A systematic mass insertion expansion for lepton violating decays in the MSSM

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A. Crivellin, ZF, W. Materkowska, U. Nierste, S. Pokorski & J. Rosiek
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Motivation

**PHYSICS**

- FCNC processes involving leptons are strictly forbidden in the SM ($m_\nu = 0$).
- LFV may serve as clue towards New Physics.
Motivation

PHYSICS

- FCNC processes involving leptons are strictly forbidden in the SM ($m_\nu = 0$).
- LFV may serve as clue towards New Physics.

MATHS

- Use tools that have not been available earlier.
- First systematic discussion of the **mass insertion approximation**.
- Recover the dependency on the Lagrangian parameters.
Upper bounds on LFV processes

<table>
<thead>
<tr>
<th>Decay</th>
<th>Experimental upper bound</th>
<th>CL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau \to e\gamma$</td>
<td>$3.3 \times 10^{-8}$</td>
<td>90%</td>
</tr>
<tr>
<td>$\tau \to \mu\gamma$</td>
<td>$4.4 \times 10^{-8}$</td>
<td>90%</td>
</tr>
<tr>
<td>$\mu \to e\gamma$</td>
<td>$5.7 \times 10^{-13}$</td>
<td>90%</td>
</tr>
<tr>
<td>$Z \to \mu e$</td>
<td>$7.5 \times 10^{-7}$</td>
<td>95%</td>
</tr>
<tr>
<td>$Z \to \mu \tau$</td>
<td>$1.2 \times 10^{-5}$</td>
<td>95%</td>
</tr>
<tr>
<td>$Z \to \tau e$</td>
<td>$9.8 \times 10^{-6}$</td>
<td>95%</td>
</tr>
<tr>
<td>$\mu \to e^- e^+ e^-$</td>
<td>$1.0 \times 10^{-12}$</td>
<td>90%</td>
</tr>
<tr>
<td>$\tau \to e^- e^+ e^-$</td>
<td>$2.7 \times 10^{-8}$</td>
<td>90%</td>
</tr>
<tr>
<td>...</td>
<td>$\sim 10^{-8}$</td>
<td></td>
</tr>
<tr>
<td>$H \to e\tau$</td>
<td>$6.1 \times 10^{-3}$</td>
<td>90%</td>
</tr>
<tr>
<td>$H \to \mu\tau$</td>
<td>$2.5 \times 10^{-3}$</td>
<td>90%</td>
</tr>
<tr>
<td>$H \to \mu e$</td>
<td>$3.6 \times 10^{-4}$</td>
<td>90%</td>
</tr>
</tbody>
</table>

Future sensitivity will improve $\rightarrow$ important to have the tools for fast, precise calculations.
Lepton flavor violating processes

Use the correlations between different processes to highlight new experimental opportunities for LFV searches.
Mass insertions
Off-diagonal elements, both flavor violating and flavor conserving, of the mass matrices of SUSY particles.

\[
\Delta_{LL}^{IJ} = \frac{(M_{LL}^{2})^{IJ}}{\sqrt{(M_{LL}^{2})^{II}(M_{LL}^{2})^{JJ}}}
\]

\[
\Delta_{RR}^{IJ} = \frac{(M_{RR}^{2})^{IJ}}{\sqrt{(M_{RR}^{2})^{II}(M_{RR}^{2})^{JJ}}}
\]

\[
\Delta_{LR}^{IJ} = \frac{A_{I}^{IJ}}{((M_{LL}^{2})^{II}(M_{RR}^{2})^{JJ})^{1/4}}
\]

\[
\Delta_{LR}^{IJ} = \frac{A_{I}^{IJ}}{((M_{LL}^{2})^{II}(M_{RR}^{2})^{JJ})^{1/4}}\]  

(1)

\[M_{LL}^{2}, M_{RR}^{2}\] - slepton soft mass matrices
\[A_{I}, A_{I}^{'}\] - trilinear terms.

Write down the amplitude in terms of penguin Wilson coefficients

\[
F^{IJ} = \frac{1}{(4\pi)^{2}} \left( F_{LL}^{IJ}\Delta_{LL}^{IJ} + F_{RR}^{IJ}\Delta_{RR}^{IJ} + F_{ALR}^{IJ}\Delta_{LR}^{IJ} + F_{BLR}^{IJ}\Delta_{LR}^{IJ} + F_{ALR}^{IJ}\Delta_{LR}^{IJ} + F_{BLR}^{IJ}\Delta_{LR}^{IJ} \right) .  
\]  

(2)
Our method

1. Expand the amplitude in flavor violating off-diagonal slepton mass insertions ($\Delta$'s) performed in the first order in $\Delta$'s.

2. Expand of $\Delta$’s coefficients in flavor conserving off-diagonal terms of all SUSY particles
   → sleptons (so-called $A$ terms)
   → gauginos

We can do this using the Flavor Expansion Theorem (FET) and the newly-developed Mathematica MassToMI package

The use of the symbolic package allows us to:

- perform the required 3rd order MI expansion in a fully automatized way.
- we do not need to make any assumptions upon degeneracy or hierarchy between SUSY particles (contrary to other analyses).
- obtain cancellations between different terms → the result is more compact.
- include terms scaling down with SUSY mass scale like $\nu^2/M^2$ or slower ($M$ - SUSY mass parameters i.e. $M_1, M_2, \mu$, diagonal soft slepton masses).
To compare our expansion we also performed calculations in the mass-eigenstates basis.

$F_{MI}$ - mass-insertion formfactor

$F_{ME}$ - mass-eigenstates formfactor

**An example: non-degenerated SUSY spectrum**

**Transition between the 2nd and 3rd generation**

**Initial setup** ([m] = GeV):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tan \beta$</td>
<td>5</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$200 + 100i$</td>
</tr>
<tr>
<td>$M_1$</td>
<td>150</td>
</tr>
<tr>
<td>$M_2$</td>
<td>300</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
A_{\mu\mu} &= A'_{\mu\mu} = 0.1 \sqrt{m_{\tilde{\mu}_L} m_{\tilde{\mu}_R}} \\
A_{\tau\tau} &= A'_{\tau\tau} = 0.1 \sqrt{m_{\tilde{\tau}_L} m_{\tilde{\tau}_R}} \\
\end{align*}
\]

(3)

Scale up to 2 TeV. Plot

\[
\Delta F = \left| \frac{F_{MI}}{F_{ME}} \right| - 1
\]

(4)

as a function of the average slepton mass.
Accuracy of MI expansion for penguin amplitudes.

L, R: effective couplings, non-degenerated SUSY spectrum (3) for both ME and MI expressions

L, R: ME expressions (3), but universal degenerated spectrum for MI expressions

\[ M - \text{average SUSY mass scale}, \ (M = M_2 = m_{\tilde{\mu}_L} = m_{\tilde{\mu}_R}) \]
Results: an example

Bounds on the mass insertions from LFV for SUSY scale $M = 400$ GeV and $\tan \beta = 2$

<table>
<thead>
<tr>
<th>Process</th>
<th>$\Delta_{IJ}^{LL}$</th>
<th>$\Delta_{IJ}^{RR}$</th>
<th>$\Delta_{IJ}^{LR}$</th>
<th>$\Delta_{IJ}^{RL}$</th>
<th>$\Delta_{IJ}^{LL'}$</th>
<th>$\Delta_{IJ}^{RR'}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau \to \mu \gamma$</td>
<td>$5.3 \cdot 10^{-1}$</td>
<td>$3.3 \cdot 10^{+0}$</td>
<td>$9.1 \cdot 10^{-2}$</td>
<td>$9.1 \cdot 10^{-2}$</td>
<td>$4.5 \cdot 10^{-2}$</td>
<td>$4.5 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>$\tau \to \mu \mu \mu$</td>
<td>$7.5 \cdot 10^{+0}$</td>
<td>$4.0 \cdot 10^{+1}$</td>
<td>$1.3 \cdot 10^{+0}$</td>
<td>$1.3 \cdot 10^{+0}$</td>
<td>$6.4 \cdot 10^{-1}$</td>
<td>$6.4 \cdot 10^{-1}$</td>
</tr>
<tr>
<td>$\tau \to \mu e^+ e^-$</td>
<td>$6.7 \cdot 10^{+0}$</td>
<td>$3.6 \cdot 10^{+1}$</td>
<td>$1.1 \cdot 10^{+0}$</td>
<td>$1.1 \cdot 10^{+0}$</td>
<td>$5.6 \cdot 10^{-1}$</td>
<td>$5.6 \cdot 10^{-1}$</td>
</tr>
<tr>
<td>$Z^0 \to \tau \mu$</td>
<td>$1.3 \cdot 10^{+3}$</td>
<td>$2.0 \cdot 10^{+5}$</td>
<td>$1.7 \cdot 10^{+4}$</td>
<td>$1.6 \cdot 10^{+4}$</td>
<td>$4.3 \cdot 10^{+3}$</td>
<td>$3.9 \cdot 10^{+3}$</td>
</tr>
<tr>
<td>$h \to \tau \mu$</td>
<td>$1.9 \cdot 10^{+2}$</td>
<td>$9.0 \cdot 10^{+2}$</td>
<td>$1.3 \cdot 10^{+3}$</td>
<td>$1.4 \cdot 10^{+3}$</td>
<td>$6.0 \cdot 10^{+0}$</td>
<td>$6.0 \cdot 10^{+0}$</td>
</tr>
</tbody>
</table>

$l \to l' \gamma$ - strongest bounds.
Non-decoupling effects in Higgs decays


Terms $\sim$ lepton Yukawa couplings $Y_l$ or to the non-holomorphic trilinear slepton soft terms $\sim A_l'$ do not decouple in the limit of heavy SUSY masses and can be potentially large.

$\rightarrow$ Related to the 2HDM structure of the MSSM.

For large SUSY masses and small mixing angles $\alpha, \beta$ the Higgs decay is more constraining than the $\mu \rightarrow e\gamma$. 
Conclusions

1. Newly developed calculation tools used to obtain a full expansion of the amplitudes in terms of mass insertions.

2. Our expansion is completely systematic in powers of $\nu^2/M^2$ and $\nu^2/M_A^2$.

3. We can observe the cancellations between different lower-order terms.

4. Also quantitatively more can be understood than using ME basis.

5. We observed the non-decoupling effects in Higgs decays, where the maximal $BR(h \rightarrow ll') \sim O(10^{-4})$, not much lower than the current experimental sensitivities.

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Sources of flavor violation in the MSSM

1. Slepton mass matrix

The slepton and sneutrino mass and mixing matrices are defined as:

\[
Z_L^\dagger \left( \begin{array}{cc}
(M_L^2)_{LL} & (M_L^2)_{LR} \\
(M_R^2)_{LR} & (M_R^2)_{RR}
\end{array} \right) Z_L = \text{diag} \left( m_{L_1}^2, \ldots, m_{L_6}^2 \right)
\]

(5)

\[
(M_L^2)_{LL} = (M_{LL}^2)^T + \frac{M_Z^2 \cos 2\beta}{2} (1 - 2c_W^2) \hat{1} + \frac{v_1^2 Y_l^2}{2}
\]

(6)

\[
(M_L^2)_{RR} = M_{RR}^2 - \frac{M_Z^2 \cos 2\beta}{2} s_W^2 \hat{1} + \frac{v_1^2 Y_l^2}{2}
\]

(7)

\[
(M_L^2)_{LR} = \frac{1}{\sqrt{2}} \left( v_2 (Y_l \mu^* - A'_l) + v_1 A_l \right)
\]

(8)

where $M_{LL}^2$, $M_{RR}^2$, $A_l$, $A'_l$ and $Y_l = -\sqrt{2} m_l/v_1$ are 3 × 3 matrices in flavor space.
2. Gaugino mass matrices

The neutralino and chargino mass and mixing matrices can be written down as:

\[ Z_N^T \begin{pmatrix} M_1 & 0 & -\frac{ev_1}{2c_W} & \frac{ev_2}{2c_W} \\ 0 & M_2 & \frac{ev_1}{2s_W} & \frac{ev_2}{2s_W} \\ -\frac{ev_1}{2c_W} & \frac{ev_2}{2s_W} & -\mu \\ 0 & -\mu & 0 \end{pmatrix} Z_N = \operatorname{diag}(m_{\chi_1^0}, \ldots, m_{\chi_4^0}) \quad (9) \]

\[ (Z_-)^T \begin{pmatrix} M_2 & \frac{ev_2}{\sqrt{2s_W}} \\ \frac{ev_1}{\sqrt{2s_W}} & \mu \end{pmatrix} Z_+ = \operatorname{diag}(m_{\chi_1}, m_{\chi_2}) \quad (10) \]
3. Higgs-slepton-slepton vertex contains the $A_I$, $A'_I$ terms

What are the non-holomorphic $A'_I$ terms?

$$\mathcal{L} \sim A'_I^{IJ} H_i^{2*} L_i^I R^J + A'_d^{IJ} H_i^{2*} Q_i^I D^J + A'_u^{IJ} H_i^{1*} Q_i^I U^J + H.c.$$ (11)

- Trilinear couplings of the scalar fields
- different in the form from the Yukawa terms in the superpotential
Divided differences

\[ f^{[0]}(x) = f(x) \]
\[ f^{[1]}(x, y) = \frac{f^{[0]}(x) - f^{[0]}(y)}{x - y} \]
\[ f^{[2]}(x, y, z) = \frac{f^{[1]}(x, y) - f^{[1]}(x, z)}{y - z} \]

\[ \ldots \quad (12) \]

symmetric under permutation of any of its \( n \) arguments

\[ f^{[k]}(x_0, \ldots, x_k) \equiv f(\{x_0, \ldots, x_k\}) \quad (13) \]

\[ g(\{x_1, x_2\}, \{y_1, y_2, y_3, y_4\}, z) \quad (14) \]
Divided difference of \( n \)-point function is a \((n+1)\)-point function

\[
B_0(m_1, \{m_2, m_3\}) = B_0(\{m_1, m_2\}, m_3) = C_0(m_1, m_2, m_3)
\]
\[
B_0(m_1, \{m_2, m_3, m_4\}) = C_0(m_1, m_2, \{m_3, m_4\}) = D_0(m_1, m_2, m_3, m_4)
\]

\[
\ldots
\]

We can now find cancellations between different terms.
→ Identify the lowest non-vanishing order of the mass insertion for every process.