3|W|^2 \right)$$

- Microscopic intuition for V → 10d sugra: quantised internal fluxes whose contribution to the vacuum energy depends on the CY moduli

$$S_{IIB} = -\frac{1}{4\kappa_{10}^2} \int_{X_{10}} e^{-2\phi}|H_3|^2 + |\tilde{F}_3|^2 + \ldots$$

$$\tilde{F}_3 = F_3 - C_0 H_3$$

$$\tau = C_0 + ie^{-\phi}$$

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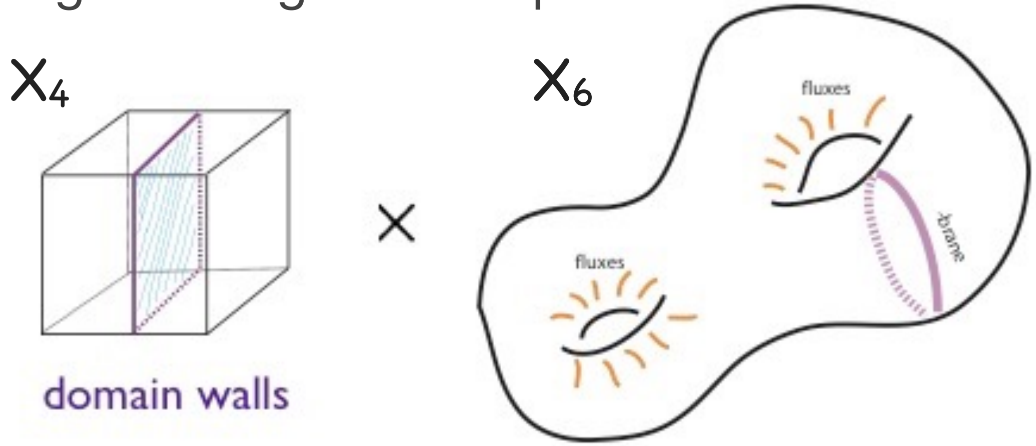
$$V_F = \frac{1}{\kappa_4^2} e^K \left(K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \overline{W} - 3|W|^2 \right)$$

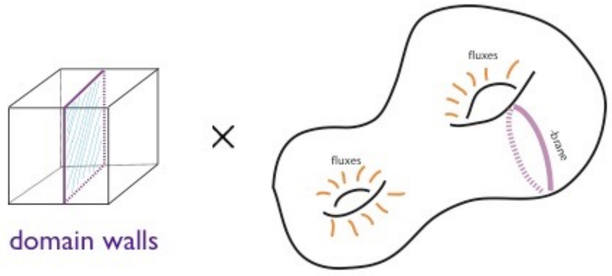
- 4d description: set of 4d Domain Walls generating the flux quanta

$$F_4^A = dC_3^A$$

4d 4-form field strength



4-forms and fluxes



- One then expects that physics behind flux compactifications is to great extent encoded in a 4d effective theory describing 4-forms

- Moreover, 4-forms appear in different proposals inspired by string theory

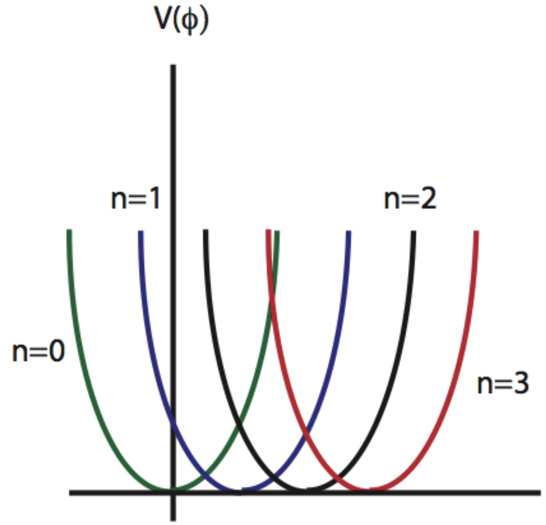
  - Bousso-Polchinski $$V_{BP} = \sum_{A,B} Z_{AB} \int_{X_4} F_4^A \star F_4^B \; + \; \Lambda_0$$

  - Kaloper-Sorbo $$\int d^4x \, |F_4|^2 + |d\phi|^2 + \phi F_4$$

  - …

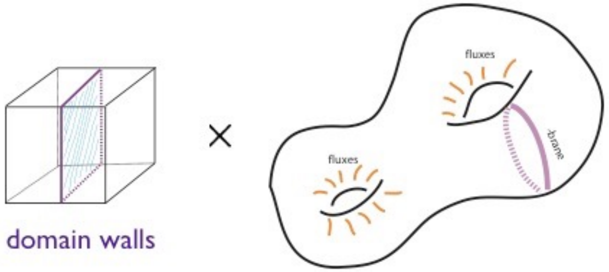
Feng et al. '00
G. Dvali et al. '04-13

$$V \; = \; \frac{1}{2}\mu^2 f_\Phi^2 (n+\phi)^2$$



multi-branched potential,
jump by crossing DW

4-forms and fluxes



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  - Kaloper-Sorbo $$\int d^4x \, |F_4|^2 + |d\phi|^2 + \phi F_4$$
  - ...

Question:

What can we learn from the structure of 4-forms in 4d string compactifications?

The type IIA potential

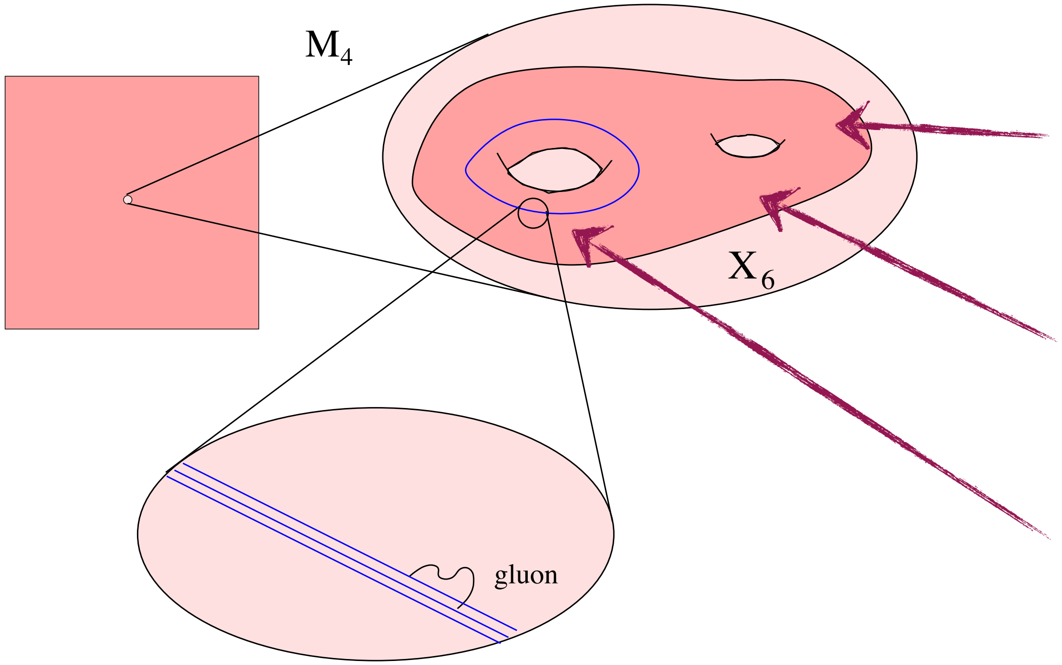
Type IIA with fluxes

- Playground: type IIA at large volume → we neglect WS and D-brane instanton effects
  - Calabi-Yau orientifold
  - D6-branes wrapping three-cycles

moduli

$$\psi^a = \phi^a + i s^a$$

axion saxion



$$b = \int_{\pi_2} B_2$$ B-field

$$c = \int_{\pi_p} C_p$$ RR potential

$$a = \int_{\pi_1} A$$ D-brane Wilson line

Type IIA with fluxes

- Playground: type IIA at large volume → we neglect WS and D-brane instanton effects
  - Calabi-Yau orientifold
  - D6-branes wrapping three-cycles
  - Internal RR fluxes F_0, F_2, F_4, F_6
  - Internal H_3 flux

moduli

$$\psi^a = \underbrace{\phi^a}_{\text{axion}} + i\underbrace{s^a}_{\text{saxion}}$$

Louis & Micu '02
Kachru & Kashani-Poor '04
Grimm & Louis '04
Grimm & Lopes '11
Kerstan & Weigand '11

→

$$\left\{ \begin{array}{l} \text{K real function of } s^a \\ \text{W holomorphic on } \psi^a \end{array} \right.$$

(4d 4-forms contribute to a part of V)

4-forms and potentials

- Alternatively → 4d Lagrangian in 4-form formalism

Bielleman, Ibanez, Valenzuela '15
Carta el al. '16

$$-\frac{1}{16}\int_{X_4} Z_{AB} F_4^A \wedge *_4 F_4^B - \frac{1}{16}\int_{X_4} Z^{AB} \varrho_A \varrho_B *_4 1 + \frac{1}{8}\int_{X_4} F_4^A \varrho_A$$

$F_4^A = dC_3^A$ runs over internal fluxes $F_4^A \leftrightarrow q_A = (\vec{q})_A \in \mathbb{Z}$

Z_{AB} depend on s^a

ϱ_A depend on ϕ^a and q_A

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Z_{AB}	depend on s^a	
ϱ_A	depend on ϕ^a and q_A	

- Integrating out the 4-forms by solving their eom: $*_4 F_4^A = Z^{AB}\varrho_B$

$$\boxed{V = \frac{Z^{AB}}{8}\varrho_A\varrho_B} \longrightarrow$$

generalisation of KS structure

$$V = \frac{1}{2}\mu^2 f_\Phi^2 (n+\phi)^2$$

4-forms and potentials

$$\boxed{V = \frac{Z^{AB}}{8} \varrho_A \varrho_B}$$

Bielleman, Ibanez, Valenzuela '15
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- Example: RR fluxes only F_0, F_2, F_4, F_6

$$\begin{tikzcd} F_0 \arrow[d, leftrightarrow] & F_2 \arrow[d, leftrightarrow] & F_4 \arrow[d, leftrightarrow] & F_6 \arrow[d, leftrightarrow] \\ m & -m^a & e_a & -e_0 \end{tikzcd}$$

- Potential generated for Kähler moduli $T^a = b^a + i\, t^a$

- Dim. red. 10d sugra democratic action → 4d 4-form for each flux

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4-forms and potentials

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  ↕ ↕ ↕ ↕

  m, $-m^a$, e_a, $-e_0$

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$$\mathbf{Z}^{-1} = 8e^K \begin{pmatrix} 4 & & & \\ & g^{ab} & & \\ & & \frac{4\mathcal{K}^2}{9} g_{ab} & \\ & & & \frac{\mathcal{K}^2}{9} \end{pmatrix}$$

$$\mathcal{K} \equiv \mathcal{K}_{abc} t^a t^b t^c \qquad \mathcal{K}_{abc} \in \mathbb{Z}$$

$$g_{ab} = \frac{3e^{\phi/2}}{2\mathcal{K}} \int_{X_6} \omega_a \wedge \star_6 \omega_b$$

$$\begin{aligned} \rho_0 &= e_0 - b^a e_a + \frac{1}{2}\mathcal{K}_{abc} m^a b^b b^c - \frac{m}{6}\mathcal{K}_{abc} b^a b^b b^c \\ \rho_a &= e_a - \mathcal{K}_{abc} m^b b^c + \frac{m}{2}\mathcal{K}_{abc} b^b b^c \\ \tilde{\rho}^a &= m^a - m b^a \\ \tilde{\rho} &= m \end{aligned}$$

saxions t^a

axions b^a and fluxes

4-forms and potentials

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- Example: RR fluxes only F_0, F_2, F_4, F_6

$$\begin{array}{cccc} F_0, & F_2, & F_4, & F_6 \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ m, & -m^a, & e_a, & -e_0 \end{array}$$

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$\mathcal{K} \equiv \mathcal{K}_{abc} t^a t^b t^c$ $\qquad$ $\mathcal{K}_{abc} \in \mathbb{Z}$

$$\Big\downarrow$$

$$V_{\rm RR} = e^K \left[4\rho_0^2 + g^{ab}\rho_a \rho_b + \frac{4}{9}\mathcal{K}^2 g_{ab} \tilde{\rho}^a \tilde{\rho}^b + \frac{1}{9}\mathcal{K}^2 \tilde{\rho}^2 \right]$$

4-forms and potentials

$$\boxed{V = \frac{Z^{AB}}{8} \varrho_A \varrho_B}$$

- One can rotate the basis of 4-forms to remove the axion dependence from ϱ

$$F_4'^{\,A} = R^A_B F^B_4 \quad \longrightarrow \quad \vec{\varrho} = \begin{pmatrix} \rho_0 \\ \rho_a \\ \tilde{\rho}^a \\ \tilde{\rho} \end{pmatrix} \to \begin{pmatrix} e_0 \\ e_a \\ m^a \\ m \end{pmatrix} = \vec{q}$$

$$\mathbf{R} = \begin{pmatrix} 1 & -b^a & \frac{1}{2}\mathcal{K}_{abc} b^b b^c & -\frac{1}{3!}\mathcal{K}_{abc} b^a b^b b^c \\ 0 & \delta^a_b & -\mathcal{K}_{abc} b^c & \frac{1}{2}\mathcal{K}_{abc} b^a b^c \\ 0 & 0 & \delta^b_a & -b^b \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{Z}^{-1} \to \mathbf{R}^t \mathbf{Z}^{-1} \mathbf{R}$$

4-forms and potentials

$$\boxed{V=\frac{Z^{AB}}{8}\varrho_A\varrho_B}$$

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$$\mathbf{Z}^{-1}\ \to\ \mathbf{R}^t\mathbf{Z}^{-1}\mathbf{R}$$

- In this basis the ϱ's are quantised fluxes and arise as integration constants

$$-\frac{1}{16}\int_{X_4} Z'_{AB}F_4'^{A}\wedge *_4F_4'^{B}$$

Louis & Micu '02

Farakos et al. '17

- The potential displays a triple factorisation:

$$V=\frac{1}{8}\,\vec{q}^{\,t}\,\mathbf{R}^t\underset{\text{saxions}}{\mathbf{Z}^{-1}}\,\underset{\text{axions}}{\mathbf{R}}\,\underset{\text{fluxes}}{\vec{q}}$$

Recap

$$\boxed{V = \frac{Z^{AB}}{8}\varrho_A\varrho_B}$$

- We have found a potential of the form

$$V = \frac{1}{8}\,\vec{q}^{\,t}\,\mathbf{R}^t\underset{\substack{\nearrow\\ \text{saxions}}}{\mathbf{Z}^{-1}}\,\underset{\substack{\uparrow\\ \text{axions}}}{\mathbf{R}}\,\underset{\substack{\nwarrow\\ \text{fluxes}}}{\vec{q}}$$

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- That V is bilinear on the fluxes is not so surprising, since we are reducing a 10d two-derivative action

- The factorisation between axions and saxions is more remarkable, and it is related to the fact that q$_A$ are quantised but non-gauge invariant fluxes, while

$$\vec{\rho} = \mathbf{R}\,\vec{q}$$

correspond to gauge invariant, non-quantised fluxes

$$\mathbf{G} = d\mathbf{C} - H\wedge\mathbf{C} + \underset{\vec{\rho}}{\boxed{\underset{\vec{q}}{\bar{\mathbf{G}}}\wedge e^B}}$$

Marolf '00

Recap

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saxions axions fluxes

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- In 4d, the entries ρ_A are axion polynomials invariant under a discrete shift symmetry. This protection makes them the basic building blocks of V.

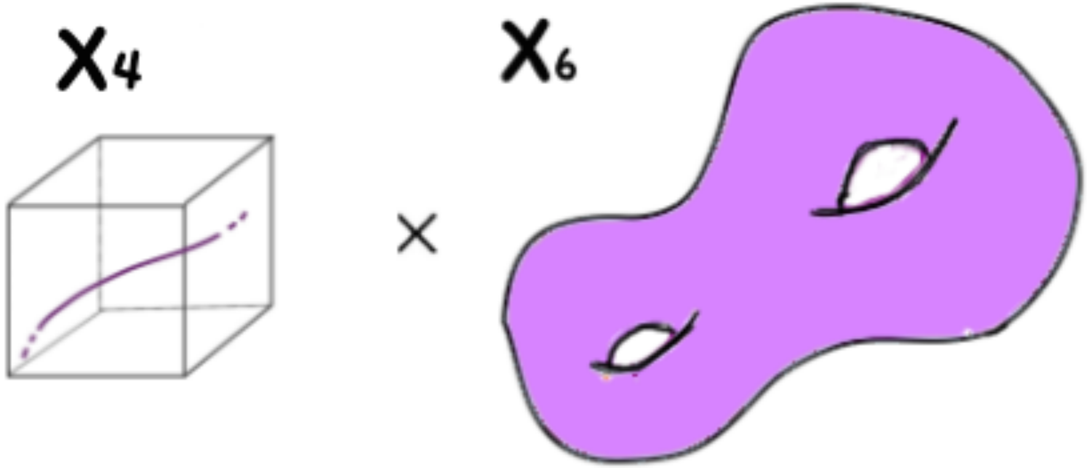
- Things become more transparent once we understand the matrix **R**…

Understanding R

R and Freed-Witten anomalies

- Microscopically, R is determined by the discrete data of the compactification. More precisely it is determined by the Freed-Witten anomalies of 4d strings

- Type IIB example: D7-brane wrapping X_6 → 4d string



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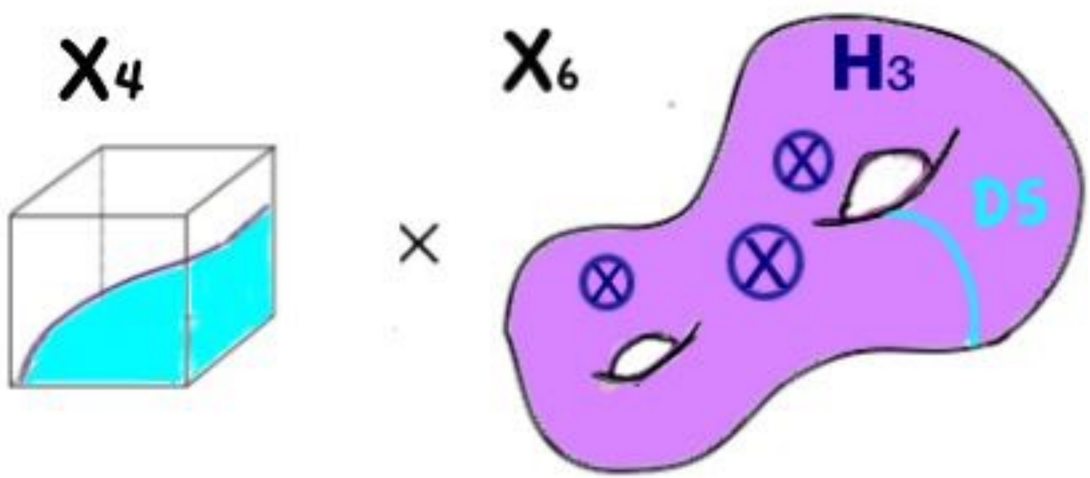

H_3 fluxes create a FW anomaly, cured by D5-branes wrapping P.D. 3-cycle

↓

4d DW ending on 4d string

Martucci & Evslin '07

Berasaluce-Gonzalez et al. '12

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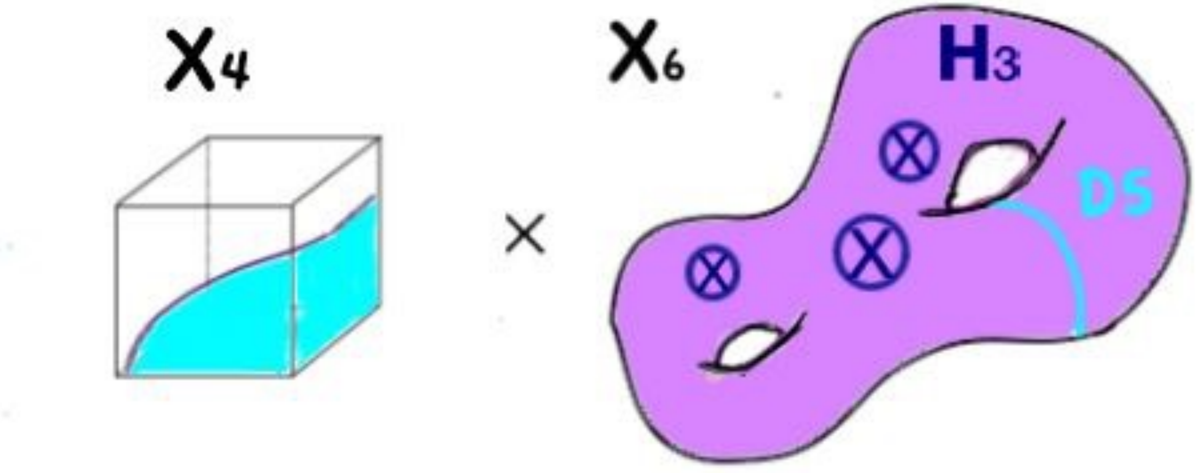

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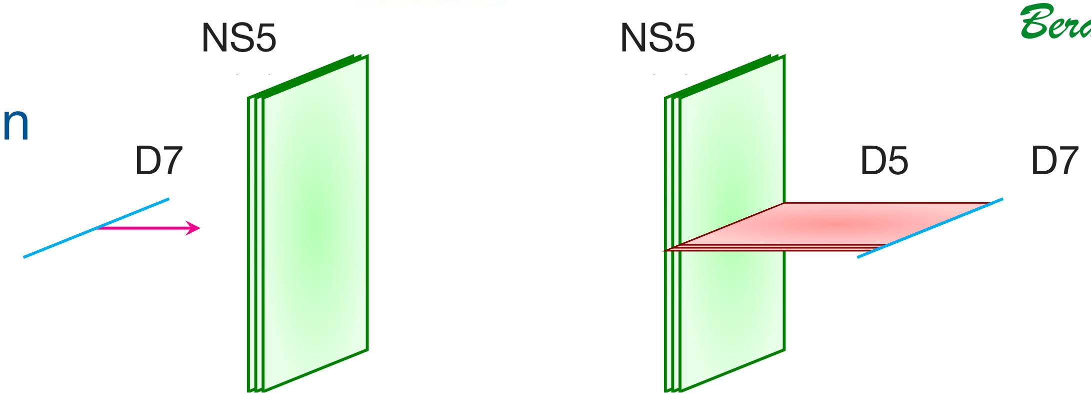
- Alternatively: Hanany-Witten effect



R and Freed-Witten anomalies

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- Macroscopically: C_0 axion lifted by a potential generated by H_3

$$W = \int_{X_6} (F_3 - \tau H_3) \wedge \Omega$$

$$\tau = C_0 + i e^{-\phi}$$

V depends on $\tilde{F}_3 = F_3 - C_0 H_3$

cures FW anomaly

axion coupled to D7-brane

creates FW anomaly

$$\downarrow$$

$$V \sim G^{ab}(f - c_0 h)_a (f - c_0 h)_b$$

FW anomalies in type IIA

Berasaluce-Gonzalez et al. '12

- In type IIA compactifications with RR fluxes, FW anomalies are developed by NS5-branes on 4-cycles:

String		**Flux**	**Domain wall**		**Rank**
type	cycle	type	type	cycle	
NS5	$[\pi_4^a] \in H_4(\mathcal{M}_6, \mathbb{Z})$	$F_4 = e_b \tilde{\omega}^b$	D2	$-$	$\int_{\pi_4^a} F_4 = e_b$
NS5	$[\pi_4^a] \in H_4(\mathcal{M}_6, \mathbb{Z})$	$F_2 = m^b \omega_b$	D4	$\pi_2 \in \text{P.D.}[F_2 \wedge \omega_a]$	$\int_{\pi_2} \omega_c = \mathcal{K}_{abc} m^b$
NS5	$[\pi_4^a] \in H_4(\mathcal{M}_6, \mathbb{Z})$	$F_0 = m$	D6	$[\pi_4^a]$	m

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For instance: F_0 creates FW anomaly cancelled by D6 DW (~ F_2)

$$V \sim \int_{X_6} |F_2 + BF_0|^2 \sim g_{ab}(m^a + mb^a)(m^b + mb^b)$$

$\tilde{\rho}^b$

cures FW anomaly

axion coupled to NS5-brane

creates FW anomaly

FW anomalies in type IIA

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NS5	$[\pi_4^a] \in H_4(\mathcal{M}_6, \mathbb{Z})$	$F_0 = m$	D6	$[\pi_4^a]$	m

Can be encoded in a matrix:

creating FW anomaly

$$P_a = \begin{array}{c} \begin{matrix} F_6 & F_4 & F_2 & F_0 \end{matrix} \\ \begin{pmatrix} 0 & \vec{\delta}_a^{\,t} & 0 & 0 \\ 0 & 0 & \mathcal{K}_{abc} & 0 \\ 0 & 0 & 0 & \vec{\delta}_a \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

nilpotent and mutually commuting

F_6 ~ $D2_{DW}$
F_4 ~ $D4_{DW}$
F_2 ~ $D6_{DW}$
F_0 ~ $D8_{DW}$

curing FW anomaly

FW anomalies in type IIA

- In type IIA compactifications with RR fluxes, FW anomalies are developed by NS5-branes on 4-cycles:

$$P_a = \begin{pmatrix} 0 & \vec{\delta}_a^{\,t} & 0 & 0 \\ 0 & 0 & \mathcal{K}_{abc} & 0 \\ 0 & 0 & 0 & \vec{\delta}_a \\ 0 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \quad \mathbf{R} = e^{-b^a P_a} = \begin{pmatrix} 1 & -b^a & \frac{1}{2}\mathcal{K}_{abc}b^b b^c & -\frac{1}{3!}\mathcal{K}_{abc}b^a b^b b^c \\ 0 & \delta^a_b & -\mathcal{K}_{abc}b^c & \frac{1}{2}\mathcal{K}_{abc}b^a b^c \\ 0 & 0 & \delta^b_a & -b^b \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

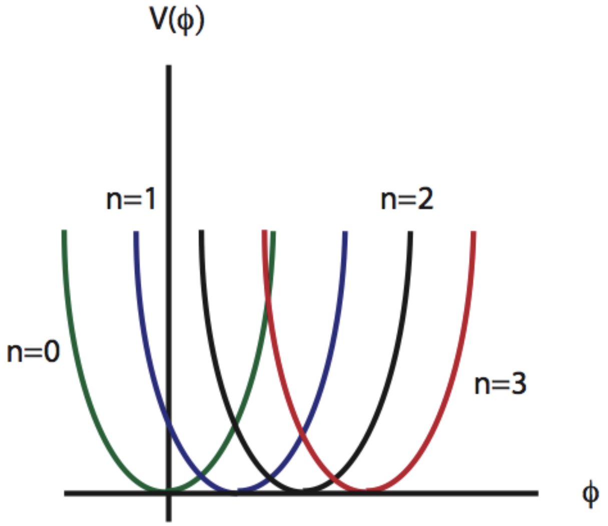
- Alternatively: **R** describes the internal DW charges induced by the B-field

→ Charge monodromy matrix

$$\begin{pmatrix} \rho_0 \\ \rho_a \\ \tilde{\rho}^a \\ \tilde{\rho} \end{pmatrix} = \begin{pmatrix} 1 & -b^a & \frac{1}{2}\mathcal{K}_{abc}b^b b^c & -\frac{1}{3!}\mathcal{K}_{abc}b^a b^b b^c \\ 0 & \delta^a_b & -\mathcal{K}_{abc}b^c & \frac{1}{2}\mathcal{K}_{abc}b^a b^c \\ 0 & 0 & \delta^b_a & -b^b \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_0 \\ e_a \\ m^a \\ m \end{pmatrix} \begin{array}{l} \mathrm{D2_{DW}} \\ \mathrm{D4_{DW}} \\ \mathrm{D6_{DW}} \\ \mathrm{D8_{DW}} \end{array}$$

R and discrete shift symmetries

- We have that
$$\vec{\rho} = \mathbf{R}\,\vec{q} = e^{-\phi^a P_a}\,\vec{q}$$
- Axions of unit period
$$\phi^a \sim \phi^a + n^a, \quad n^a \in \mathbb{Z}$$
$$\mathbf{R}(\vec{\phi} + \vec{n}) = \mathbf{R}(\vec{\phi}) \cdot e^{-n^a P_a}$$
- Compensated by a shift of flux quanta so that ρ remains invariant
$$\vec{q} \longrightarrow \vec{q}\,' = e^{n^a P_a}\vec{q}$$
   - Generalisation of the discrete symmetry in KS
   - Multi-branched scalar potential



ρ and the superpotential

- Since the discrete shift symmetry is of gauge nature, the superpotential W is also invariant under it

- In fact W and ρ$_A$ contain the same information (in the large volume limit):

Gukov '99

Taylor & Vafa '99

$$W = \Pi^{\,t}(\psi)\cdot\vec{q}$$

$$\downarrow$$

In our case:

$$W = e_0 - e_a T^a + \frac{1}{2}\mathcal{K}_{abc}m^aT^bT^c - m\frac{1}{6}\mathcal{K}_{abc}T^aT^bT^c$$

$$\Pi^{\,t} = (1, -T^a, \frac{1}{2}\mathcal{K}_{abc}T^aT^b, -\frac{1}{6}\mathcal{K}_{abc}T^aT^bT^c)$$

$$W = [\mathbf{R}(\phi)^{t\,-1}\Pi(\psi)]^{\,t}\cdot\vec{\rho} = [\Pi(s)]^{\,t}\cdot\vec{\rho} = e^{is^a\frac{\partial}{\partial\phi^a}}\left[\Pi^{\,t}(0)\cdot\vec{\rho}\right] \equiv e^{is^a\frac{\partial}{\partial\phi^a}}\rho_0$$

inv. & flux independent
no exponential dep. in ϕ

holomorphicity of W

$$\rho_0 = e_0 - b^ae_a + \frac{1}{2}\mathcal{K}_{abc}m^ab^bb^c - \frac{m}{6}\mathcal{K}_{abc}b^ab^bb^c$$

Recap

$$V = \frac{1}{8}\,\vec{q}^{\,t}\,\mathbf{R}^t\underset{\substack{\nearrow\\ \text{saxions}}}{\mathbf{Z}^{-1}}\,\underset{\substack{\uparrow\\ \text{axions}}}{\mathbf{R}}\,\underset{\substack{\nwarrow\\ \text{fluxes}}}{\vec{q}}$$

- The axion-dependent matrix **R** defining the invariants ρ_A is specified by discrete, topological data of the compactification. In particular by the FW anomalies developed by 4d string defects.

- In particular, each 4d string corresponds to an integer valued, nilpotent matrix P_a that acts on fluxes and axions and leaves the ρ_A invariant

$$\boxed{\mathbf{R} = e^{-\phi^a P_a}}$$

See also I. Valenzuela's talk

- This generalises the discrete shift symmetry and multi-branched structure of the KS potential. The corrections to V must be functions of ρ_A and not of V. In principle there could be a correction of the form

$$\kappa^{ABC}\rho_A\rho_B\rho_C$$

- All this information is gathered in a master polynomial $\rho_0 = W|_{s=0}$

- All these results hold when we add NS-fluxes and D6-branes…

Generalisations

Adding H_3 flux

$$V = \frac{Z^{AB}}{8}\varrho_A\varrho_B$$

- New 4d strings get FW anomalies (D4's) ↔ new axions enter V : $\xi^K = \int_{\Lambda_k} C_3$
- **Z** larger and no longer definite positive

$$V = V_{\mathrm{RR}} + V_{\mathrm{NS}} + V_{\mathrm{loc}}$$

$$\sum_\alpha [\Pi_\alpha] - m[\Pi_H] = 4[\Pi_{\mathrm{O6}}]$$

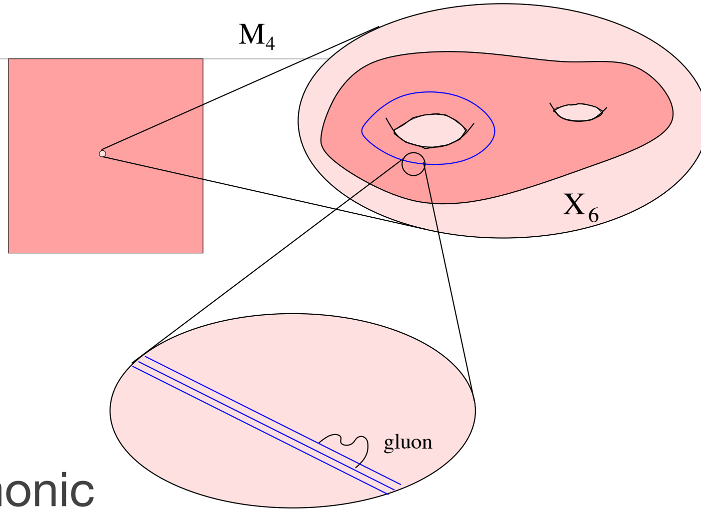
D6-brane deficit

$$\mathbf{Z}^{-1} = 8\,e^K \begin{pmatrix} 4 & & & & \\ & g^{ab} & & & \\ & & \frac{4}{9}\mathcal{K}^2 g_{ab} & & \\ & & & \frac{1}{9}\mathcal{K}^2 & \frac{2}{3}\mathcal{K}n'^I \\ & & & \frac{2}{3}\mathcal{K}n'^J & 4c^{IJ} \end{pmatrix}$$

$$c_{IJ} \sim \int_{X_6} \alpha_I \wedge \star_6 \alpha_J$$

→ Allows to obtain AdS_4 vacua

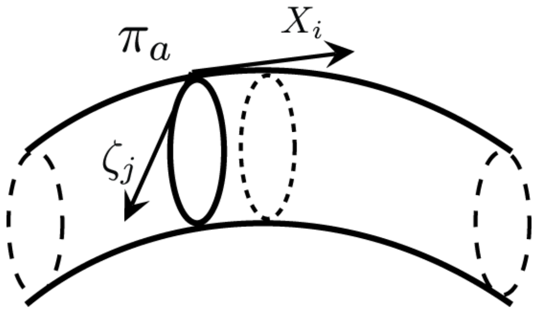
Adding D6-branes



- New fluxes: n_F, $n_a{}^i$
- New fields
  - 3-cycle deformations X
  - Wilson lines A
- Both classes of moduli are counted by the number of harmonic 1-forms in Π_3. We then have $b_1(\Pi_3)$ complex moduli

$$\Phi = \int_{\Sigma_2} J_c - \int_{\pi_1} A = T^a f_a - \theta$$

$$J_c = B + iJ = T^a \omega_a$$

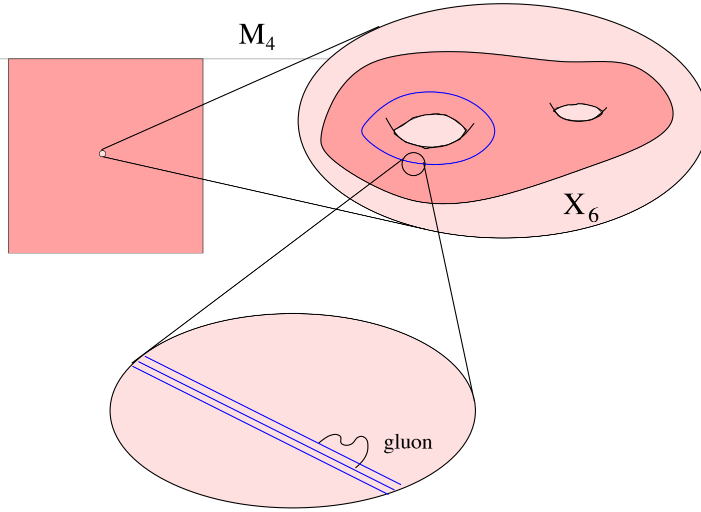


- For each complex modulus/harmonic 1-form we have one non-trivial 2-cycle in Π_3, on which we can switch on a flux n_F. Moreover, such 2-cycle may be trivial or non-trivial in the compactification six-manifold → flux $n_a{}^i$

F.M., Regalado, Zoccarato '14
Carta et al. '16

$$W = e_0 - e_a T^a + \frac{1}{2}\mathcal{K}_{abc} m^a T^b T^c - m \frac{1}{6}\mathcal{K}_{abc} T^a T^b T^c - h_K N^K - \boxed{\Phi^i (n_{F\,i} - n_{a\,i} T^a)}$$

Adding D6-branes



- The structure of the potential is

$$V = V_{\rm RR+CS} + V_{\rm NS} + V_{\rm DBI} + V_{\rm loc}$$

which again can be rewritten as

$$V = \frac{1}{8}\,\vec{q}^{\,t}\,\mathbf{R}^t\mathbf{Z}^{-1}\,\mathbf{R}\,\vec{q}$$

saxions axions fluxes

- **R** depends on the 4d axions $b^a\,,\quad \hat{\theta}^i = b^a f_a^i - \theta^i\,,\quad \xi^K = \mathrm{Re}\,N^K = \xi'^K - (\dots)$
- **Z**$^{-1}$ not always invertible

Metric fluxes

- One can also consider type IIA with RR, NS and metric fluxes

- Example worked out: **Z**$_2$ x **Z**$_2$ orientifold

- Everything works the same $V = \frac{1}{8}\,\vec{q}^{\,t}\,\mathbf{R}^t\mathbf{Z}^{-1}\,\mathbf{R}\,\vec{q}$ with q_A now containing the metric fluxes

  saxions ↗ axions ↑ ↖ fluxes

- **Z**$^{-1}$ obtained from standard N=1 formula is a priori not invertible: only when the Bianchi identities for the fluxes are imposed

  - $V_{\text{4d SUGRA}}$ matches V
  - **Z**$^{-1}$ is invertible (needed for 4-form formalism)

Villadoro & Zwirner '05

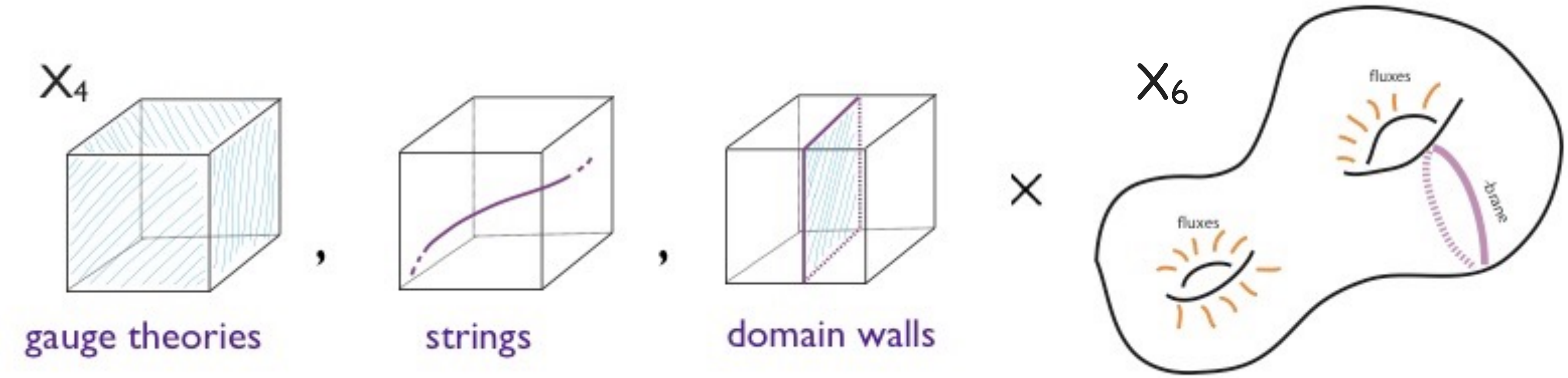
↓

Criterion for consistency

Conclusions

$$\boxed{V = \frac{Z^{AB}}{8}\varrho_A\varrho_B}$$

- We have computed the classical scalar potential for type IIA CY orientifolds in the presence of RR+NS fluxes and D6-branes
- The CY condition is not essential, but it gives us the spectrum of moduli, internal fluxes and 4d defects



Conclusions

$$\boxed{V = \frac{Z^{AB}}{8}\varrho_A\varrho_B}$$

- We have computed the classical scalar potential for type IIA CY orientifolds in the presence of RR+NS fluxes and D6-branes
- The CY condition is not essential, but it gives us the spectrum of moduli, internal fluxes and 4d defects
- We have found a bilinear structure and a triple factorisation between saxions, unit period axions and quantised fluxes

$$V = \frac{1}{8}\vec{q}^{\,t}\,\mathbf{R}^t\mathbf{Z}^{-1}\,\mathbf{R}\,\vec{q}$$

saxions axions fluxes

- Microscopically, this comes from gauge invariance (gauge inv. fluxes, FW, HW)
- Macroscopically, this translates into a discrete shift symmetry that relates different branches of the scalar potential V, and defines the invariants ρ_A that any flux-dependent quantity must defend on, even after UV corrections
- To connect with the 4-form formalism $\mathbf{Z}^{-1}$ must be invertible. This seems to be related to the consistency conditions between different fluxes.

Instituto de Física Teórica UAM-CSIC presents:

Vistas over the Swampland

Madrid, 19-21 September 2018

https://workshops.ift.uam-csic.es/swampland

Swamp lookouts

- N. Arkani-Hamed (IAS - Princeton)
- T. Banks (Santa Cruz & Rutgers U.)
- R. Blumenhagen (MPI - Munich)
- T. Crisford (DAMTP - Cambridge)
- U. Danielsson (Uppsala U.)
- A. Hebecker (Heidelberg U.)
- M. Kleban (New York U.)
- D. Lüst (LMU & MPI - Munich)
- M. Montero (ITP - Utrecht)
- E. Palti (MPI - Munich)
- M. Reece (Harvard U.)
- G. Remmen (UC - Berkeley)
- T. Rudelius (IAS - Princeton)
- G. Shiu (UW - Madison)
- P. Soler (Heidelberg U.)
- C. Vafa (Harvard U.)
- I. Valenzuela (ITP - Utrecht)
- T. Van Riet (KU Leuven)

Swamp rangers

L. E. Ibáñez
F. Marchesano
A. M. Uranga

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