See also [W. Buchmüller's & H. Abe's talks]

Effects of fixed-point localized μ-terms in flux compactifications

Yoshiyuki Tatsuta (DESY)

based on arXiv:1806.10369

In collaboration with
Hiroyuki Abe (Waseda U.) ← ***prev. speaker***
Makoto Ishida (Waseda U.)

The SM & flux compactification

- **Properties "behind" the SM:**
  - Gauge theory
  - Higgs mechanism
  - Chiral matters
  - Three generations of fermions
  - Yukawa couplings

$\vdots$

The SM & flux compactification

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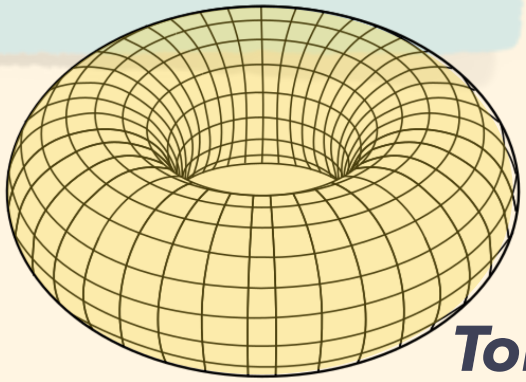
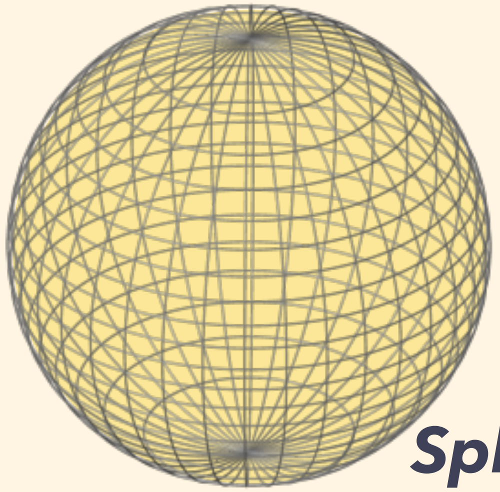
⋮

Compact manifolds with const. flux background

e.g.,



Torus T^2



Sphere S^2

Usually, to lead to these properties, flux compactification plays important roles.

The SM & flux compactification

- **Properties "behind" the SM:**
  - Gauge theory
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pursue EW breaking in the MSSM

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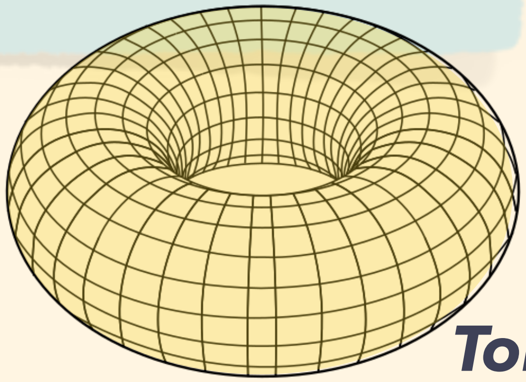

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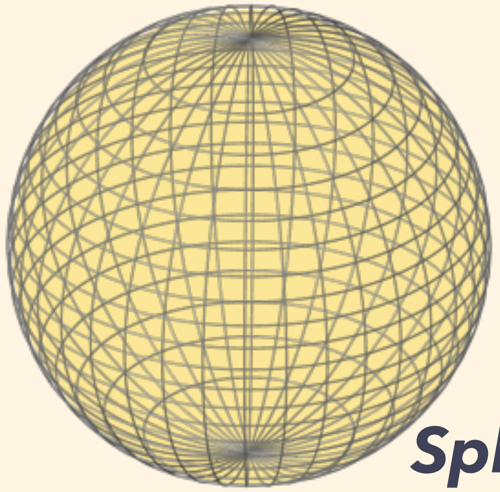
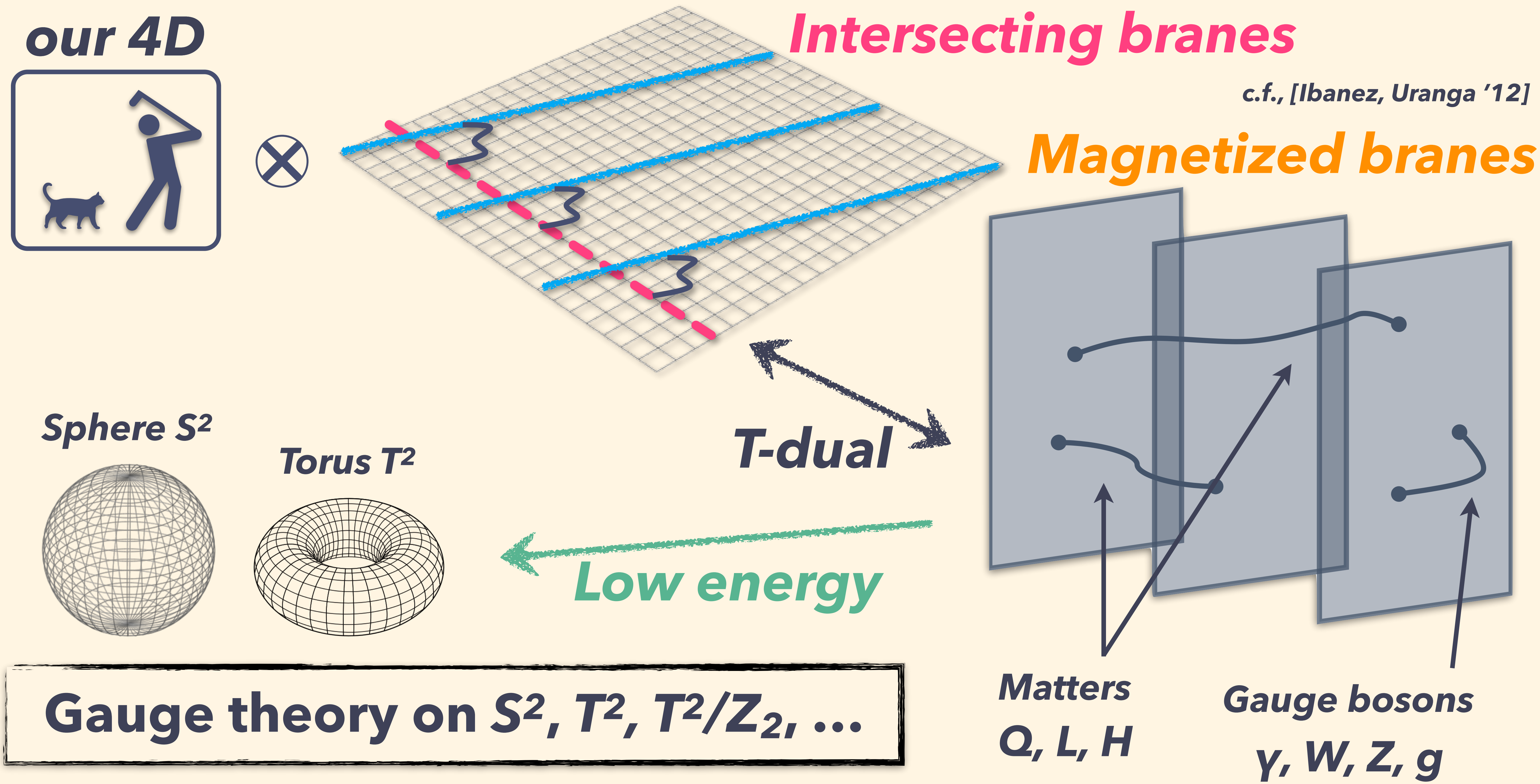


Sphere S^2

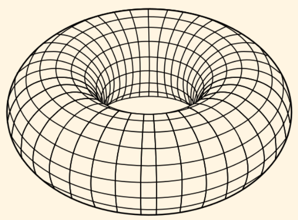
Usually, to lead to these properties, flux compactification plays important roles.

As effective theory of strings

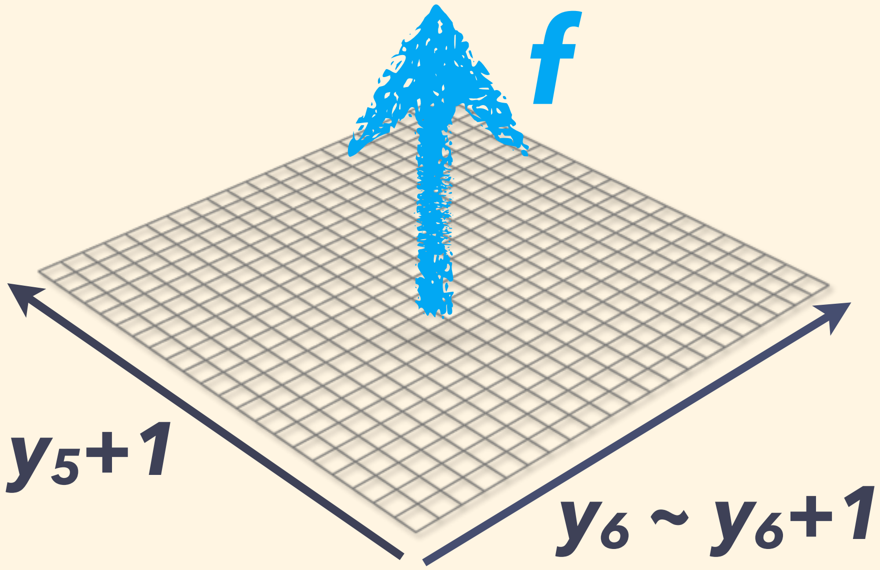
- Higher dimensional gauge theory with flux bg. can appear as effective theories of superstring theory, e.g.,



Harmonic oscillation in QM



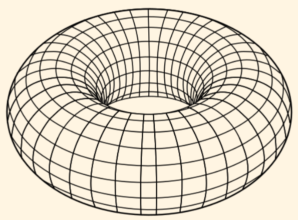
- We review 6D U(1) gauge theory with flux bg.
  - Extracting 2D as **torus** → 2D QM
- Flux: $\langle F_{56} \rangle = 2\pi M \equiv f$ is given by extra components of vector potential: $A_5 = -f y_6$ & $A_6 = 0$.
- Analogy to harmonic oscillation:



$$H = \frac{1}{2m}\left[(P_5 + f y_6)^2 + P_6^2\right]$$

Harmonic oscillation in QM

Torus T^2



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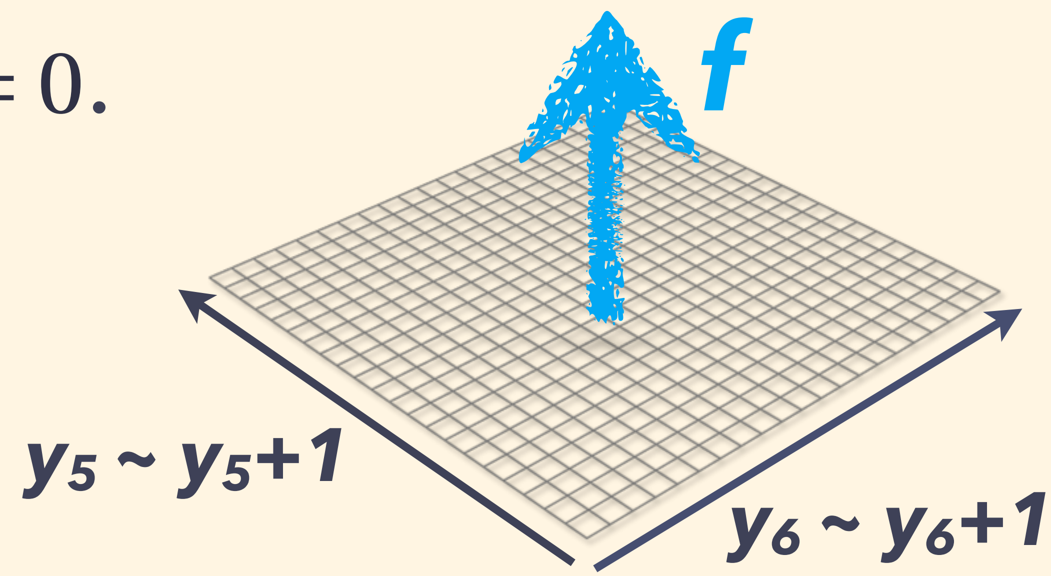


$$H = \frac{1}{2m}\left[(P_5 + f y_6)^2 + P_6^2\right]$$

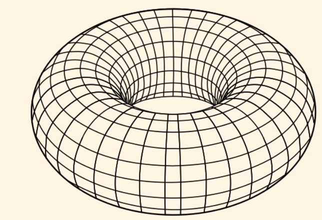
$$[H, P_5] = 0 \quad \to \quad P_5 = 2\pi j \qquad\qquad H = \frac{1}{2m}\left[f^2 (y_6 + j/M)^2 + P_6^2\right]$$

... simultaneously diagonalizable **for j = 0, 1, ..., M−1** ***w/ shifted position by j/M***

KK momenta = Landau levels.

Kaluza-Klein decomposition

Torus T^2



- KK mass spectrum:

scalar $$m_n^2 = \frac{4\pi M}{\mathscr{A}}\left(n+\frac{1}{2}\right)$$

spinor $$m_n^2 = \frac{4\pi M}{\mathscr{A}} n$$

analogous to
$H \sim \hbar\omega_c(n + 1/2)$

(e.g., by embedding our setup into 10D SYM or so)

We assume SUSY and "1/2"-term vanishes in scalars.

- KK modes (zero modes):

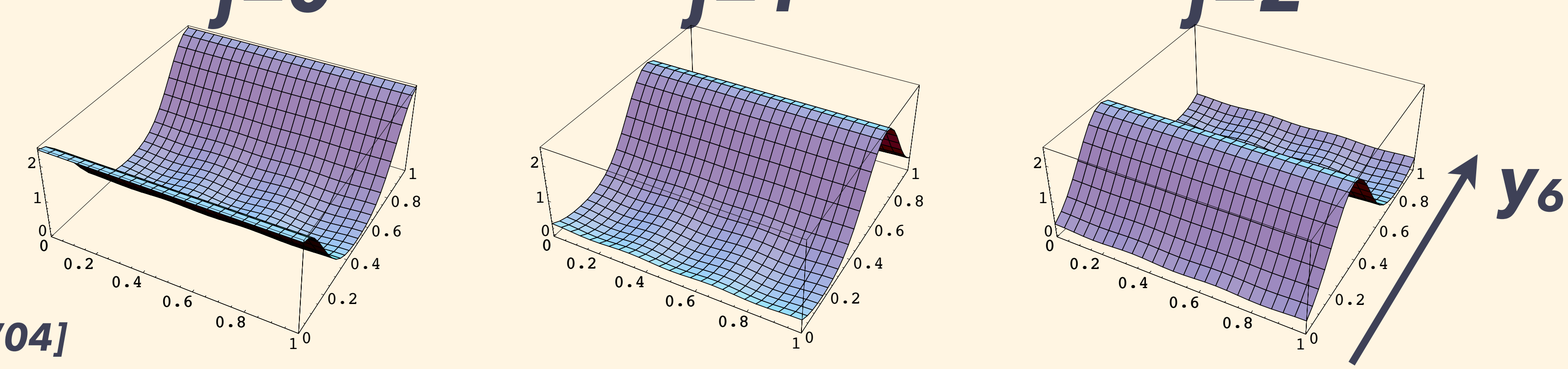
Gaussian

$$\psi_0^j \sim e^{2\pi i \cdot j y_5} e^{-\pi M \,\mathrm{Im}\,\tau (y_6 + j/M)^2} \qquad j = 0, 1, \ldots, M-1$$

family ←

$M = 3 \rightarrow \psi^0, \psi^1, \psi^2$

j=0 j=1 j=2



[Cremades, Ibanez, Uranga '04]

The μ-problem in the MSSM

- SUSY Higgs potential: ***μ*-term** is

$$W = \boxed{\mu} H_u H_d$$

mass dimension +1

- To realize radiative EW symmetry breaking at ~ 100 GeV, we need a small value of ***μ***, e.g., ~ 10^3 GeV (= 1 TeV).

(as well as soft terms. In this talk, we just focus on μ-term.)

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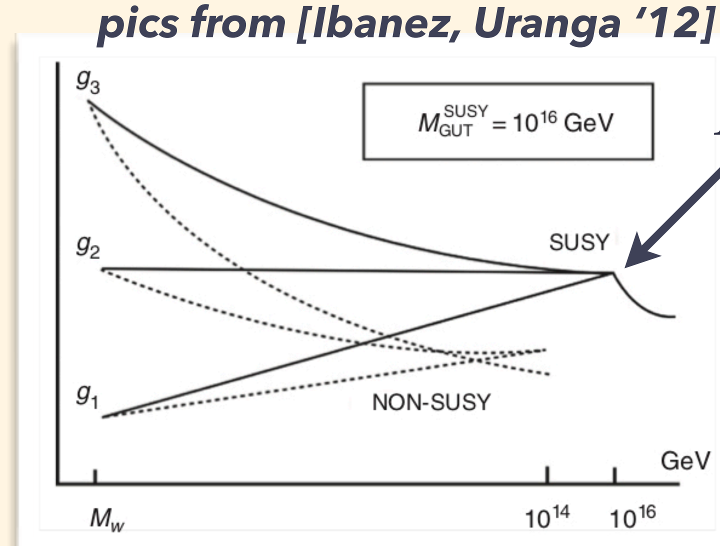
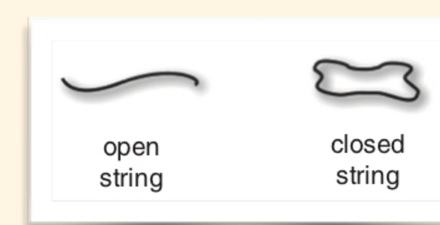
pics from [Ibanez, Uranga '12]



$M_{\rm unif} \sim 10^{16}$ GeV



μ

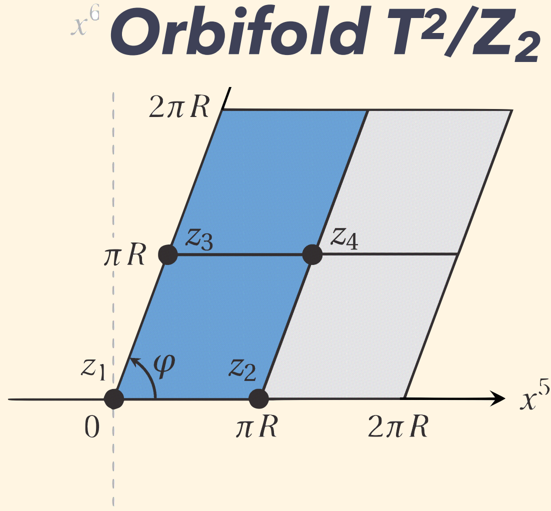
Why so smaller than M_{unif} or M_P?

[Kim, Nilles, '84]

10^3 **~10^{16} GeV** ***E***

Idea in this project

Orbifold T^2/Z_2



- Our idea is as follows:

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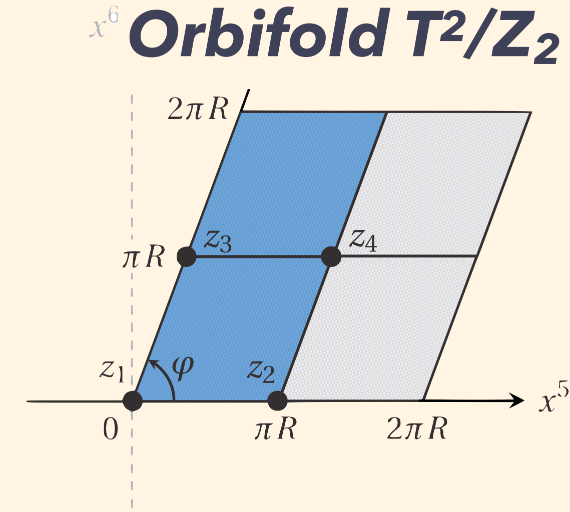
We put 6D μ-term:

$$W_{6D} = \mu\, H_u\, H_d\, \delta^2(z - \zeta)$$

Orbifold T^2/Z_2



fixed points

$$z_1 = 0, \quad z_2 = \frac{1}{2}$$
$$z_3 = \frac{\tau}{2}, \quad z_4 = \frac{1+\tau}{2}$$

Idea in this project

- Our idea is as follows:

$$z = y_5 + i y_6$$

$$\underbrace{\psi^{j}_{0,\eta}}_{T^2/Z_2} = \underbrace{\psi^{j}(z)}_{T^2} + \eta\psi^{j}(-z)$$

$$\sim e^{-\pi M (\mathrm{Im}\, z)^2 \mathrm{Im}\,\tau}$$

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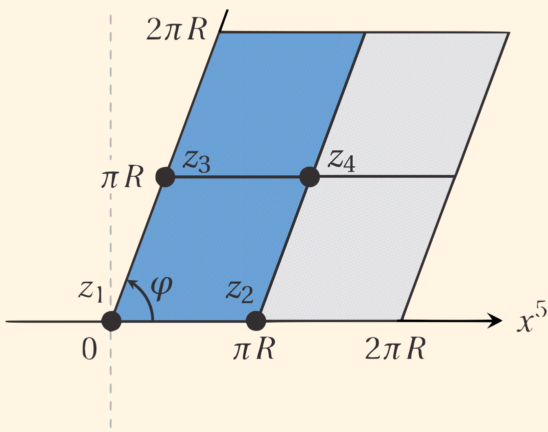
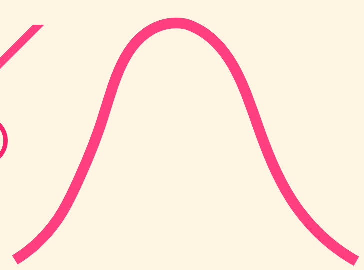


wavefunc.

$H_u^{4D} \otimes$



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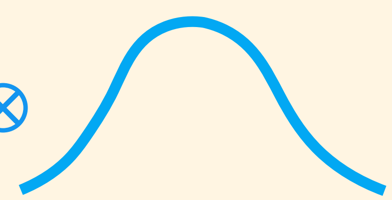

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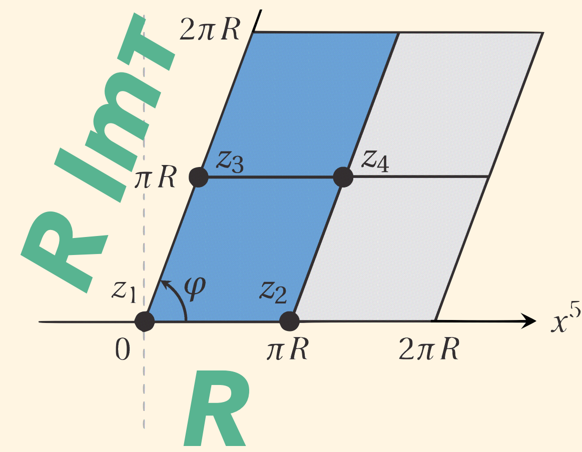

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$$W_{6D} = \mu H_u H_d \delta^2(z - \zeta)$$

dimensional reduction:

$$\int_{T^2/Z_2} d^2 z :$$

wavefunc.

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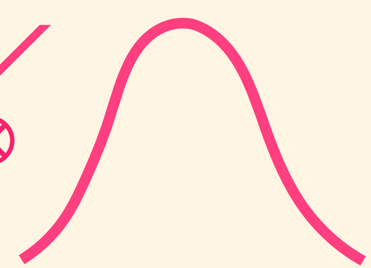

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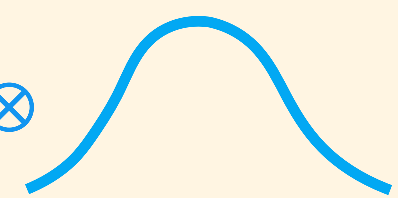

fixed points

$z_1 = 0, \quad z_2 = \frac{1}{2}$

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$$\boxed{W_{\text{eff}} \sim \mu e^{-c\, \mathrm{Im}\, \tau} H_u^{4D} H_d^{4D}}$$

effective μ-term in 4D

Numerical analysis

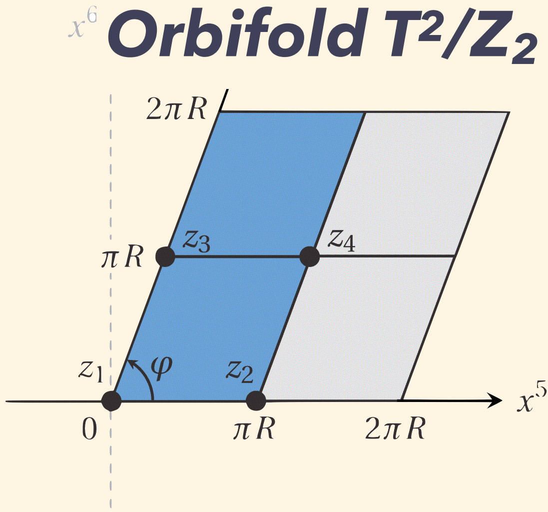
- Effective μ-parameter:

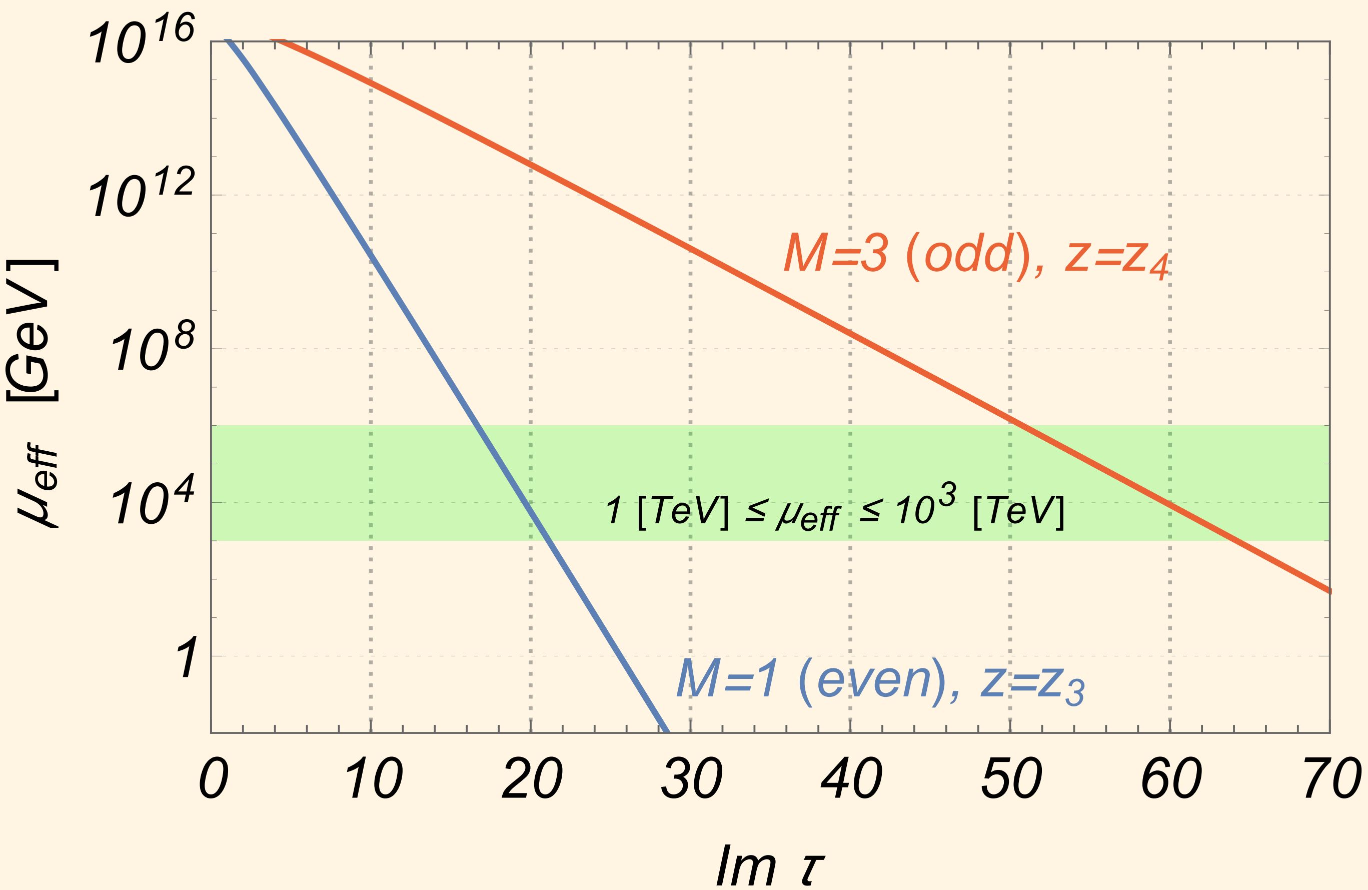
"c" depends on setups

$$\mu_{\rm eff} \sim M_C \exp(-c\,{\rm Im}\,\tau)\ {\rm GeV}$$

- D.o.f.'s: ***M*** & position of fixed points ***ζ***

Orbifold T^2/Z_2





As sample,

$\boldsymbol{M_C \sim 1/\sqrt{A} \sim 10^{16}}$ **GeV**

Setup I: $M = 1$, $\zeta = z_3$

Setup II: $M = 3$, $\zeta = z_4$

We need specific setups, but this is an interesting mechanism to lead to hierarchy.

How about multiple Higgses?

- Unfortunately, the setup tends to generate multiple Higgs doublets, if the Higgs field is charged under U(1).
- What's happen in cases with **multiple Higgs pairs?**
- $\boldsymbol{\mu_{eff}}$: light & $\boldsymbol{\mu_2}$: heavy eigenvalues of 2-by-2 μ-matrix
  - ...even if we set μ-matrix ~ M_C ~10^{16} GeV

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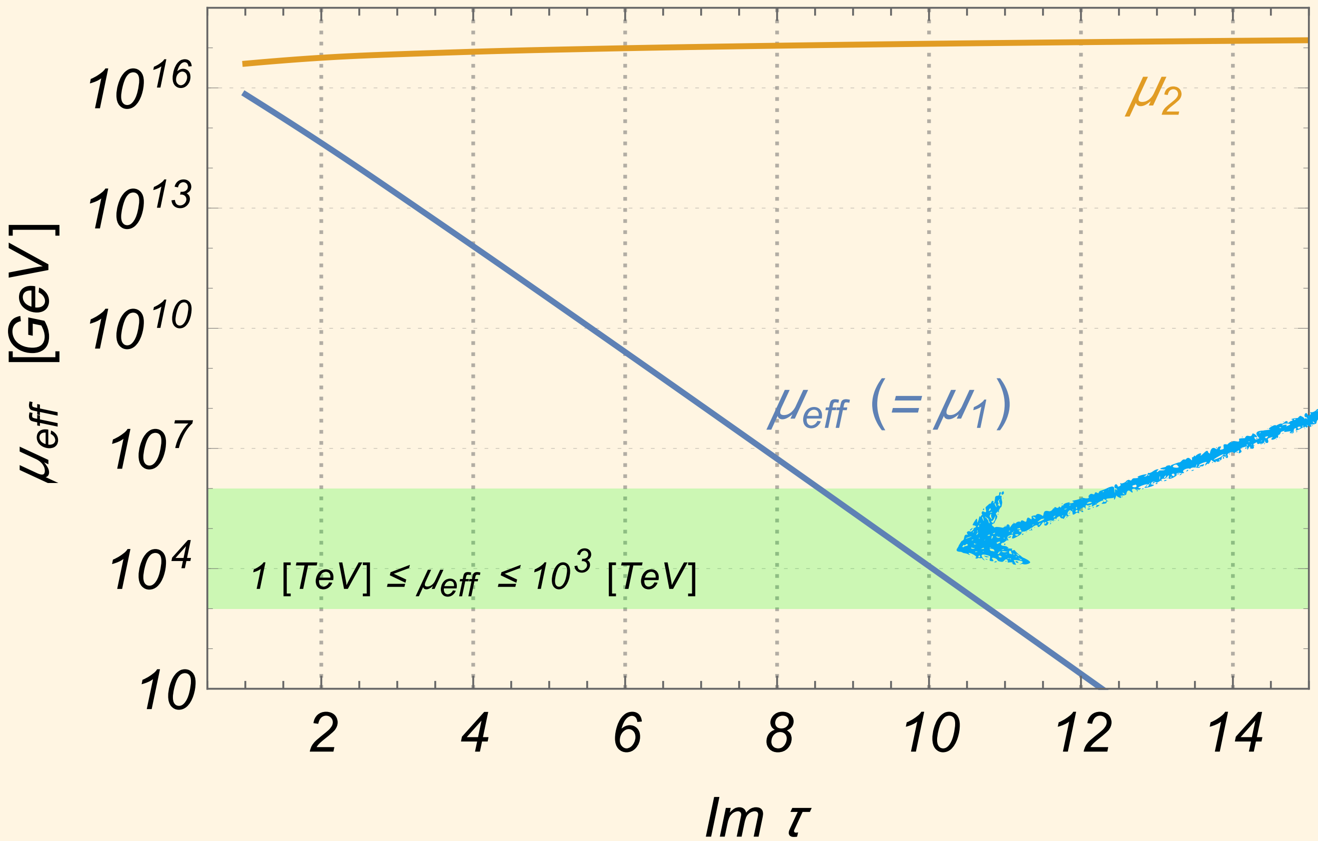


$\sim 10^{16}$ GeV

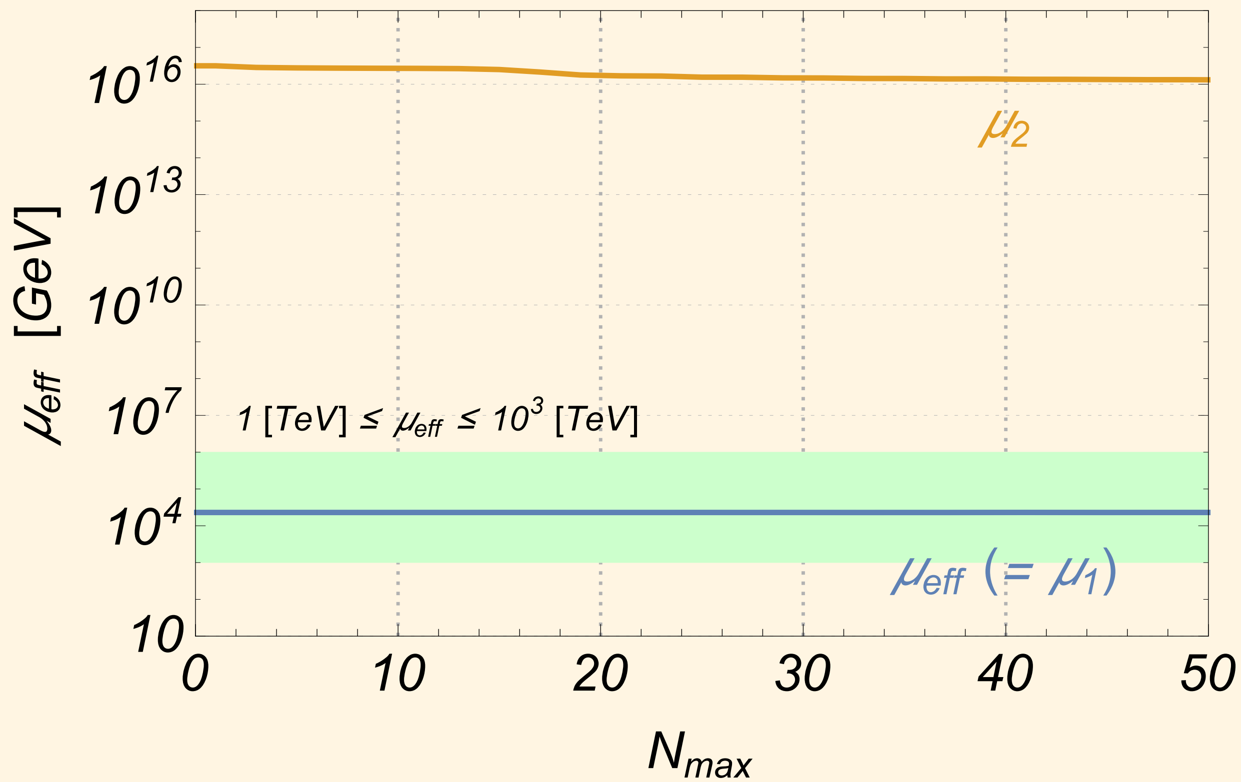
$\sim 10^{16} e^{-c\,\mathrm{Im}\,\tau}$ GeV

If $\mathrm{Im}\,\tau = \mathcal{O}(10)$ **,**

$\boldsymbol{\mu_{eff}}$ **~ TeV** $\ll$ $\boldsymbol{\mu_2}$ **~ 10^{16} GeV**

Momentum truncation

- So far, we focus only on massless modes. But, there are many massive KK modes. **Corrections from them?**
  - KK tower should be truncated at some KK level $n = N_{\max}$.
  - We call its scale **cutoff scale** $\Lambda \equiv m_{N_{\max}}$ $(= \sqrt{4\pi M N_{\max}/\mathcal{A}})$.



two pairs of MSSM Higgs:

Setup:
$M = 2$, Z_2 parity = +1

Effective μ-parameter is independent on "cutoff".

Summary

- Flux compactification plays important roles in the context of extra dim QFT & string phenomenology & cosmology.
  - Chiral matters, their families, …
- We have investigated effects of localized μ-terms on T^2/Z_2 fixed points.
  - In single Higgs, $\mathrm{Im}\,\tau = \mathcal{O}(10)$ → **small *μ*-term** around TeV
  - In multi Higgses, **split mechanism**: $\mu_{\rm eff} \ll \mu_2 < \dots$

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Flux can give an effectively small μ-term.
It would be a possibility to solve so-called μ-problem.

Thank you!

Backup

Why brane-localized masses?

- We seem to be ready. Why brane-localized masses?

formal interests

- A way to treat mass terms localized at fixed points
- Regularization of fixed points?
- Relation b/w # of modes, especially, zero-modes?

- Making unwanted modes sufficiently heavy
- Applications of mass terms to phenomenology

pheno. interests

Localized mass: 6D scalar

- Introducing a mass term localized at a fixed point z_i.
  - 6D scalar Lagrangian:

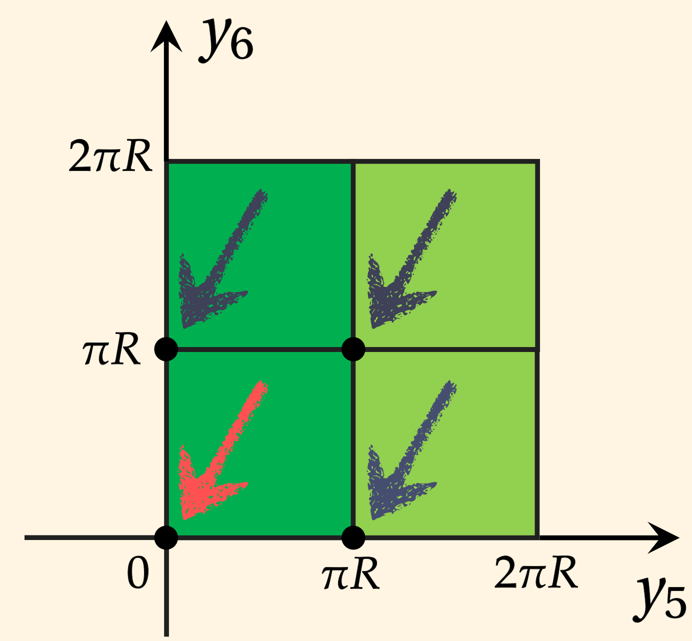
$$\mathscr{L} = -|D_M \Phi(x,z)|^2 - h|\Phi(x,z)|^2 \delta^2(z - z_i)$$

4D kin.

KK masses

localized mass

$$\Phi(x,z) = \sum_n \varphi_n(x) \otimes \phi_n(z)$$



$$z_1 = 0, \quad z_2 = \frac{1}{2}$$
$$z_3 = \frac{\tau}{2}, \quad z_4 = \frac{1+\tau}{2}$$

  - After dimensional reduction,

$$(M_{\mathrm{eff}})_{\{n,j\},\{n',j'\}} = m_n^2 \delta_{n,n'} \delta_{j,j'} + h\,(\phi^j(z_i))^* \phi^{j'}(z_i)$$

diagonal: KK masses

off diagonal: give by h & w.f.'s

Let's look at the simplest case

- Focus on **3-by-3** KK matrix = **two** lowest modes & **a** 1st KK

$\psi_n^{\,j}$ ← *degeneracy* (j)
← *KK level* (n)

$\psi_0^0(z), \psi_0^1(z)$ $\qquad$ $\psi_1(z)$

lowests

$$(v_1, v_2, v_3) = (\psi_0^0, \psi_0^0, \psi_1)\Big|_{z=0}$$

$$M^{KK}_{eff} \sim \begin{array}{cc} & \begin{array}{ccc} \psi_0^0(z) & \psi_0^1(z) & \psi_1(z) \end{array} \\ \begin{array}{c} \psi_0^0(z) \\ \psi_0^1(z) \\ \psi_1(z) \end{array} & \begin{pmatrix} m_0^2 & & \\ & m_0^2 & \\ & & m_1^2 \end{pmatrix} \end{array} + h \begin{pmatrix} |v_1|^2 & (v_1)^* v_2 & (v_1)^* v_3 \\ (v_2)^* v_1 & |v_2|^2 & (v_2)^* v_3 \\ (v_3)^* v_1 & (v_3)^* v_2 & |v_3|^2 \end{pmatrix}$$

1st

(Here, for η=1)

$v_i \sim O(0.1)$

- … gives eigen values approximately:

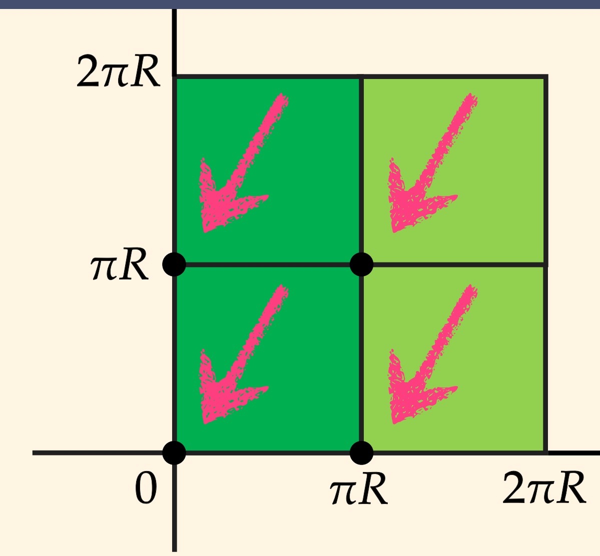
$\lambda_1 = m_0^2, \quad \lambda_2 = m_0^2, \quad \lambda_3 = m_1^2$

A $n=0$ mode is uplifted.

$\lambda'_1 = m_0^2, \quad \lambda'_2 = m_0^2(1+O(h)), \quad \lambda'_3 = m_1^2(1+O(h))$

Generic analysis w/ multi-masses

- We have four fixed points on T^2/Z_2.
  - For simplicity, focus on lowest modes:



$$M^{KK}_{eff} \sim \underbrace{\begin{pmatrix} m_0^2 & & & \\ & m_0^2 & & \\ & & \ddots & \\ & & & m_0^2 \end{pmatrix}}_{\textit{degenerate}} \underbrace{+ h_1 (v^1)^* v^1 + h_2 (v^2)^* v^2 + \dots}_{\textit{rank 4 at most}}$$

We can uplift four light exotics at most in eff. theory.

$$\left\{\begin{matrix} m_0^2 \\ m_0^2 \\ m_0^2 \\ m_0^2 \end{matrix}\right. \xrightarrow{\textit{mass}} \left\{\begin{matrix} m_0^2 \\ m_0^2 \\ m_0^2 \\ m_0^2(1+O(h_i)) \end{matrix}\right. \xrightarrow{\textit{mass}} \left\{\begin{matrix} m_0^2 \\ m_0^2 \\ m_0^2(1+O(h_i)) \\ m_0^2(1+O(h_i)) \end{matrix}\right. \xrightarrow{\textit{mass}} \cdots$$

Localization of modes

- Localization profiles of modes uplifted by a localized mass
  - Lowest modes:

light: localization *dark: delocalization*

$\psi'^{0}_{0}(z)$: m_0^2 $\psi'^{1}_{0}(z)$: $m_0^2(1+O(h))$ $\psi'_1(z)$: $m_1^2(1+O(h))$

Modes unaffected by localized mass are away from the fixed points.