DAI-FREED ANOMALIES IN PARTICLE PHYSICS

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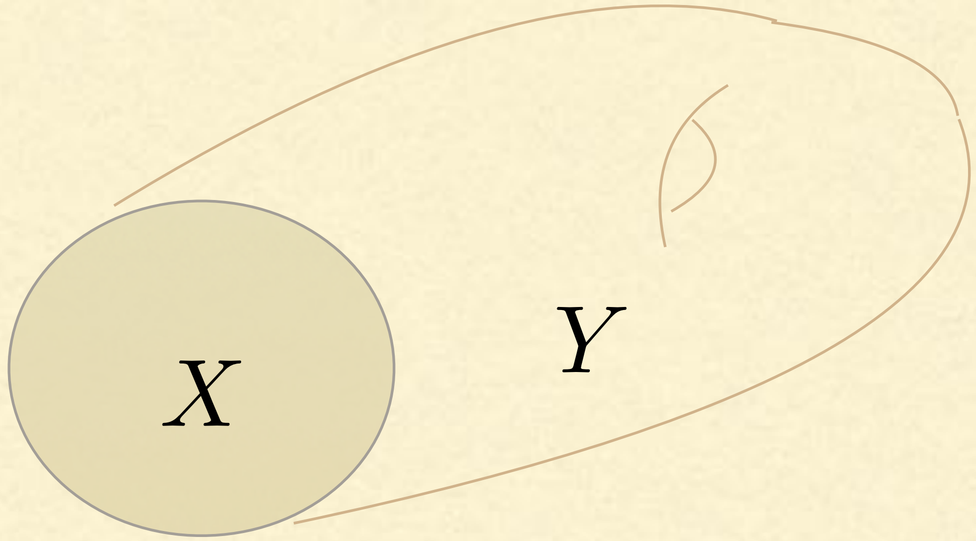
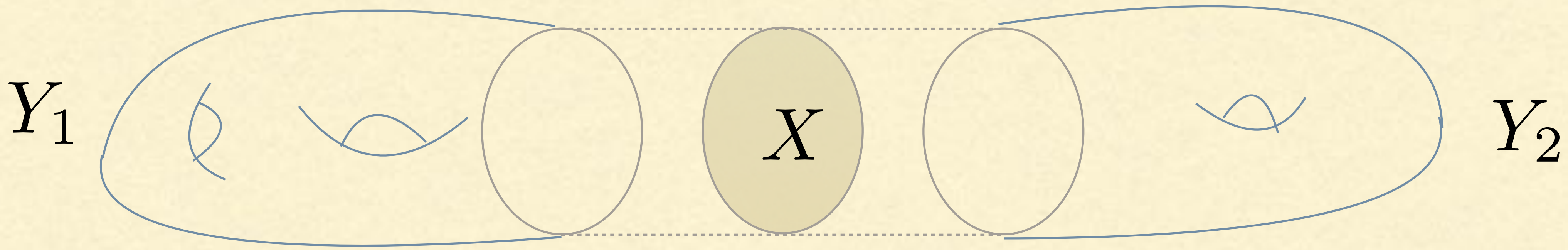
QUICK RECAP

- Iñaki just explained the **Dai-Freed** formalism to compute anomalies of fermion systems in d dimensions.



- Anomalies cancel if $\exp(2\pi i\, \eta_Y)$ is independent of the choice of Y



- To ensure this, we must have $\mathbf{\exp(2\pi i\, \eta_Y)=1}$ on any allowed (d+1) manifold.

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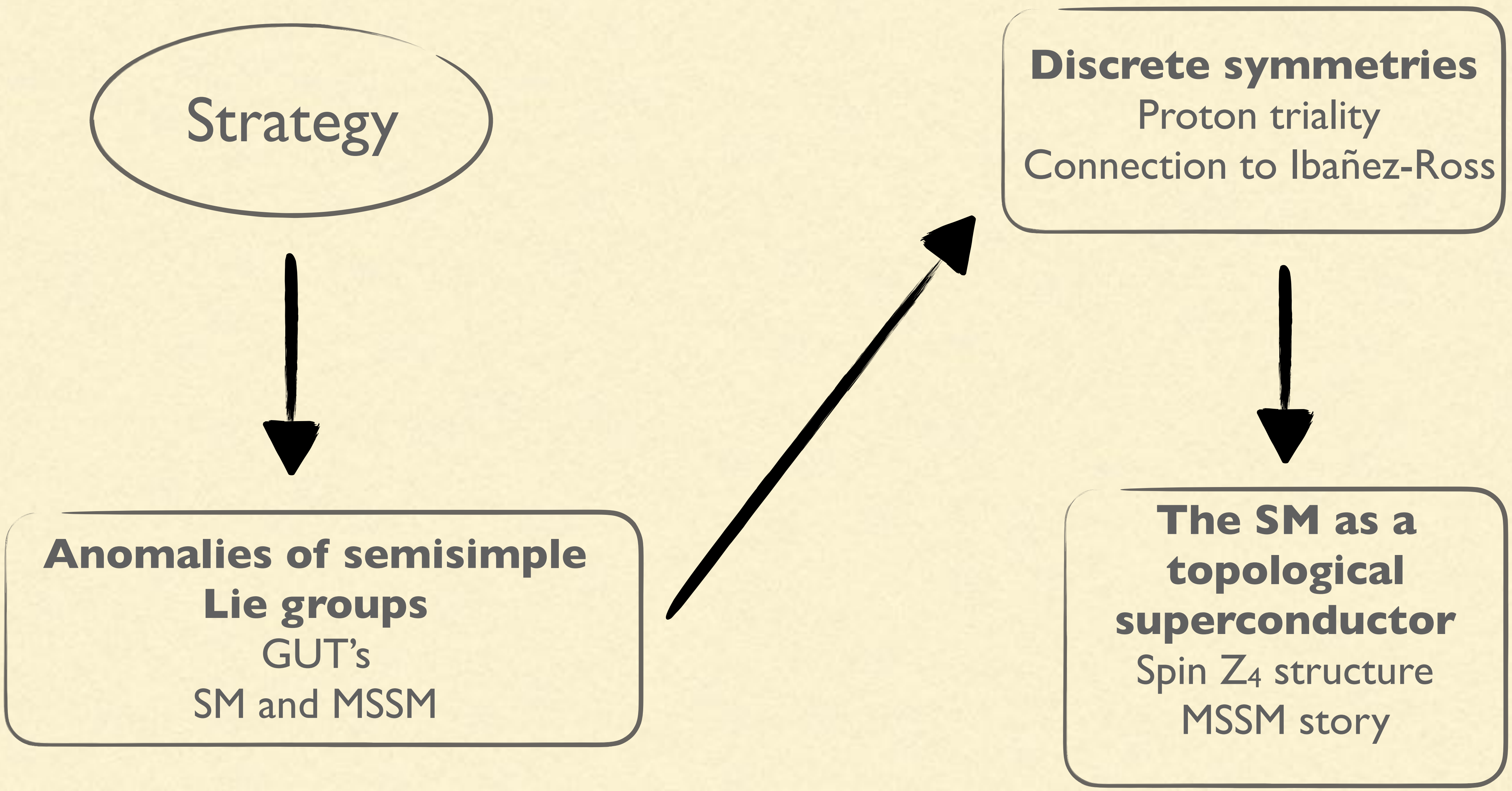
Is the Standard Model Dai-Freed anomalous?

PLAN OF THE TALK



- Once local anomalies cancel, η is a **bordism invariant:**

$$\exp(2\pi i \eta_{Y_1}) = \exp(2\pi i \eta_{Y_2})$$

- **Bordism** is an equivalence relation, which defines **bordism groups**

$$\Omega^{Spin}_{d+1}(BG)$$

These classify (d+1)-dimensional manifolds, with a principal G-bundle, modulo bordism (bundle extends over bordism too)

- Computed using **AHSS**.

- η is a **group homomorphism** from the relevant bord. group to U(1).

GENERAL STRATEGY

- Compute relevant bordism group
    - If it vanishes, there is no new anomaly.
- Find a nontrivial manifold Y, compute η.
    - If it vanishes, there is no new anomaly.
- If η is nonvanishing, there is an anomaly.

SEMISIMPLE LIE GROUPS

$\mathbf{G}$	$\mathbf{\Omega_d^{Spin}(BG)}$								
	0	1	2	3	4	5	6	7	8
$SU(2)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$4\mathbb{Z}$
$SU(n>2)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	0	–	–	–
$USp(2k>2)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$5\mathbb{Z}$
$U(1)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}$	0	$\mathbb{Z} \oplus \mathbb{Z}$	0	–	–	–
$Spin(n \geq 8)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	0	–	–	–
$SO(n \geq 3)$	$\mathbb{Z}$	$\mathbb{Z}_2$	$e(\mathbb{Z}_2, \mathbb{Z}_2)$	0	$e(\mathbb{Z}, \mathbb{Z} \oplus \mathbb{Z}_2)$	0	–	–	–
E_6, E_7, E_8	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
G_2	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	0	–	–	–
F_4	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$2\mathbb{Z}$	0	0	0	–

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THE STANDARD MODEL

- Experiments only probe the gauge **algebra** of the SM. There are four possibilities [Tong '17…]

$$\frac{SU(3)\times SU(2)\times U(1)}{\Gamma}, \quad \Gamma\in\{1,\mathbb{Z}_2,\mathbb{Z}_3,\mathbb{Z}_6\}$$

- The discrete group Γ acts trivially on the SM fermions.
- $\Gamma = Z_6$ is "maximal": Includes bundles for any other choice of Γ.
    - This is also the group that embeds in SU(5).

- The SM fermion spectrum falls into **SU(5) representations.**
- Any (**SU(3) x SU(2) x U(1))/**Z_6 bundle is a **SU(5) bundle** too!
- As far as anomalies are concerned, the SM is **equivalent** to the SU(5) GUT. But since

$$\Omega_5^{Spin}(BSU(5)) = 0$$

The SM is free of Dai-Freed anomalies

- Similar situation for Spin(10).

- To get an anomaly, we need to look at **more general** spaces.
- What about the SM in **non-orientable spaces?**
  - Only makes sense if one assumes CP breaking in SM is **spontaneous.**
  - Need a Pin structure to define fermions, which can change cob. groups, e.g. $\Omega_6^{Pin-}=Z_{16}$, but $\Omega_6^{Spin}=0$.
  - Majorana masses require a Pin^+ structure [Berg et al '00].

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  - Majorana masses require a Pin^+ structure [Berg et al '00].
- We have again

$$\Omega_5^{Pin^+}(BSU(5)) = \Omega_5^{Pin^-}(BSU(5)) = 0$$

- Last try: SM + right-handed neutrinos + gauged (B-L).
- Since all fermion charges under (B-L) are odd, we can now consider the SM on **Spin^c manifolds** or even **Pin^c manifolds**

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- Since all fermion charges under (B-L) are odd, we can now consider the SM on **Spin^c manifolds** or even **Pin^c manifolds**
- Still,

$$\Omega_5^{Spin^c}(BSU(5)) = \Omega_5^{Pin^c}(BSU(5)) = 0$$

so we find no anomalies in the SM.

DISCRETE CYCLIC GROUPS

- Lucky! Bordism groups & η invariants already computed by mathematicians [Bahri-Gilkey ‘ 87, Gilkey ’89, Gilkey-Botvinnik ’94] both for Spin and Spin^c cases. They are **nontrivial.**

- We can compare with **known anomalies** of discrete symmetries. [Ibañez-Ross‘ 91]. These were originally obtained by demanding that the Z_n embeds in a U(1).

$$2\sum s_i \equiv 0 \bmod N, \quad \sum s_i^3 \equiv 0 \bmod N$$

- Only **linear** constraints are UV-independent [Banks-Dine‘ 91]

- There is a nontrivial **Dai-Freed** anomaly coming from evaluating the η in a generalized lens space [Bahri-Gilkey '86, Gilkey '89, Gilkey-Botvinnik '94].

- We get constraints which are cubic in the charges: A "remnant" of the **cubic Ibañez-Ross constraint**

$$\sum -4s_i^3 + (N^2+3)s_i \equiv \mod 24N$$

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$$\sum -4s_i^3 + (N^2+3)s_i \equiv \mod 24N$$

- Are these UV-sensitive? YES

  - Topological GS term that can forbid some of the bundles [Ibañez '92, Garcia-Etxebarria-Hayashi-Ohmori-Tachikawa-Yonekura '17]

- For Z_3,

$$\sum s_i \equiv 0 \bmod 3 \qquad \text{(Linear IR)}$$
$$\sum s_i \equiv 0 \bmod 9 \qquad \text{(Dai-Freed)}$$

This one has phenomenological consequences: Proton triality (and also hexality) in the MSSM is a IR-anomaly free Z_3 symmetry, but it has a mod 9 anomaly

Q	u	d	l	e	H	$\bar{H}$
0	-1	1	-1	-1	1	-1
0	-2	-5	-5	1	5	5

$$\sum_{MSSM} s_i = 3 \bmod 9$$

- Dai-Freed anomaly cancellation requires 3k generations.
- Consistent with previous results [Dreiner et al. '04]: U(1) embedding of proton triality only with gen. dependent charges.

SM = TOP. SUPERCONDUCTOR

- **Topological superconductor**: 1st example of Dai-Freed anomaly [Kapustin-Thorgren-Turzillo-Wang '14, Witten 015, Hsieh-Cho-Ryu '16]
    - T-invariant 3d fermions. Global grav. anomaly requires multiple of 8.
    - Dai-Freed enhances to a multiple of 16, because

$$\Omega_4^{Pin^+} = \mathbb{Z}_{16}$$

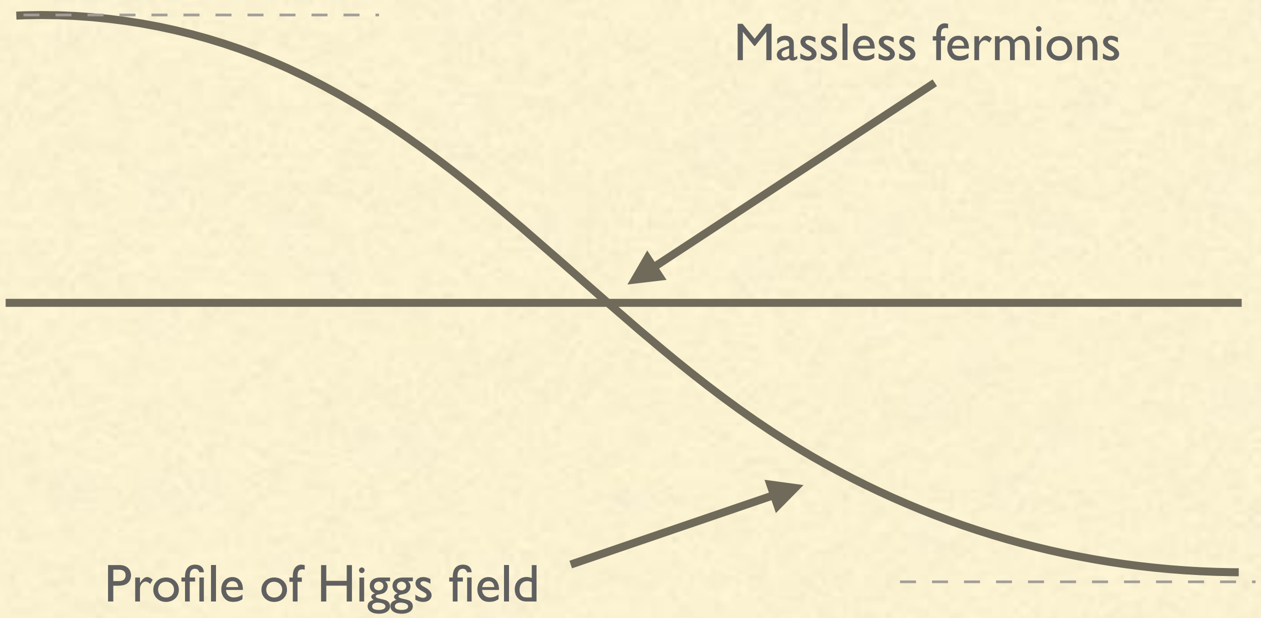
- # of fermions/ generation in SM + rh neutrinos = 16. **Not a coincidence!**

- The SM + rh neutrinos has a Z_4 symmetry (center of Spin(10)) that acts on every fermion by multiplication by i.

- We can use this to put the SM on manifolds with a $Spin^{\mathbb{Z}_4}$ structure [Tachikawa-Yonekura '18]. Transition functions of the spinors in (Spin x Z_4)/Z_2

- The **Smith homomorphism** maps

$$\Omega_{d+1}^{Spin^{\mathbb{Z}_4}} \to \Omega_d^{Pin^+}$$



- Physical interpretation: Higgsing the Z_4 w. a nontrivial bundle, there is a 3d locus with massless Pin^+ fermions.

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- Total: 16! **Dai-Freed anomaly vanishes.**

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- # of fermionic superpartners:
    - 12 gauginos (8+3+1)
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- Total: 16! **Dai-Freed anomaly vanishes.**
- Only works because of the detailed structure of SM: Dim. of gauge group + EWSB sector.
- No obvious relation to GUT's. Related to reflections of compactification manifold? [Tachikawa-Yonekura '18]

CONCLUSIONS

- We've explored a new kind of anomaly in four dimensional gauge theories of phenomenological interest.
- SM and GUT's are anomaly free. Can put SM on non-Spin manifolds (related to topological superconductor).
- New anomalies for discrete symmetries e.g. proton triality.
- Outlook
    - We only checked a few theories!
    - Is the Z_4 in the MSSM telling us something?

DZIĘKI!