Global aspects of T-branes

Raffaele Savelli

University of Rome - Tor Vergata

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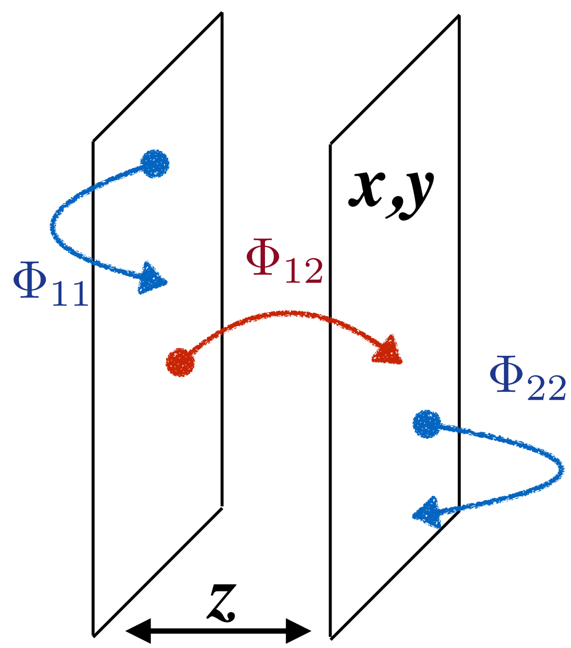
Based on work with:

F. Marchesano & S. Schwieger

arXiv:1707.03797 & to appear…

The Higgs field

- In Type IIB at weak g_s, describe D7 systems by 8d SYM.



$\boldsymbol{\Phi} = \boldsymbol{\Phi}_1 + i\,\boldsymbol{\Phi}_2$ (Higgs field)

Adjoint scalar describing D7's normal deformations.

$\langle\boldsymbol{\Phi}\rangle$'s eigenvalues encode D7's locations.

- $\langle\boldsymbol{\Phi}\rangle = 0$ ➠ U(2) stack at $z = 0$
- $\langle\boldsymbol{\Phi}\rangle = \begin{pmatrix} x & 0 \\ 0 & -x \end{pmatrix}$ ➠ U(1)xU(1), $D7_1 \cap D7_2$ at $z = x = 0$
- $\langle\boldsymbol{\Phi}\rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ ➠ U(1), D7's not moved but bound: T-brane

[Cecotti, Cordova, Heckman, Vafa `10]; [Donagi, Katz, Sharpe `03]

T-branes

- Host of peculiar brane-model-building phenomena:
  - $<\Phi> = \begin{pmatrix} 0 & y \\ 0 & 0 \end{pmatrix}$ ➠ "nilpotent" matter.
  - $<\Phi> = \begin{pmatrix} 0 & 1 \\ y & 0 \end{pmatrix}$ ➠ monodromies.
  - $<\Phi> = \begin{pmatrix} 0 & x \\ y & 0 \end{pmatrix}$ ➠ matter localized at points.
- Common feature: Physics missed by spectral data.
- $[\Phi, \Phi^\dagger] \neq 0 \quad \Rrightarrow$ At most 8 supercharges preserved.

So far…

- Intriguing structure & attractive applications:

    - T-branes in F-theory $\Rrightarrow$ singularities.

      [Anderson, Heckman, Katz `13]
      [Collinucci, RS `14]
      [Anderson, Heckman, Katz, Schaposnik `17]

    - Probing T-branes $\Rrightarrow$ monopole deformations.

      [Heckman, Tachikawa, Vafa, Wecht `10]
      [Collinucci, Giacomelli, RS, Valandro `16]

    - α' corrections to T-brane vacua. [Marchesano, Schwieger `16]

    - Realistic GUT Yukawa's from T-branes.

      [Cecotti, Cordova, Heckman, Vafa; Chiou, Faraggi, Tatar, Walters; Carta, Font, Ibáñez, Marchesano, Regalado, Zoccarato `10-`16]

    - De Sitter from T-branes. [Cicoli, Quevedo, Valandro `15]

BPS equations

- All previous studies were local
  - Miss crucial info, like existence of T-brane vacua !
- Minimal Susy in 4d $\nRightarrow$ 7-brane stack wrapping S
  - $S \subset CY_3$ Compact Kähler Surface w/ $V_G \to S$
- Stack in isolation:

$$\mathbb{F}^{(0,2)} = \bar{\partial}_A \Phi = 0 \qquad \mathrm{F-terms}$$

$$2\mathbb{F}^{(1,1)} \wedge J = -[\Phi, \Phi^\dagger] \qquad \mathrm{D-terms}$$

V_G field-strength

$H^{2,0}$ (S , adj(G))

A no-go theorem

- T-branes generically involve non-harmonic flux
  - Non-zero vev's for KK-modes of gauge field.
- Take $V_G = L \oplus L^{-1}$ & $\Phi = \begin{pmatrix} \mathbf{0} & \boldsymbol{m} \\ \boldsymbol{p} & \mathbf{0} \end{pmatrix}$

$m \to H^0(S, L^2 \otimes K_S)$

$p \to H^0(S, L^{-2} \otimes K_S)$

$$\Rightarrow \int_S \begin{pmatrix} F & 0 \\ 0 & -F \end{pmatrix} \wedge J = \begin{pmatrix} \sum_p |p|^2 - \sum_m |m|^2 & 0 \\ 0 & \sum_m |m|^2 - \sum_p |p|^2 \end{pmatrix}$$

- If e.g. $\int F \wedge J > 0$, $\exists$ modes of type p & $Vol\{p=0\} \geq 0$

$\Rightarrow$ $-2\int F \wedge J - \int R \wedge J \geq 0$ $\Rightarrow$ $\int R \wedge J < 0$

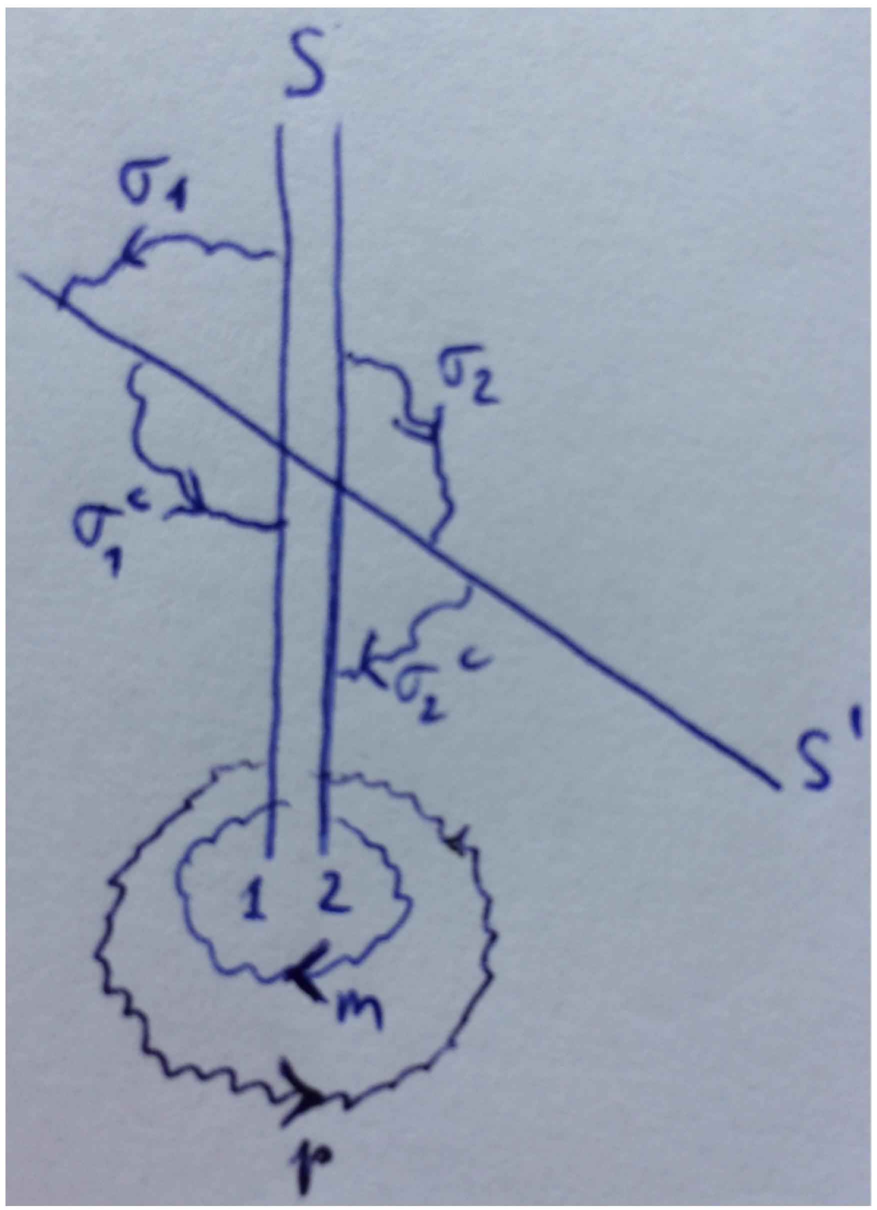
Forbids positive and zero Ricci-curvature !

A no-go theorem

- Same conclusion when $\int F \wedge J < 0$
- Valid for any gauge group so long as $[\Phi, \Phi^\dagger]$ is Cartan
- Valid at Large Volume & for $\int F \wedge J \ll Vol\{S\}$
  - T-brane stable where $Vol\{p=0\} < Vol\{S \cap S\}$ [Sebastian's talk]
- String phenomenologists love to wrap 7-branes on del Pezzo's
  - Must find ways to evade the no-go !

Defect fields

- Massless fields at $S \cap S'$ ➠ 8d SYM w/ 6d defect



F & D -terms modified :

$$\bar{\partial}_A \Phi = \mu_{\mathbb{C}}(\sigma, \sigma^c) \wedge \delta_{S\cap S'}$$

$$2\mathbb{F}^{(1,1)} \wedge J + [\Phi, \Phi^\dagger] = [\mu_{\mathbb{R}}(\sigma, \bar{\sigma}) - \mu_{\mathbb{R}}(\sigma^c, \bar{\sigma}^c)]\ J \wedge \delta_{S\cap S'}$$

Moment maps:

$\mu_{\mathbb{C}}$ → adj-valued (1,0)-form

$\mu_{\mathbb{R}}$ → adj-valued scalar

Holomorphic scenario

- Turn on $\langle\sigma_2\rangle$ only, U(1) charge: $q(\sigma_2) = -q(p)/2 = 1$
  - $\mu_{\mathbb{C}} = 0$ ➠ Φ still holomorphic
  - $\mu_{\mathbb{R}} = -|\sigma_2|^2$ ➠ Flips sign of FI term !
- Now: $\int_S R \wedge J \leq -2 \int_S F \wedge J = -2 \sum_p |p|^2 + \mathrm{Vol}(S \cap S') \sum_{\sigma_2} |\sigma_2|^2 > 0$
- Existence of σ_2 is harmless:
  - $deg(\mathrm{L}) - deg(\mathrm{L}') \geq 1 - g(S \cap S')$

Meromorphic scenario

- Turn on $\langle\sigma_1\rangle$ & $\langle\sigma_2^c\rangle$, $q(\sigma_1) = q(\sigma_2^c) = q(p)/2 = -1$
  - $\mu_{\mathbb{C}} = \sigma_1\,\sigma_2^c$ ➠ $\boldsymbol{\Phi}$ is meromorphic
  - $p \sim \sigma_1\,\sigma_2^c\,/\,s'$ ➠ simple pole along $S \cap S'$

- Now existence condition on p is relaxed:
  - $-\,2\int F \wedge J\; -\; \int R \wedge J\; +\, Vol\{S \cap S'\}\; \geq 0$

- $\int F \wedge J$ becomes more positive, due to $\mu_{\mathbb{R}}$:
  - $$2\int_S F \wedge J = 2\sum_p |p|^2 + \mathrm{Vol}(S \cap S')\left(\sum_{\sigma_1} |\sigma_1|^2 + \sum_{\sigma_2^c} |\sigma_2^c|^2\right)$$

Conclusions

- T-branes don't like positively curved 4-cycles.
- T-branes generally intersect other 7-branes in compact space.
- Coupling to fields at intersection avoids no-go.
    - Higgs field can develop poles. How about non-simple poles?
- D-terms heavily corrected for small volumes & large field vev's.
    - Need different techniques to analyze stability.
    - But hints for validity of no-go.