LVS flat directions and inflaton field range

Pramod Shukla
IFT, Madrid

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(In collaboration with M. Cicoli, D. Ciupke, C. Mayrhofer)
Three phases in the story

• everything looks great!
• trapped with “something" in the middle
• end! ?!
Why LVS ??

In general, $K$ and $W$ can have several corrections induced from various sources,

$$K = K_0 + K_{\alpha'} + K_{g_s} + ...,$$
$$W = W_0 + W_{np}^{n=1} + W_{np}^{n=2} + ......$$

- Effects of string loop-corrections [Berg, Haack, Kors], [Cicoli, Conlon, Quevedo], [Berg, Haack, Pajer], [Berg, Haack, Kang, Sjörs], [Haack, Kang].
- Effects of $\alpha'$-corrections [Becker, Becker, Haack, Louis], [Grimm, Savelli, Weissenbacher], [Bonetti, Weissenbacher], [Minasian, Pugh, Savelli].
- Higher derivatives ($F^4$)-corrections to scalar potential [Ciupke, Louis, Westphal].

Time evolution of the knowledge of these unknown corrections has been quite uncertain.

- And issue of viability !
- Extensions of “No scale structure” [von Gersdorff, Hebecker], [Berg,Haack, Kors], [Cicoli,Conlon,Quevedo], [Pedro, Rummel,Westphal], .......

Attractive features of LVS:

- The CY volume $\mathcal{V}$ is dynamically stabilized to exponentially large values, and works as a good expansion parameter, e.g. in $V_{\alpha'}, V_{g_s}, V_{F^4}, ..$
- Controlled breaking of the (sub-)leading order symmetries $\Rightarrow$ step-by-step computations with analytic (or at least numerical) control; good for moduli stabilization, creating some mass hierarchies, useful e.g. for single field inflation.
- Useful control over (un)known $\alpha'$ and $g_s$ corrections ?
LVS flat directions and reduced Moduli space

Underlying logic and the detailed insights behind the LVS:

\[ K = -2 \ln \left( \mathcal{V} + \frac{\hat{\xi}}{2} \right), \quad \hat{\xi} = -\frac{\chi(X)\zeta(3)}{2(2\pi)^3 g_s^{3/2}}, \quad W = W_0 + \sum_{i \in I} A_i e^{-a_i T_i}, \]

\[ \mathcal{V} = \frac{1}{3!} \int_X J \wedge J \wedge J = \frac{1}{6} k_{ijk} t_i t_j t_k \quad \text{where} \quad k_{ijk} = \int_X \hat{D}_i \wedge \hat{D}_j \wedge \hat{D}_k. \]

\[ \tau_i = \frac{1}{2!} \int_X \hat{D}_i \wedge J \wedge J = \frac{1}{2} k_{ijk} t_j t_k, \quad T_i = \tau_i + i \int_{D_i} C_4. \]

\[ V = \sum_{i,j \in I} a_i a_j A_i A_j K^{i\bar{j}} \frac{e^{-(a_i \tau_i + a_j \tau_j)}}{\mathcal{V}^2} - \sum_{i \in I} 4A_i W_0 a_i \tau_i \frac{e^{-a_i \tau_i}}{\mathcal{V}^2} + \frac{3 \hat{\xi} W_0^2}{4\mathcal{V}^3}, \]

where

\[ K^{i\bar{j}} = -\frac{4}{9} \left( \mathcal{V} + \frac{\hat{\xi}}{2} \right) k_{ijk} t^k + \frac{4 \mathcal{V} - \hat{\xi}}{\mathcal{V} - \hat{\xi}} \tau_i \tau_j \quad \mathcal{V} \gg \hat{\xi} \quad -\frac{4}{9} \mathcal{V} k_{ijk} t^k + 4 \tau_i \tau_j. \]

How to make the leading order terms in each of the three pieces compete ??
LVS flat directions and reduced Moduli space

Requirements for LVS vacua:

1. $X$ has negative Euler number $\chi(X) < 0$.

2. $X$ features at least one divisor $D_s$ which supports non-perturbative effects and can be made ‘small’, i.e. the CY volume $V$ does not become zero or negative when $\tau_s \to 0$.

3. The element $K^{s\bar{s}}$ of the inverse Kähler metric scales as (for $V \gg \tau_s^{3/2} \sim \hat{\xi}$):

$$K^{s\bar{s}} \simeq \lambda V \sqrt{\tau_s}, \quad \lambda \simeq O(1)$$

Diagonal divisor $D_s$:

$$k_{ssi} k_{ssj} = k_{sss} k_{sij} \quad \forall i, j,$$

$$\tau_s = \frac{1}{2} k_{sij} t^i t^j = \frac{1}{2 k_{sss}} k_{ssi} t^i k_{ssj} t^j = \frac{1}{2 k_{sss}} \left( k_{ssi} t^i \right)^2 \implies "ddP".$$  

LVS vacua is generically determined by:

$$\frac{3 \hat{\xi}}{2} \equiv -\frac{3}{2} \frac{\chi(X) \zeta(3)}{2 (2\pi)^3 g_s^{3/2}} \simeq \sum_{i=1}^{n_s} \sqrt{\frac{2}{d_i}} \tau_{0,i}^{3/2}, \quad \mathcal{V}_0 \simeq \sqrt{\frac{2}{d_i}} \frac{W_0}{A_i} \frac{\sqrt{\tau_{0,i}}}{4 a_i} e^{a_i \tau_{0,i}} \quad \forall i = 1, \ldots, n_s.$$  

$\implies$ Reduced moduli space $\mathcal{M}_r$ with $\text{dim}(\mathcal{M}_r) = (h^{1,1} - n_s - 1)$.  

A snapshot of Fiber Inflation [Cicoli, Burgess, Quevedo’08]

The CYs used for this class of models are so-called ‘weak’ swiss-cheese CYs which have

\[ V = \gamma_b \tau_b \sqrt{\tau_f} - \gamma_s \tau_s^{3/2}, \quad W = W_0 + A_s e^{-a_s T_s}. \]

The direction in the \((\tau_b - \tau_f)\)-plane orthogonal to the overall volume \( V \) is still flat and is lifted by two-types of string-loop corrections.

- **KK-type string-loop corrections:** \( K_{gs}^{kk} = g_s \sum_i \frac{C_{i}^{kk} t_i}{\sqrt{V}} \): can arise via KK string exchange among non-intersecting stacks of \( D3/D7 \)-brane and \( O3/O7 \)-planes.

- **Winding-type string-loop corrections:** \( K_{gs}^{w} = \sum_i \frac{C_{i}^{w}}{\sqrt{V} t_i} \): can arise via winding exchange between stacks of \( D7/O7 \) intersecting along a non-contractible 1-cycle.

After extended no-scale, the leading order \( \tau_f \) dependent terms in the scalar potential:

\[
V(\tau_f) \approx \frac{g_s |W_0|^2}{\sqrt{V}^2} \left[ \frac{A_1}{\tau_f^2} - \frac{A_2}{\sqrt{V} \sqrt{\tau_f}} + \frac{A_3 \tau_f}{\sqrt{V}^2} \right],
\]

\[ \tau_f \equiv e^{\frac{2}{\sqrt{3}} \phi} = \langle \tau_f \rangle e^{\frac{2}{\sqrt{3}} \varphi}, \quad \epsilon \approx \frac{3}{2} \eta^2, \quad r \sim 6(n_s - 1)^2; \]

\[ P_s \sim 2.3 \times 10^{-9}, \quad N_e \sim 60, \quad n_s \approx 0.96, \quad r \approx 0.007. \]

Many nice features for phenomenology (see Cicoli’s talk).

Global embeddings ???
Strategy and minimal requirements

• Searching for some $K3$-fibred CY threefolds with $h^{11} = 3$ and having a diagonal del-Pezzo divisor (to support LVS).

• Choice of involutions, tadpole cancellations, and Brane-setting.

• Ensuring that the possible brane settings have enough structure (being parallel or intersecting) to generate "appropriate" string-loop corrections.

• Incorporating the effects of recently proposed higher derivative corrections.

• Moduli stabilization and Inflationary dynamics along with the numerics to fit the values without violating the assumptions made.

• Chiral global embedding: repeating above steps with appropriate and consistent choice of fluxes using some $K3$-fibred CY threefolds with $h^{11} \geq 3$ and having a shrinkable del-Pezzo.

• Kähler cone conditions and size of the reduced moduli space.

We investigated all the $CY_3$ with $h^{1,1} = 3$ by considering all the 244 polytopes of the Kreuzer-Skarke list using the CY database [Altman, Gray, He, Jejjala, Nelson].

• # of CYs = 526 in which there are 305 distinct geometries.

• $K3$-fibred along with at least one diagonal $dP$: # of CYs = 43. Thanks to [Oguiso’ 92]'s theorem.
Minimal global embedding: [Cicoli, Muia, PS’16]

Let us consider a CY threefold defined by a hypersurface with the following Toric data,

<table>
<thead>
<tr>
<th>CY Hyp.</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
<th>$x_6$</th>
<th>$x_7$</th>
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<td>1</td>
<td>0</td>
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<td>4</td>
</tr>
<tr>
<td>8</td>
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<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

$dP_8$  $NdP_{10}$  $SD2$  $NdP_{15}$  $NdP_{13}$  $K3$  $SD1$

$$SR = \{x_1x_5, x_1x_6x_7, x_2x_3x_4, x_2x_6x_7, x_3x_4x_5\}$$

with $(h^{2,1}, h^{1,1}) = (99, 3)$, Euler number $\chi = -192$, and the volume form being,

$$\mathcal{V} = 9 t_f t_b^2 + \frac{t_s^3}{6} = \frac{\tau_b \sqrt{\tau_f}}{6} - \frac{\sqrt{2} \tau_s^{3/2}}{3}; \quad t_s = -\sqrt{2} \sqrt{\tau_s}, \quad t_f = \frac{\tau_b}{6 \sqrt{\tau_f}}, \quad t_b = \frac{\sqrt{\tau_f}}{3}.$$

Involution, tadpole cancellations and Brane setting: Involution $\sigma : x_3 \to -x_3$

$D7$ tadpole: $8[O7] \equiv 8[D_3] = 8[D_2] + 8[D_5], \quad D7$ not on top of $O7$

$D3$ tadpole: $N_{D3} + \frac{N_{\text{flux}}}{2} + N_{\text{gauge}} = \frac{N_{O3}}{4} + \frac{\chi(O7)}{12} + \sum_a \frac{N_a (\chi(D_a) + \chi(D'_a))}{48}$

$$= \frac{5}{4} + \frac{35}{12} + \frac{8(16 + 13)}{48} = 9 \quad \text{some space for background fluxes.}$$
Curves at the intersection of two divisors

<table>
<thead>
<tr>
<th></th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
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<th>$D_6$</th>
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<td>$P^1$</td>
<td>$T^2$</td>
<td>$2P^1$</td>
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<td>$C_4$</td>
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<tr>
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<td>$T^2$</td>
<td>$C_2$</td>
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<td>$P^1$</td>
<td>$C_2$</td>
<td>$C_{14}$</td>
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<tr>
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<td>$C_4$</td>
<td>$C_{14}$</td>
<td>$C_5$</td>
<td>$C_5$</td>
<td>$C_{10}$</td>
<td>$C_{82}$</td>
</tr>
</tbody>
</table>

$O7 \cap D_5 = \mathbb{P}^1$, $O7 \cap D_2 = T^2$, $D_2 \cap D_5 = C_2$

\[
V_{g_s}^W = -2 \left( \frac{g_s}{8\pi} \right) \sum_i \frac{W_0^2}{V^3} \frac{C_i^W}{t_i^\cap} \]

\[
= -2 \left( \frac{g_s}{8\pi} \right) |W_0|^2 \left[ \frac{C_1^W}{6t_b} + \frac{C_2^W}{(t_s + 6t_b)} + \frac{C_3^W}{2(t_f - t_b)} \right].
\]

The KK loop-corrections and $F^4$-corrections, are given as:

\[
V_{KK}^{g_s} = g_s^2 \left( \frac{g_s}{8\pi} \right) \frac{W_0^2}{V^2} \left[ \frac{(C_{f KK}^K)^2}{4\tau_f^2} + \frac{(C_{b KK}^K)^2}{72V^2} \right] \left( 1 - 6 \frac{C_{f KK}^K}{C_{b KK}^K} \sqrt{\frac{2\tau_s}{\tau_f}} + \frac{C_{f KK}^K}{C_{b KK}^K} \left( \frac{2\tau_s}{\tau_f} \right)^{3/2} \right)
\]

\[
\Pi_i(D_i) = \int_{CY} c_2(CY) \wedge D_i, \quad \text{and} \quad V_{F^4} = \frac{\lambda |W_0|^4 \Pi_i t_i^i}{V^4} \quad [\text{Ciupke, Louis, Westphal}]
\]

\[
V_{F^4} = - \left( \frac{g_s}{8\pi} \right)^2 \frac{\lambda W_0^4}{g_s^{3/2}V^3} \left[ \frac{24}{\tau_f} + \frac{8\sqrt{2\tau_s}^{3/2}}{\tau_f V} + \frac{36\sqrt{\tau_f}}{V} - \frac{10\sqrt{2}\sqrt{\tau_s}}{V} \right].
\]

\[
V_{\text{inf}} = \frac{AW_0^2}{\langle \tau_f \rangle^2 V^2} \left( C_{dS} + e^{-2k\varphi} - 4e^{-k\varphi/2} + \mathcal{R}_1 e^{k\varphi} + \mathcal{R}_2 e^{k\varphi/2} \right),
\]

where $k = \frac{2}{\sqrt{3}}$, $C_{dS} = 3 - \mathcal{R}_1 - \mathcal{R}_2$ to obtain a Minkowski (or slightly dS) vacuum.
Inflationary dynamics

For $g_s = 0.1$ and $V = 10^3$, $R_1 = 10^{-6}$ and with reasonable choices of the underlying parameters $C_W = 90$, $C_{f\text{KK}} = 65$, $C_{b\text{KK}} = 0.58$ we have,

| $R_2$   | $n_s$ | $r$ | $|W_0|$ | $|\lambda|$ | $\delta = \frac{H^2}{m_{pl}^2}$ |
|---------|-------|-----|--------|-------------|-------------------------------|
| 0       | 0.964 | 0.007 | 5.7    | 0           | 0.167                         |
| $7 \cdot 10^{-4}$ | 0.970 | 0.008 | 6.1    | $1.5 \cdot 10^{-3}$ | 0.169                        |
| $1.5 \cdot 10^{-3}$ | 0.977 | 0.012 | 6.7    | $2.7 \cdot 10^{-3}$ | 0.171                        |

- **Control over $\alpha'$ expansion**: Using $\chi(CY) = -192$, $\xi = -\frac{\chi(CY) \zeta(3)}{2(2\pi)^3} = 0.465$ and $\zeta = \frac{\xi}{2g_s^{3/2} V} = 0.0074 \ll 1$. Also $\langle \tau_f \rangle \simeq 60.4 \gg \langle \tau_s \rangle \simeq 2.9$, so that the corrections proportional to $\langle \tau_s \rangle / \tau_f$ can be self-consistently justified to be neglected.

- **Shift in $\chi$ (Savelli’s et al)**, $\chi_{\text{eff}} := \chi(CY) + \chi_{\text{shift}} = \chi(CY) + 2 \int_{CY} D_{O7}^3$. We find that the sign of $\chi_{\text{eff}}$ remains the same as of that of $\chi$, and also the inclusion of such effects is small; e.g. $\{\chi = -192, \xi = 0.465, \zeta = 0.0074\}$ changes into $\chi_{\text{eff}} = -190, \xi_{\text{eff}} = 0.460, \zeta_{\text{eff}} = 0.0073\}$. 

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**A concrete example of the “weaker" swiss-cheese**

Let us consider the following toric data for a Calabi Yau threefold which produces a volume form of kind $V = \gamma_1 \sqrt{\tau_1 \tau_2 \tau_3} - \gamma_2 \tau_3^{3/2}$,

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
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</tr>
</tbody>
</table>

$SR = \{x_1 x_4, x_1 x_6, x_1 x_7, x_2 x_7, x_3 x_6, x_4 x_5 x_8, x_2 x_3 x_5 x_8\}$

with $(h^{2,1}, h^{1,1}) = (98, 4)$ and Euler number $\chi = -188$. This corresponds to polytope ID #1206 in the database of [Altman, Gray, He, Jejjala, Nelson].

- The Intersection Polynomial and volume form:

  $$I_3 = 2 D_1^3 + 2 D_4 D_6 D_7 \quad \Longrightarrow \quad V = t_1^3 + 2 t_4 t_6 t_7 = \frac{\sqrt{\tau_4 \tau_6 \tau_7}}{\sqrt{2}} - \frac{1}{3} \tau_1^{3/2}.$$  

- Useful for chiral global embedding of fibre inflation by using appropriate fluxes.

- Gauge fluxes $\Rightarrow \tau_4 \sim \tau_6$, and back to the fibre inflation.

[Cicoli, Ciupke, Diaz, Guidetti, Muia, PS’17]. (see Guidetti’s talk)
Looks nice so far, BUT !

New challenge from the Kähler cone conditions: \( \frac{\Delta \phi}{M_p} \leq O(1) \ln \mathcal{V} \)

Relevance of Kähler cone and some estimates:

\[
\mathcal{V} = \frac{a}{2} t_f t_b^2 + \frac{b}{6} t_s^3 = \frac{\tau_b \sqrt{t_f}}{\sqrt{2} a} - \frac{\sqrt{2} \tau_s^3}{3 \sqrt{b}}; \quad a, b > 0.
\]

One type of Kaehler cone conditions which appear in concrete model can be given as:

\[
t_s < 0, \quad t_f + t_s > 0, \quad t_b + t_s > 0 \quad \implies \quad \frac{a \tau_s}{b} < \tau_f < \frac{\sqrt{b} \mathcal{V}}{\sqrt{2} \tau_s} + \frac{\tau_s}{3}
\]

\[
\implies \frac{\sqrt{3}}{2} \ln \left( \frac{a \tau_s}{b} \right) < \phi < \frac{\sqrt{3}}{2} \ln \left( \frac{\sqrt{b} \mathcal{V}}{\sqrt{2} \tau_s} + \frac{\tau_s}{3} \right).
\]
Modified checklist

• (Chiral) Global Embedding:
  1. \( K3 \)-fibred CY with diagonal del-Pezzo ✓
  2. tadpole cancellations ✓
  3. generation of ‘appropriate’ string-loop corrections to drive inflation ✓
  4. moduli stabilization leading to \( \mathcal{V} \approx 10^3 \) with \( W_0 \approx \mathcal{O}(1) \) and controlled values of \( C_{gs} \) needed to match the COBE normalization ✓
  5. a chiral visible sector (though not MSSM-like) is realized ✓

Higher derivative effects are delicate, and can be useful or dangerous!

• New challenge from the Kähler cone conditions: \( \frac{\Delta \phi}{M_p} \leq \mathcal{O}(1) \ln \mathcal{V} \)
  1. \( \mathcal{V} \approx 10^3 \) with \( W_0 \approx \mathcal{O}(1) \), which turns out to be needed to match the COBE normalization, does not result in sufficient inflaton field range, i.e. this does not lead to sufficient e-foldings to drive inflation.
  2. \( \mathcal{V} \geq 5 \times 10^4 \) can result in sufficient inflaton field range to drive inflation BUT the COBE normalization conditions are not met for \( \lambda \approx 10^{-3} \).
  3. If one increases \( W_0 \) to match the COBE with \( \mathcal{V} \geq 5 \times 10^4 \) then higher derivative effects are dangerous for \( \lambda \approx 10^{-3} \); For \( \lambda \leq 10^{-6} \) things are okay ✓.
  4. If one increases \( C_{gs} \) (which involves a factor of \( g_s^2 \)) to match the COBE with \( \mathcal{V} \geq 5 \times 10^4 \) then \( V_{gs} \approx V_{LVS} \), i.e. single field approximation?

Only approximate Kähler cone is known, hope for improvements!
Inflaton range [Cicoli, Ciupke, Mayrhofer, PS’18]

We consider the reduced moduli space of all the LVS models realized with CY orientifolds having $h^{1,1} = 3$, and one superpotential contribution:

\[
\text{Reduced moduli space } (\mathcal{M}_r) \\
\text{Leading order moduli stabilization: } \\
\mathcal{V}(\tau_i) = \mathcal{V}_0, \quad \tau_s = \tau_0 \\
\text{Kähler cone restrictions: } \\
\int_{C_i} J > 0, \quad \text{Vol}(\mathcal{M}_r) = \int_{\mathcal{M}_r} \ast 1_r.
\]

Geometry/size of the (reduced) moduli space depend on the (reduced) metric and the Kähler cone conditions (KCC): While metric can be read-off from the intersection tensor, KCC can only be approximated:

\[
M_A \supseteq M_X \supseteq M_\cap \iff \mathcal{K}_A \subseteq \mathcal{K}_X \subseteq \mathcal{K}_\cap \iff \text{Vol}(\mathcal{M}_{A,r}) \leq \text{Vol}(\mathcal{M}_r) \leq \text{Vol}(\mathcal{M}_{\cap,r}).
\]

Estimates for $M_A = M_\cap$ and otherwise. In most of the cases, one has the following analytic field range:

\[
\tau_{\min} \equiv a \tau_0 \ < \ \tau \ < \ f_1(\mathcal{V}_0, \tau_0) \equiv \tau_{\max},
\]

where $a \geq 0$ and $f_1$ is a homogeneous function of degree 1 in the 4-cycle moduli that is strictly positive for all $\mathcal{V}_0 > 0$ and $\tau_0 > 0$. 

Scanning results for LVS using CY threefolds with $h^{1,1} = 3$

Classification of models with LVS flat directions:

- $n_{\text{ddP}} = 2$, $n_{K3f} = 0$ leading to: $V = \alpha \tau_b^{3/2} - \beta_1 \tau_s^{3/2} - \beta_2 \tau_{s2}^{3/2}$ (SSC)
- $n_{\text{ddP}} = 1$, $n_{K3f} = 1$ leading to: $V = \alpha \sqrt{\tau_f} \tau_b - \beta \tau_s^{3/2}$ (K3-fibered)
- $n_{\text{ddP}} = 1$, $n_{K3f} = 0$ leading to:
  1. $V = f_{3/2}(\tau_1, \tau_2) - \beta \tau_s^{3/2}$ (Structureless)
  2. $V = \alpha \tau_b^{3/2} - \beta_1 \tau_s^{3/2} - \beta_2 (\gamma_1 \tau_s + \gamma_2 \tau_*)^{3/2}$ (SSC-like)

Here $\alpha > 0$, $\beta_s = \frac{1}{3} \sqrt{\frac{2}{d_s}}$, $\gamma_1 > 0$, $\gamma_2 > 0$ and $d_s$ degree of the del-Pezzo divisor $D_s$.

Note that the combination $\gamma_1 D_s + \gamma_2 D_*$ does not correspond to a smooth divisor, and so there is no choice of basis of smooth divisors where $V$ takes the standard strong Swiss cheese (SSC) form.

<table>
<thead>
<tr>
<th>$h^{1,1}$</th>
<th>$n_{\text{CY}}$</th>
<th>$n_{\text{LVS}}$</th>
<th>$%$</th>
<th>$n_{\text{ddP}} = 1$</th>
<th>$n_{\text{ddP}} = 2$</th>
<th>$n_{\text{ddP}} = 3$</th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>39</td>
<td>22</td>
<td>56.4%</td>
<td>22</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>305</td>
<td>132</td>
<td>43.3%</td>
<td>93</td>
<td>39</td>
<td>–</td>
</tr>
<tr>
<td>4</td>
<td>1997</td>
<td>749</td>
<td>37.5%</td>
<td>464</td>
<td>261</td>
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<table>
<thead>
<tr>
<th>$h^{1,1}$</th>
<th>$n_{\text{CY}}$</th>
<th>$n_{\text{LVS}}$</th>
<th>SSC</th>
<th>K3 fibred</th>
<th>SSC-like</th>
<th>Structureless</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>305</td>
<td>132</td>
<td>39</td>
<td>43</td>
<td>36</td>
<td>14</td>
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</table>
Compactness of the reduced moduli space

Computation of the reduced moduli space size via restricting the moduli space metric:

\[ ds^2 = g_{ij} \, d\tau_i \, d\tau_j \quad \text{with} \quad g_{ij} = 2 \frac{\partial^2 K}{\partial T_i \partial T_j} = \frac{1}{2} \frac{\partial^2 K}{\partial \tau_i \partial \bar{\tau}_j}, \quad \Delta \phi = \int_{\tau_{\text{min}}}^{\tau_{\text{max}}} ds_r. \]

Except for the structureless case, the CY examples in all the other three classes of LVS examples, one can analytically estimate the size of reduced moduli space \( \mathcal{M}_r \):

- **SSC:**
  \[
  ds_r^2 = \frac{3\beta_2}{4\mathcal{V}_0 \sqrt{\tau}} \left( \frac{1 + \beta_1 \epsilon}{1 + \beta_1 \epsilon + \beta_2 \frac{\tau^{3/2}}{\mathcal{V}_0}} \right) \, d\tau^2 \quad \text{with} \quad \epsilon \equiv \frac{\tau^{3/2}}{\mathcal{V}_0} \ll 1.
  \]
  \[
  \Delta \phi = \frac{2}{\sqrt{3}} \sqrt{1 + \beta_1 \epsilon} \, \text{Arcsinh} \left( f_1^{3/4} \sqrt{\frac{\beta_2}{\mathcal{V}_0 (1 + \beta_1 \epsilon)}} \right).
  \]

- **K3-fibred:**
  \[
  ds_r^2 = \frac{3}{4\tau^2} (1 + \beta \epsilon) \, d\tau^2, \quad \Delta \phi = \frac{\sqrt{3}}{2} \sqrt{1 + \beta \epsilon} \, \ln \left( \frac{f_1(\mathcal{V}_0, \tau_0)}{a \, \tau_0} \right)
  \]
  \[
  f_1(\mathcal{V}_0, \tau_0) = b \, \mathcal{V}_0^{2/3} + \mathcal{O}(\epsilon), \quad f_1(\mathcal{V}_0, \tau_0) = b \frac{\mathcal{V}_0}{\sqrt{\tau_0}}.
  \]

- **SSC:** similar as the first case.

- **Structureless:** Numerical tests show the compactness.
Some numerical estimates for $g_s = 0.1$

Both large field as well as small field inflation are possible

<table>
<thead>
<tr>
<th></th>
<th>$\mathcal{V}_0$</th>
<th>SSC ($M_p$)</th>
<th>K3 fibred ($M_p$)</th>
<th>SSC-like ($M_p$)</th>
<th>structureless ($M_p$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle \text{Vol}(\mathcal{M}_{A,r}) \rangle$</td>
<td>$10^3$</td>
<td>0.58</td>
<td>2.27</td>
<td>0.43</td>
<td>0.57</td>
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<tr>
<td></td>
<td>$10^4$</td>
<td>0.67</td>
<td>3.62</td>
<td>0.55</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>$10^5$</td>
<td>0.76</td>
<td>4.98</td>
<td>0.62</td>
<td>0.97</td>
</tr>
<tr>
<td>$\langle \text{Vol}(\mathcal{M}_{\cap,r}) \rangle$</td>
<td>$10^3$</td>
<td>0.71</td>
<td>2.47</td>
<td>0.70</td>
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<tr>
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<td>0.82</td>
<td>0.80</td>
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<tr>
<td></td>
<td>$10^5$</td>
<td>0.91</td>
<td>5.17</td>
<td>0.89</td>
<td>0.97</td>
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<tr>
<td>$\text{max}(\text{Vol}(\mathcal{M}_{A,r}))$</td>
<td>$10^3$</td>
<td>1.44</td>
<td>3.31</td>
<td>0.87</td>
<td>1.48</td>
</tr>
<tr>
<td></td>
<td>$10^4$</td>
<td>1.91</td>
<td>5.29</td>
<td>1.38</td>
<td>2.41</td>
</tr>
<tr>
<td></td>
<td>$10^5$</td>
<td>2.38</td>
<td>7.29</td>
<td>1.87</td>
<td>2.79</td>
</tr>
</tbody>
</table>

Conjecture: $\frac{\Delta \phi}{M_p} \leq \mathcal{O}(1) \ln \mathcal{V}, \quad \text{Vol}(\mathcal{M}_r) \lesssim \left[ \ln \left( \frac{M_p}{\Lambda} \right) \right]^{\dim(\mathcal{M}_r)}$. 
Field range for \( g_s = 0.1 \) and three sets of CY volume \( \mathcal{V} \)

\[ \mathcal{V}_0 = 10^3 \]

\[ \mathcal{V}_0 = 10^4 \]

\[ \mathcal{V}_0 = 10^5 \]
Field range for three sets of string-coupling $g_s$ and volume $\mathcal{V}$

- $g_s = 0.1, \quad 10^3$
- $g_s = 0.1, \quad 10^4$
- $g_s = 0.1, \quad 10^5$
- $g_s = 0.2, \quad 10^3$
- $g_s = 0.2, \quad 10^4$
- $g_s = 0.2, \quad 10^5$
- $g_s = 0.3, \quad 10^3$
- $g_s = 0.3, \quad 10^4$
- $g_s = 0.3, \quad 10^5$
Conclusions and outlook

• **Phase 1**: (Chiral) Global Embedding
  1. $K^3$-fibred CY with diagonal del-Pezzo ✓
  2. tadpole cancellations ✓
  3. generation of ‘appropriate’ string-loop corrections to drive inflation ✓
  4. moduli stabilization leading to $V \sim 10^3$ with $W_0 \sim O(1)$ and controlled values of $C_{gs}$ needed to match the COBE normalization ✓

• **Phase 2**:
  1. a chiral visible sector (though not MSSM-like) is realized ✓
  2. higher derivative effects are delicate, and can be useful or dangerous!

• **Phase 3**: some improvement in the Kähler cone approximations, enough ??.

• **Other possible solutions**:
  1. To explore the curvaton possibility
  2. Multi-field analysis to seek ‘new’ vacua besides LSV ??
  3. Revisit the higher derivative effects to get a complete story ??
  4. Back-reaction and stretching of the Kähler cone [Landete, Shiu ’18]??

Thanks for the attention!