LVS flat directions and inflaton field range

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Three phases in the story

- everything looks great !
- trapped with “something" in the middle
- end ! ? !

Why LVS ??

In general, K and W can have several corrections induced from various sources,

$$K = K_0 + K_{\alpha'} + K_{g_s} + ..., \qquad W = W_0 + W_{np}^{n=1} + W_{np}^{n=2} + ...$$

- Effects of string loop-corrections [Berg, Haack, Kors], [Cicoli, Conlon, Quevedo], [Berg, Haack, Pajer], [Berg, Haack, Kang, Sjörs], [Haack, Kang].
- Effects of α'-corrections [Becker, Becker, Haack, Louis], [Grimm, Savelli, Weissenbacher], [Bonetti, Weissenbacher], [Minasian, Pugh, Savelli].
- Higher derivatives (F^4)-corrections to scalar potential [Ciupke, Louis, Westphal].

Time evolution of the knowledge of these unknown corrections has been quite uncertain.

- And issue of viability !
- $\mathbb{E}$xtentions of "No scale structure" [von Gersdorff, Hebecker], [Berg,Haack, Kors], [Cicoli,Conlon,Quevedo], [Pedro, Rummel,Westphal], ... !

Attractive features of LVS:

- The CY volume $\mathcal{V}$ is dynamically stabilized to exponetially large values, and works as a good expansion parameter, e.g. in $V_{\alpha'}, V_{g_s}, V_{F^4}, ..$
- Controlled breaking of the (sub-)leading order symmetries $\Longrightarrow$ step-by-step computations with analytic (or at least numerical) control; good for moduli stabilization, creating some mass hierarchies, useful e.g. for single field inflation.
- Useful control over (un)known α' and g_s corrections ?

LVS flat directions and reduced Moduli space

Underlying logic and the detailed insights behind the LVS:

$$K = -2\,\ln\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right), \;\; \hat{\xi} = -\frac{\chi(X)\zeta(3)}{2(2\pi)^3 g_s^{3/2}}, \qquad W = W_0 + \sum_{i\in I} A_i\, e^{-a_i T_i}\,,$$

$$\mathcal{V} = \frac{1}{3!}\int_X J\wedge J\wedge J = \frac{1}{6}\,k_{ijk}\,t^i t^j t^k \qquad \text{where} \qquad k_{ijk} = \int_X \hat{D}_i\wedge\hat{D}_j\wedge\hat{D}_k\,.$$

$$\tau_i = \frac{1}{2!}\int_X \hat{D}_i\wedge J\wedge J = \frac{1}{2}\,k_{ijk}\,t^j t^k\,, \quad T_i = \tau_i + \mathrm{i}\int_{D_i} C_4\,.$$

$$V = \sum_{i,j\in I} a_i a_j A_i A_j\, K^{i\bar{j}}\,\frac{e^{-(a_i\tau_i + a_j\tau_j)}}{\mathcal{V}^2} - \sum_{i\in I} 4A_i W_0\, a_i\tau_i\,\frac{e^{-a_i\tau_i}}{\mathcal{V}^2} + \frac{3\,\hat{\xi}\,W_0^2}{4\mathcal{V}^3}\,,$$

where

$$K^{i\bar{j}} = -\frac{4}{9}\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right)k_{ijk}\,t^k + \frac{4\,\mathcal{V} - \hat{\xi}}{\mathcal{V} - \hat{\xi}}\,\tau_i\tau_j \overset{\mathcal{V}\gg\hat{\xi}}{\simeq} -\frac{4}{9}\,\mathcal{V}\,k_{ijk}\,t^k + 4\,\tau_i\tau_j\,.$$

How to make the leading order terms in each of the three pieces compete ??

LVS flat directions and reduced Moduli space

Requirements for LVS vacua:

1. X has negative Euler number $\chi(X) < 0$.
2. X features at least one divisor D_s which supports non-perturbative effects and can be made 'small', i.e. the CY volume $\mathcal{V}$ does not become zero or negative when $\tau_s \to 0$.
3. The element $K^{s\bar{s}}$ of the inverse Kähler metric scales as (for $\mathcal{V} \gg \tau_s^{3/2} \sim \hat{\xi}$):

$$K^{s\bar{s}} \simeq \lambda \mathcal{V} \sqrt{\tau_s}\,, \qquad \lambda \simeq \mathcal{O}(1)$$

Diagonal divisor D_s:

$$k_{ssi}\, k_{ssj} = k_{sss}\, k_{sij} \qquad \forall\, i, j\,,$$

$$\tau_s = \frac{1}{2}\, k_{sij}\, t^i t^j = \frac{1}{2k_{sss}}\, k_{ssi} t^i\, k_{ssj} t^j = \frac{1}{2k_{sss}} \left(k_{ssi} t^i\right)^2 \implies \quad \text{“}ddP\text{”}.$$

LVS vacua is generically determined by:

$$\frac{3\,\hat{\xi}}{2} \equiv -\frac{3}{2}\,\frac{\chi(X)\zeta(3)}{2(2\pi)^3 g_s{}^{3/2}} \simeq \sum_{i=1}^{n_s} \sqrt{\frac{2}{d_i}}\, \tau_{0,i}^{3/2}, \qquad \mathcal{V}_0 \simeq \sqrt{\frac{2}{d_i}}\, \frac{W_0}{A_i}\, \frac{\sqrt{\tau_{0,i}}}{4\,a_i}\, e^{a_i \tau_{0,i}} \quad \forall\, i = 1, ..., n_s\,.$$

$\implies$ Reduced moduli space $\mathcal{M}_r$ with $dim(\mathcal{M}_r) = (h^{1,1} - n_s - 1)$.

A snapshot of Fiber Inflation [Cicoli, Burgess, Quevedo'08]

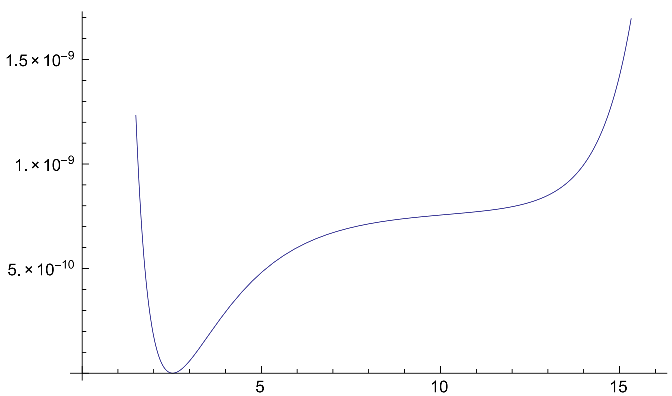
The CYs used for this class of models are so-called 'weak' swiss-cheese CYs which have

$$\mathcal{V} = \gamma_b\, \tau_b \sqrt{\tau_f} - \gamma_s\, \tau_s^{3/2}, \qquad W = W_0 + A_s\, e^{-a_s\, T_s}.$$

The direction in the $(\tau_b - \tau_f)$-plane orthogonal to the overall volume $\mathcal{V}$ is still flat and is lifted by two-types of string-loop corrections.

- KK-type string-loop corrections: $K^{kk}_{gs} = g_s \sum_i \frac{C_i^{kk}\, t_i^{\perp}}{\mathcal{V}}$: can arise via KK string exchange among non-intersecting stacks of $D3/D7$-brane and $O3/O7$-planes.
- Winding-type string-loop corrections: $K^{w}_{gs} = \sum_i \frac{C_i^{w}}{\mathcal{V}\, t_i^{\cap}}$: can arise via winding exchange between stacks of D7/O7 intersecting along a non-contractible 1-cycle.

After extended no-scale, the leading order τ_f dependent terms in the scalar potential:



$$V(\tau_f) \simeq \frac{g_s\, |W_0|^2}{\mathcal{V}^2}\left[\frac{A_1}{\tau_f^2} - \frac{A_2}{\mathcal{V}\sqrt{\tau_f}} + \frac{A_3\, \tau_f}{\mathcal{V}^2}\right],$$

$$\tau_f \equiv e^{\frac{2}{\sqrt{3}}\phi} = \langle \tau_f \rangle\, e^{\frac{2}{\sqrt{3}}\varphi}, \quad \epsilon \simeq \frac{3}{2}\eta^2, \; r \sim 6(n_s - 1)^2\,;$$

$$P_s \sim 2.3 \times 10^{-9}, \; N_e \sim 60, \; n_s \simeq 0.96, \; r \simeq 0.007.$$

Many nice features for phenomenology (see Cicoli's talk).

Global embeddings ???

Strategy and minimal requirements

- Searching for some $K3$-fibred CY threefolds with $h^{11} = 3$ and having a diagonal del-Pezzo divisor (to support LVS).
- Choice of involutions, tadpole cancellations, and Brane-setting.
- Ensuring that the possible brane settings have enough structure (being parallel or intersecting) to generate "appropriate" string-loop corrections.
- Incorporating the effects of recently proposed higher derivative corrections.
- Moduli stabilization and Inflationary dynamics along with the numerics to fit the values without violating the assumptions made.
- Chiral global embedding: repeating above steps with appropriate and consistent choice of fluxes using some $K3$-fibred CY threefolds with $h^{11} \geq 3$ and having a shrinkable del-Pezzo.
- Kähler cone conditions and size of the reduced moduli space.

We investigated all the CY_3 with $h^{1,1} = 3$ by considering all the 244 polytopes of the Kreuzer-Skarke list using the CY database [Altman, Gray, He, Jejjala, Nelson].

- $\#$ of CYs $= 526$ in which there are 305 distinct geometries.
- $K3$-fibred along with at least one diagonal dP: $\#$ of CYs $= 43$.
  Thanks to [Oguiso' 92]'s theorem.

Minimal global embedding: [Cicoli, Muia, PS'16]

Let us consider a CY threefold defined by a hypersurface with the following Toric data,

CY Hyp.	x_1	x_2	x_3	x_4	x_5	x_6	x_7
6	0	0	1	1	1	0	3
8	0	1	1	1	0	1	4
8	1	0	1	0	1	1	4
	dP_8	NdP_{10}	$SD2$	NdP_{15}	NdP_{13}	$K3$	$SD1$

$$\text{SR} \quad = \quad \{x_1 x_5,\, x_1 x_6 x_7,\, x_2 x_3 x_4,\, x_2 x_6 x_7,\, x_3 x_4 x_5\}$$

with $(h^{2,1}, h^{1,1}) = (99, 3)$, Euler number $\chi = -192$, and the volume form being,

$$\mathcal{V} = 9\, t_f\, t_b^2 + \frac{t_s^3}{6} = \frac{\tau_b \sqrt{\tau_f}}{6} - \frac{\sqrt{2}\, \tau_s^{3/2}}{3}\,; \qquad t_s = -\sqrt{2}\sqrt{\tau_s},\ t_f = \frac{\tau_b}{6\sqrt{\tau_f}},\ t_b = \frac{\sqrt{\tau_f}}{3}\,.$$

Involution, tadpole cancellations and Brane setting: Involution $\sigma : x_3 \to -x_3$

$$D7 \text{ tadpole}: \quad 8\,[O7] \equiv 8\,[D_3] = 8\,[D_2] + 8\,[D_5], \qquad \text{D7 not on top of O7}$$

$$D3 \text{ tadpole}: \quad N_{D3} + \frac{N_{\text{flux}}}{2} + N_{\text{gauge}} = \frac{N_{O3}}{4} + \frac{\chi(O7)}{12} + \sum_a \frac{N_a\,(\chi(D_a) + \chi(D_a'))}{48}$$

$$= \frac{5}{4} + \frac{35}{12} + \frac{8(16+13)}{48} = 9 \quad \text{some space for background fluxes.}$$

Curves at the intersection of two divisors

	D_1	D_2	D_3	D_4	D_5	D_6	D_7
D_1	$\mathcal{C}_2$	$\mathbb{T}^2$	$\mathbb{T}^2$	$\mathcal{C}_2$	$\emptyset$	$\emptyset$	$\mathcal{C}_4$
D_2	$\mathbb{T}^2$	$\mathbb{P}^1$	$\mathbb{T}^2$	$2\,\mathbb{P}^1$	$\mathcal{C}_2$	$\emptyset$	$\mathcal{C}_4$
D_3	$\mathbb{T}^2$	$\mathbb{T}^2$	$\mathcal{C}_2$	$\mathbb{P}^1$	$\mathbb{P}^1$	$\mathcal{C}_2$	$\mathcal{C}_{14}$
D_4	$\mathcal{C}_2$	$2\,\mathbb{P}^1$	$\mathbb{P}^1$	$6\,\mathbb{P}^1$	$\mathbb{P}^1$	$\mathcal{C}_2$	$\mathcal{C}_5$
D_5	$\emptyset$	$\mathcal{C}_2$	$\mathbb{P}^1$	$\mathbb{P}^1$	$4\,\mathbb{P}^1$	$\mathcal{C}_2$	$\mathcal{C}_5$
D_6	$\emptyset$	$\emptyset$	$\mathcal{C}_2$	$\mathcal{C}_2$	$\mathcal{C}_2$	$\emptyset$	$\mathcal{C}_{10}$
D_7	$\mathcal{C}_4$	$\mathcal{C}_4$	$\mathcal{C}_{14}$	$\mathcal{C}_5$	$\mathcal{C}_5$	$\mathcal{C}_{10}$	$\mathcal{C}_{82}$

$O7 \cap D_5 = \mathbb{P}^1$, $O7 \cap D_2 = \mathbb{T}^2$, $D_2 \cap D_5 = \mathcal{C}_2$

$$V^{\rm W}_{g_s} = -2\left(\frac{g_s}{8\pi}\right)\sum_i \frac{W_0^2\, C_i^{\rm W}}{\mathcal{V}^3\, t_i^{\cap}}$$

$$= -2\left(\frac{g_s}{8\pi}\right)\frac{|W_0|^2}{\mathcal{V}^3}\left[\frac{C_1^{\rm W}}{6\,t_b} + \frac{C_2^{\rm W}}{(t_s + 6\,t_b)} + \frac{C_3^{\rm W}}{2\left(t_f - t_b\right)}\right].$$

The KK loop-corrections and F^4-corrections, are given as:

$$V^{\rm KK}_{g_s} = g_s^2\left(\frac{g_s}{8\pi}\right)\frac{W_0^2}{\mathcal{V}^2}\left[\frac{(C_f^{\rm KK})^2}{4\tau_f^2} + \frac{(C_b^{\rm KK})^2\tau_f}{72\mathcal{V}^2}\left(1 - 6\frac{C_s^{\rm KK}}{C_b^{\rm KK}}\sqrt{\frac{2\tau_s}{\tau_f}} + \frac{C_f^{\rm KK}}{C_b^{\rm KK}}\left(\frac{2\tau_s}{\tau_f}\right)^{3/2}\right)\right],$$

$$\Pi_i(D_i) = \int_{CY} c_2(CY)\wedge D_i, \quad \text{and} \quad V_{F^4} = \frac{\lambda\,|W_0|^4\,\Pi_i\,t^i}{\mathcal{V}^4}$$ [Ciupke, Louis, Westphal]

$$V_{F^4} = -\left(\frac{g_s}{8\pi}\right)^2\frac{\lambda\,W_0^4}{g_s^{3/2}\mathcal{V}^3}\left[\frac{24}{\tau_f} + \frac{8\sqrt{2}\tau_s^{3/2}}{\tau_f\,\mathcal{V}} + \frac{36\sqrt{\tau_f}}{\mathcal{V}} - \frac{10\sqrt{2}\sqrt{\tau_s}}{\mathcal{V}}\right].$$

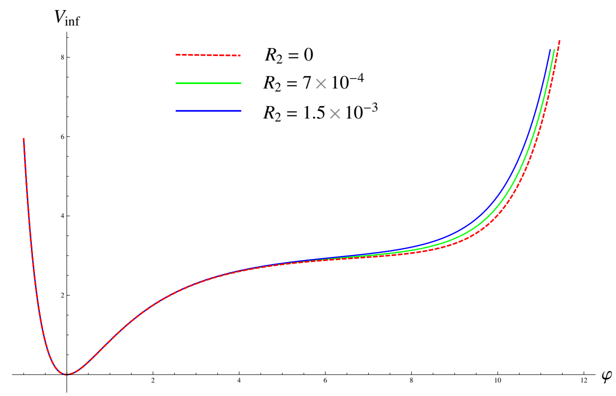
$$V_{\rm inf} = \frac{AW_0^2}{\langle\tau_f\rangle^2\mathcal{V}^2}\left(C_{\rm dS} + e^{-2k\varphi} - 4e^{-\frac{k\varphi}{2}} + \mathcal{R}_1\,e^{k\varphi} + \mathcal{R}_2\,e^{\frac{k\varphi}{2}}\right),$$

where $k = \frac{2}{\sqrt{3}}$, $C_{\rm dS} = 3 - \mathcal{R}_1 - \mathcal{R}_2$ to obtain a Minkowski (or slightly dS) vacuum.

Inflationary dynamics

For $g_s = 0.1$ and $\mathcal{V} = 10^3$, $\mathcal{R}_1 = 10^{-6}$ and with reasonable choices of the underlying parameters $C_{\rm W} = 90$, $C_f^{\rm KK} = 65$, $C_b^{\rm KK} = 0.58$ we have,

$\mathcal{R}_2$	n_s	r	$\|W_0\|$	$\|\lambda\|$	$\delta = \frac{H^2}{m_{\mathcal{V}}^2}$
0	0.964	0.007	5.7	0	0.167
$7 \cdot 10^{-4}$	0.970	0.008	6.1	$1.5 \cdot 10^{-3}$	0.169
$1.5 \cdot 10^{-3}$	0.977	0.012	6.7	$2.7 \cdot 10^{-3}$	0.171



- Control over α' expansion: Using $\chi(CY) = -192$, $\xi = -\frac{\chi(CY)\,\zeta(3)}{2\,(2\pi)^3} = 0.465$ and $\zeta = \frac{\xi}{2 g_s^{3/2} \mathcal{V}} = 0.0074 \ll 1$. Also $\langle \tau_f \rangle \simeq 60.4 \gg \langle \tau_s \rangle \simeq 2.9$, so that the corrections proportional to $\langle \tau_s \rangle / \tau_f$ can be self-consistently justified to be neglected.
- Shift in χ (Savelli's et al), $\chi_{\rm eff} := \chi(CY) + \chi_{\rm shift} = \chi(CY) + 2 \int_{CY} D_{O7}^3$. We find that the sign of $\chi_{\rm eff}$ remains the same as of that of χ, and also the inclusion of such effects is small; e.g. $\{\chi = -192,\ \xi = 0.465,\ \zeta = 0.0074\}$ changes into $\chi_{\rm eff} = -190,\ \xi_{\rm eff} = 0.460,\ \zeta_{\rm eff} = 0.0073\}$.

A concrete example of the "weaker" swiss-cheese

Let us consider the following toric data for a Calabi Yau threefold which produces a volume form of kind $\mathcal{V} = \gamma_1 \sqrt{\tau_1 \tau_2 \tau_3} - \gamma_2 \tau_s^{3/2}$,

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
4	0	0	0	1	1	0	0	2
4	0	0	1	0	0	1	0	2
4	0	1	0	0	0	0	1	2
8	1	0	0	1	0	1	1	4
	dP_7	NdP_{11}	NdP_{11}	$K3$	NdP_{11}	$K3$	$K3$	

$$\text{SR} = \{x_1x_4,\, x_1x_6,\, x_1x_7,\, x_2x_7,\, x_3x_6,\, x_4x_5x_8,\, x_2x_3x_5x_8\}$$

with $(h^{2,1}, h^{1,1}) = (98, 4)$ and Euler number $\chi = -188$. This corresponds to polytope ID $\#1206$ in the database of [Altman, Gray, He, Jejjala, Nelson].

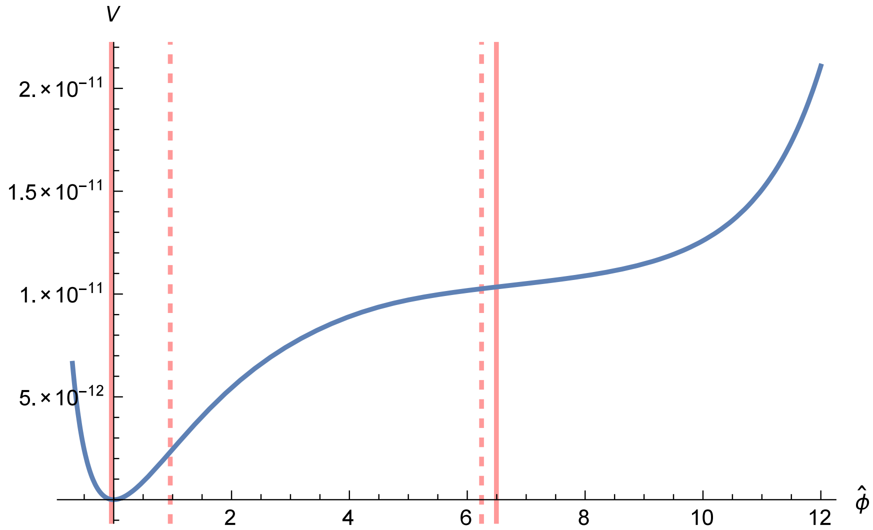
- The Intersection Polynomial and volume form:

  $I_3 = 2\, D_1^3 + 2\, D_4\, D_6\, D_7 \qquad \Longrightarrow \mathcal{V} = \frac{t_1^3}{3} + 2\, t_4\, t_6\, t_7 = \frac{\sqrt{\tau_4\, \tau_6\, \tau_7}}{\sqrt{2}} - \frac{1}{3}\, \tau_1^{3/2}.$
- Useful for chiral global embedding of fibre inflation by using appropriate fluxes.
- Gauge fluxes $\Longrightarrow \tau_4 \sim \tau_6$, and back to the fibre inflation.

[Cicoli, Ciupke, Diaz, Guidetti, Muia, PS'17]. (see Guidetti's talk)

Looks nice so far, BUT !!

New challenge from the Kähler cone conditions: $\frac{\Delta\phi}{M_p} \leq \mathcal{O}(1)\,\ln\mathcal{V}$



Relevance of Kähler cone and some estimates:

$$\mathcal{V} = \frac{a}{2}\,t_f\,t_b^2 + \frac{b}{6}\,t_s^3 = \frac{\tau_b\,\sqrt{\tau_f}}{\sqrt{2\,a}} - \frac{\sqrt{2}\,\tau_s^3}{3\,\sqrt{b}}; \quad a, b > 0\,.$$

One type of Kaehler cone conditions which appear in concrete model can be given as:

$$t_s < 0, \quad t_f + t_s > 0, \quad t_b + t_s > 0 \quad \Longrightarrow \quad \frac{a\,\tau_s}{b} < \tau_f < \frac{\sqrt{b}\,\mathcal{V}}{\sqrt{2\,\tau_s}} + \frac{\tau_s}{3}$$

$$\Longrightarrow \frac{\sqrt{3}}{2}\,\ln\left(\frac{a\,\tau_s}{b}\right) < \phi < \frac{\sqrt{3}}{2}\,\ln\left(\frac{\sqrt{b}\,\mathcal{V}}{\sqrt{2\,\tau_s}} + \frac{\tau_s}{3}\right)\,.$$

Modified checklist

- (Chiral) Global Embedding:
  1. $K3$-fibred CY with diagonal del-Pezzo ✓
  2. tadpole cancellations ✓
  3. generation of 'appropriate' string-loop corrections to drive inflation ✓
  4. moduli stabilization leading to $\mathcal{V} \simeq 10^3$ with $W_0 \simeq \mathcal{O}(1)$ and controlled values of $\mathcal{C}_{g_s}$ needed to match the COBE normalization ✓
  5. a chiral visible sector (though not MSSM-like) is realized ✓

  Higher derivative effects are delicate, and can be useful or dangerous !
- New challenge from the Kähler cone conditions: $\frac{\Delta\phi}{M_p} \leq \mathcal{O}(1)\, \ln \mathcal{V}$

1. $\mathcal{V} \simeq 10^3$ with $W_0 \simeq \mathcal{O}(1)$, which turns out to be needed to match the COBE normalization, does not result in sufficient inflaton field range, i.e. this does not lead to sufficient e-foldings to drive inflation.
2. $\mathcal{V} \geq 5 \times 10^4$ can result in sufficient inflaton field range to drive inflation BUT the COBE normalization conditions are not met for $\lambda \simeq 10^{-3}$.
3. If one increases W_0 to match the COBE with $\mathcal{V} \geq 5 \times 10^4$ then higher derivative effects are dangerous for $\lambda \simeq 10^{-3}$; For $\lambda \leq 10^{-6}$ things are okay ✓ .
4. If one increases C_{g_s} (which involves a factor of g_s^2) to match the COBE with $\mathcal{V} \geq 5 \times 10^4$ then $V_{g_s} \simeq V_{LVS}$, i.e. single field approximation ?

Only approximate Kähler cone is known, hope for improvements !

Inflaton range [Cicoli, Ciupke, Mayrhofer, PS'18]

We consider the reduced moduli space of all the LVS models realized with CY orientifolds having $h^{1,1} = 3$, and one superpotential contribution:

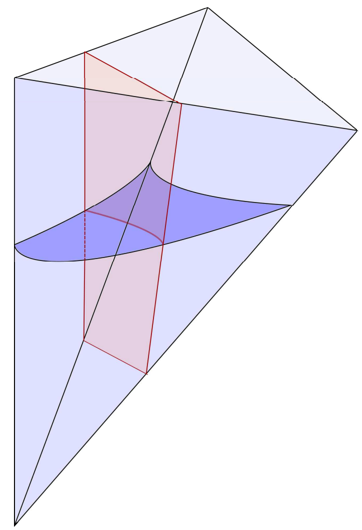
Reduced moduli space ($\mathcal{M}_r$)

Leading order moduli stabilization:

$$\mathcal{V}(\tau_i) = \mathcal{V}_0, \qquad \tau_s = \tau_0$$

Kähler cone restrictions:

$$\int_{C_i} J > 0, \quad \text{Vol}\,(\mathcal{M}_r) = \int_{\mathcal{M}_r} *\mathbf{1}_r\,.$$



Geometry/size of the (reduced) moduli space depend on the (reduced) metric and the Kähler cone conditions (KCC): While metric can be read-off from the intersection tensor, KCC can only be approximated:

$$M_A \supseteq M_X \supseteq M_\cap \Longleftrightarrow \mathcal{K}_A \subseteq \mathcal{K}_X \subseteq \mathcal{K}_\cap \Longleftrightarrow \text{Vol}\,(\mathcal{M}_{A,r}) \leq \text{Vol}\,(\mathcal{M}_r) \leq \text{Vol}\,(\mathcal{M}_{\cap,r})\,.$$

Estimates for $M_A = M_\cap$ and otherwise. In most of the cases, one has the following analytic field range:

$$\tau_{\text{min}} \equiv a\,\tau_0 \;<\; \tau \;<\; f_1(\mathcal{V}_0, \tau_0) \equiv \tau_{\text{max}}\,,$$

where $a \geq 0$ and f_1 is a homogeneous function of degree 1 in the 4-cycle moduli that is strictly positive for all $\mathcal{V}_0 > 0$ and $\tau_0 > 0$.

Scanning results for LVS using CY threefolds with $h^{1,1} = 3$

Classification of models with LVS flat directions:

- $n_{\rm ddP} = 2$, $n_{\rm K3f} = 0$ leading to: $\mathcal{V} = \alpha\, \tau_b^{3/2} - \beta_1\, \tau_{s_1}^{3/2} - \beta_2\, \tau_{s_2}^{3/2}$ (SSC)
- $n_{\rm ddP} = 1$, $n_{\rm K3f} = 1$ leading to: $\mathcal{V} = \alpha\, \sqrt{\tau_f}\ \tau_b - \beta\, \tau_s^{3/2}$ ($K3$-fibered)
- $n_{\rm ddP} = 1$, $n_{\rm K3f} = 0$ leading to:
  - (i). $\mathcal{V} = f_{3/2}(\tau_1, \tau_2) - \beta\, \tau_s^{3/2}$ (Structureless)
  - (ii). $\mathcal{V} = \alpha\, \tau_b^{3/2} - \beta_1\, \tau_s^{3/2} - \beta_2\, (\gamma_1\, \tau_s + \gamma_2\, \tau_*)^{3/2}$ (SSC-like)

Here $\alpha > 0,\ \beta_s = \frac{1}{3}\sqrt{\frac{2}{d_s}},\ \gamma_1 > 0,\ \gamma_2 > 0$ and d_s degree of the del-Pezzo divisor D_s.
Note that the combination $\gamma_1 D_s + \gamma_2 D_*$ does not correspond to a smooth divisor, and so there is no choice of basis of smooth divisors where $\mathcal{V}$ takes the standard strong Swiss cheese (SSC) form.

$h^{1,1}$	$n_{\rm CY}$	$n_{\rm LVS}$	%	$n_{\rm ddP} = 1$	$n_{\rm ddP} = 2$	$n_{\rm ddP} = 3$
2	39	22	56.4%	22	–	–
3	305	132	43.3%	93	39	–
4	1997	749	37.5%	464	261	24

$h^{1,1}$	$n_{\rm CY}$	$n_{\rm LVS}$	SSC	K3 fibred	SSC-like	Structureless
3	305	132	39	43	36	14

Compactness of the reduced moduli space

Computation of the reduced moduli space size via restricting the moduli space metric:

$$\mathrm{d}s^2 = g_{ij}\,\mathrm{d}\tau_i \mathrm{d}\tau_j \quad \text{with} \quad g_{ij} = 2\,\frac{\partial^2 K}{\partial T_i \partial \bar{T}_j} = \frac{1}{2}\,\frac{\partial^2 K}{\partial \tau_i \partial \tau_j}\,, \quad \Delta\phi = \int_{\tau_{\min}}^{\tau_{\max}} \mathrm{d}s_r\,.$$

Except for the structureless case, the CY examples in all the other three classes of LVS examples, one can analytically estimate the size of reduce modli space $\mathcal{M}_r$:

- SSC:

$$\mathrm{d}s_r^2 = \frac{3\beta_2}{4\mathcal{V}_0\sqrt{\tau}}\left(\frac{1+\beta_1\,\epsilon}{1+\beta_1\,\epsilon+\beta_2\,\frac{\tau^{3/2}}{\mathcal{V}_0}}\right)\mathrm{d}\tau^2 \qquad \text{with} \qquad \epsilon \equiv \frac{\tau_0^{3/2}}{\mathcal{V}_0} \ll 1\,.$$

$$\Delta\phi = \frac{2}{\sqrt{3}}\sqrt{1+\beta_1\,\epsilon}\ \mathrm{Arcsinh}\left(f_1^{3/4}\sqrt{\frac{\beta_2}{\mathcal{V}_0\,(1+\beta_1\,\epsilon)}}\right)\,.$$

- $K3$-fibred:

$$\mathrm{d}s_r^2 = \frac{3}{4\tau^2}\,(1+\beta\,\epsilon)\,\mathrm{d}\tau^2, \quad \Delta\phi = \frac{\sqrt{3}}{2}\sqrt{1+\beta\,\epsilon}\ \ \ln\left(\frac{f_1(\mathcal{V}_0,\tau_0)}{a\,\tau_0}\right)$$

$$f_1(\mathcal{V}_0,\tau_0) = b\,\mathcal{V}_0^{2/3} + \mathcal{O}(\epsilon), \qquad f_1(\mathcal{V}_0,\tau_0) = b\,\frac{\mathcal{V}_0}{\sqrt{\tau_0}}\,.$$

- SSC: similar as the first case.
- Structureless: Numerical tests show the compactness.

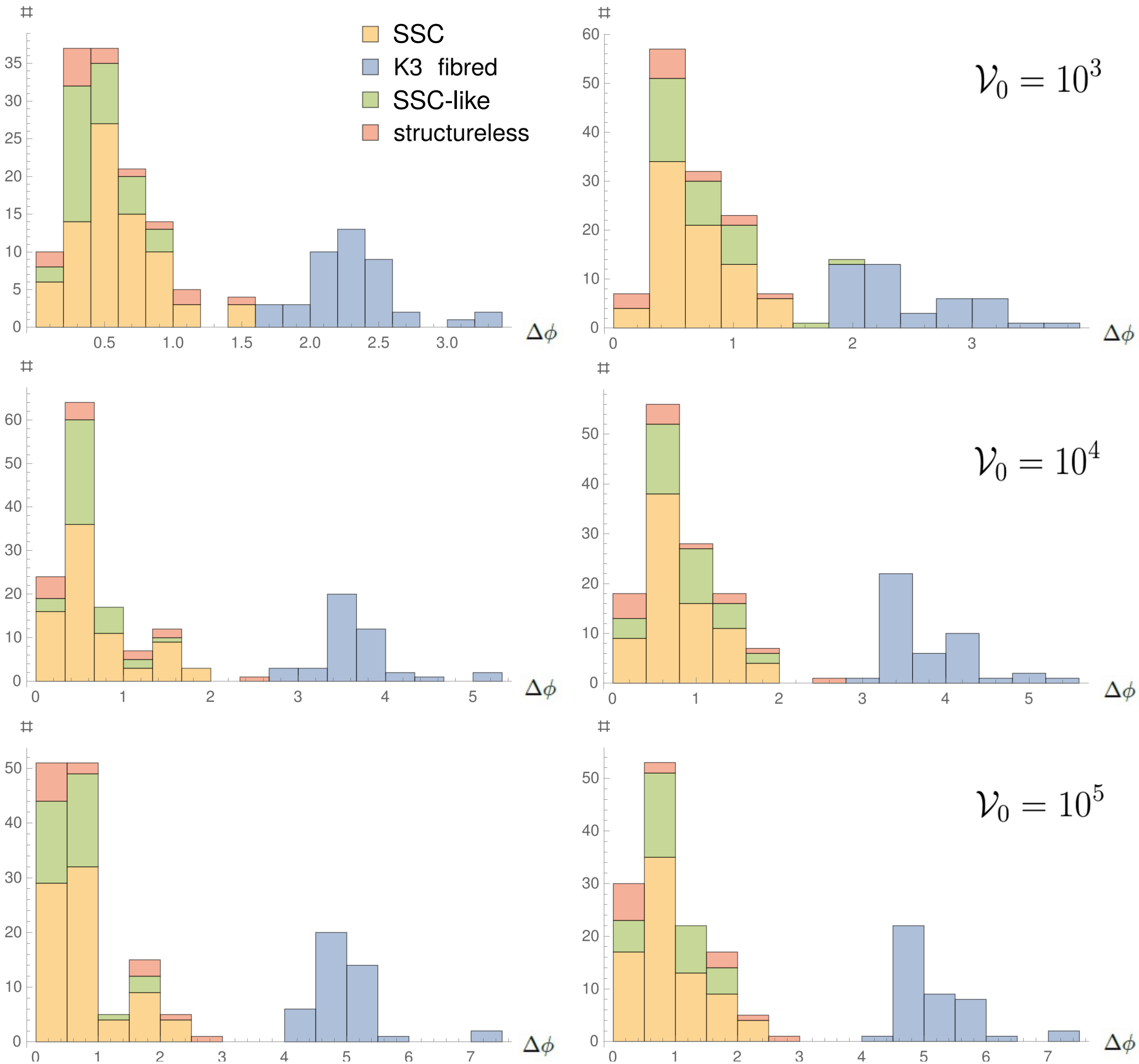
Some numerical estimates for $g_s = 0.1$

Both large field as well as small field inflation are possible

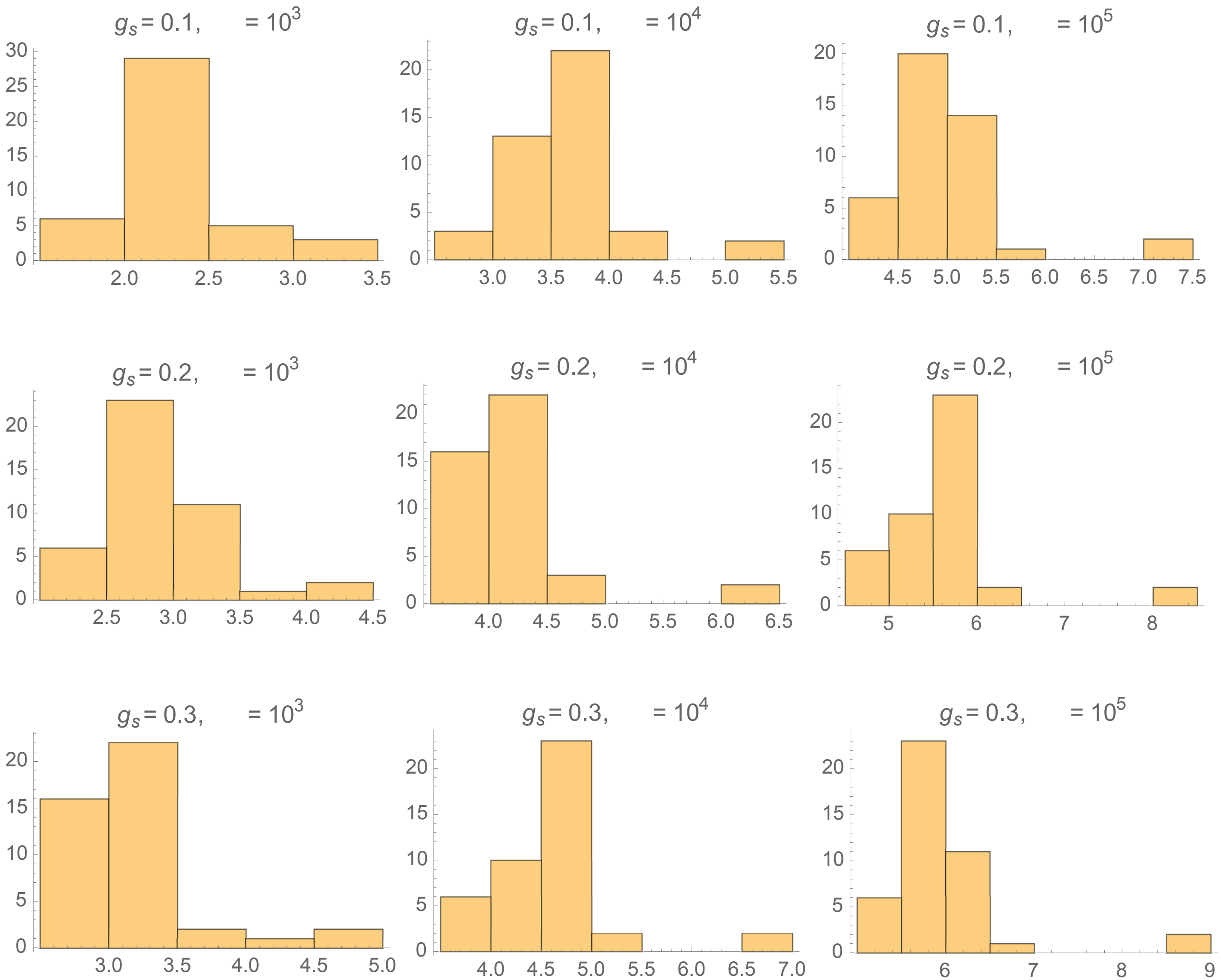
	$\mathcal{V}_0$	SSC (M_p)	K3 fibred (M_p)	SSC-like (M_p)	structureless (M_p)
$\langle \mathrm{Vol}\,(\mathcal{M}_{A,r}) \rangle$	10^3	0.58	2.27	0.43	0.57
	10^4	0.67	3.62	0.55	0.80
	10^5	0.76	4.98	0.62	0.97
$\langle \mathrm{Vol}\,(\mathcal{M}_{\cap,r}) \rangle$	10^3	0.71	2.47	0.70	0.57
	10^4	0.81	3.81	0.82	0.80
	10^5	0.91	5.17	0.89	0.97
$\max(\mathrm{Vol}\,(\mathcal{M}_{A,r}))$	10^3	1.44	3.31	0.87	1.48
	10^4	1.91	5.29	1.38	2.41
	10^5	2.38	7.29	1.87	2.79

Conjecture : $\dfrac{\Delta\phi}{M_p} \leq \mathcal{O}(1)\,\ln\mathcal{V}\,, \qquad \mathrm{Vol}(\mathcal{M}_r) \lesssim \left[\ln\left(\dfrac{M_p}{\Lambda}\right)\right]^{\dim(\mathcal{M}_\mathrm{r})}.$

Field range for $g_s = 0.1$ and three sets of CY volume $\mathcal{V}$



Field range for three sets of string-coupling g_s and volume $\mathcal{V}$



Conclusions and outlook

- Phase 1: (Chiral) Global Embedding
  1. $K3$-fibred CY with diagonal del-Pezzo ✓
  2. tadpole cancellations ✓
  3. generation of 'appropriate' string-loop corrections to drive inflation ✓
  4. moduli stabilization leading to $\mathcal{V} \simeq 10^3$ with $W_0 \simeq \mathcal{O}(1)$ and controlled values of $\mathcal{C}_{g_s}$ needed to match the COBE normalization ✓
- Phase 2:
  1. a chiral visible sector (though not MSSM-like) is realized ✓
  2. higher derivative effects are delicate, and can be useful or dangerous !
- Phase 3: some improvement in the Kähler cone approximations, enough ??.
- Other possible solutions:
  1. To explore the curvaton possibility
  2. Multi-field analysis to seek 'new' vacua besides LSV ??
  3. Revisit the higher derivative effects to get a complete story ??
  4. Back-reaction and stretching of the Kähler cone [Landete, Shiu '18]??

Thanks for the attention !