D-brane description of charged black holes with the help of the Dixmier-Douady invariant

Tetiana V. Obikhod

Institute for Nuclear Rsearch NAS of Ukraine, 03028, Kiev, Prosp. Nauki 47, Ukraine

Plan of the talk

- The resolution of the black hole problem;
- Dp-branes as supergravity solutions;
- Dp-branes as Chan-Paton vector bundles;
- Classification of Dp-branes with twisted K-groups;
- Conclusions.

The resolution of the black hole problem

$$S = \frac{A_H}{4G_N \hbar}$$

$$dM = T_H dS$$

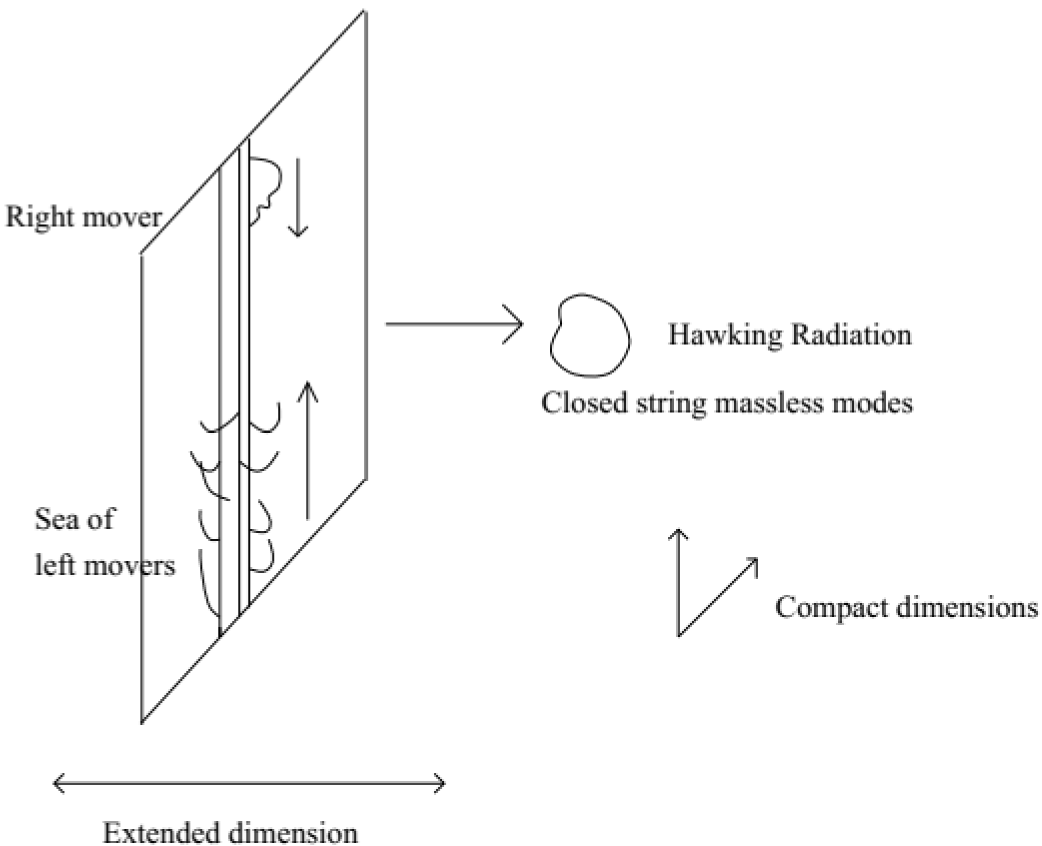
$\Downarrow$

Increases
First and second

Laws of thermodynamics

BUT

Hawking radiation leads to
decreasing of black hole mass
and so does the area
of the horizon

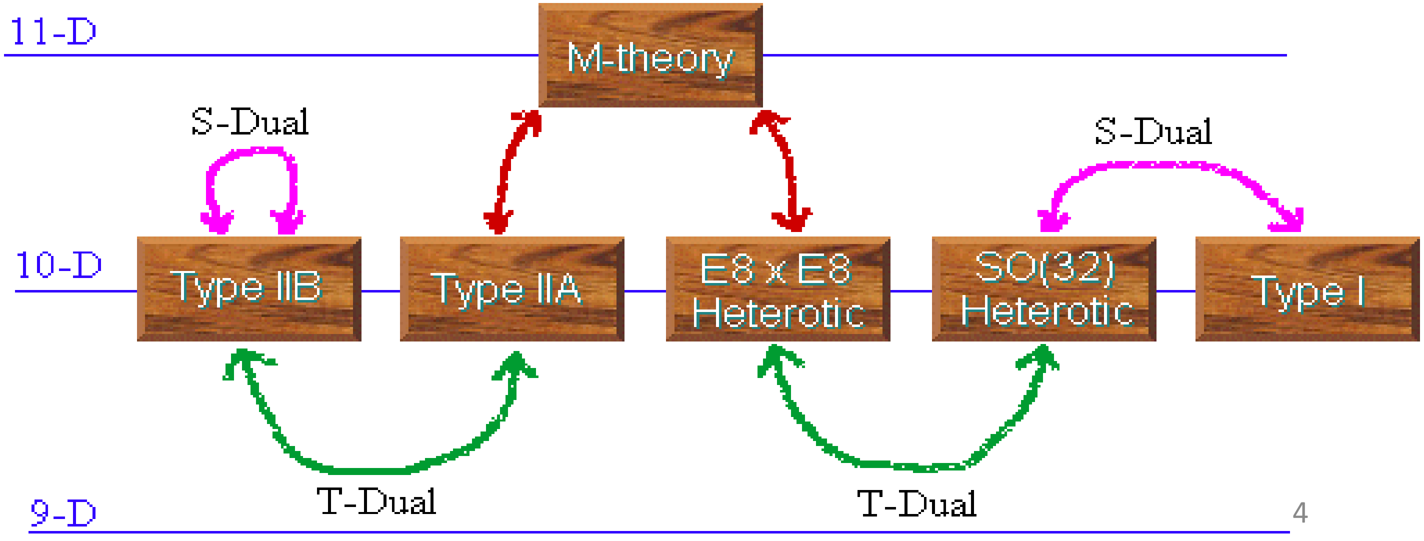


M THEORY

Yang-Mills theory is embedded in type I or heterotic theory. How to connect **IIA** and **HE** – when *g* is large?

M-theory and type IIB theory are connected through the invariance of moduli space under U duality group at toroidally compactification.

an eleventh dimension of size *g*L. This new dimension is a circle in the **IIA** case and a line interval in the **HE** case



BRANE WORLDS

D-branes are objects with strings which end on them. Standard model can live on D-branes. The observable Universe is a set of 3-branes, which is embedded in a space with 6 additional spatial dimensions.

Properties:

- the dependence on the coupling constant, tension is given by $T_{Dp} = 2\pi m_s^{p+1}/g_s$;
- carry the charge that couples to gauge field in the RR sector of the theory.

Polchinski-Grothendieck Ansatz

(Chien-Hao Liu and Shing-Tung Yau)

A D-brane is geometrically a locus in space-time that serves as the boundary condition for open strings. Through this, open strings dictate also the fields and their dynamics on D-branes. In particular, when a collection of D-branes are stacked together, the fields on the D-brane that govern the deformation of the brane are enhanced to matrix-valued, cf. Polchinski in [Po: vol. I, Sec. 8.7]. This open-string-induced phenomenon on D-branes, when re-read from Grothendieck's contravariant equivalence between the category of geometries and the category of algebras, says that D-brane world-volume carries an Azumaya-type noncommutative structure. I.e.

· Polchinski-Grothendieck Ansatz : D-brane has a geometry that is generically locally associated to algebras of the form Mr(R), where R is an R-algebra.

Dp-branes as supergravity solutions

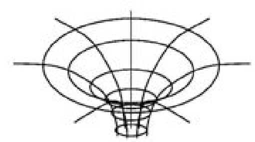
$$ds^2 = H^{-1/2}(r)[-dt^2 + d\vec{x}^2] + H^{1/2}(r)[dr^2 + r^2 g_{ij} dx^i dx^j]$$

where $H(r) = 1 + \frac{L^4}{r^4}, \qquad L^4 = 4\pi g_s N (\alpha')^2$

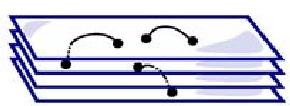
★ gauge/gravity duality

cf)

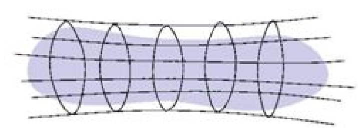
particle



Solution of Einstein eq.



D-brane



Supergravity solution

<u>Conjecture</u>

The information of the gauge theory on the D-brane is somehow encoded in the supergravity solution.

ADS/CFT DUALITY

In 1997 Maldacena proposed a new class of dualities (or equivalences) – for example, between a certain 4d QFT called N = 4 super Yang-Mills theory and Type IIB superstring theory in the 10d geometry $AdS_5 \times S^5$. SYM includes in Hilbert space the states of type IIB on $AdS_5 \times S^5$.

It is associated to the conformal boundary of the 10d or 11d spacetime. Since the QFT is conformally invariant (CFT), this is called an AdS/CFT duality.

Dp-branes as supergravity solutions

In the papers [hep-th-1310.5996, 1302.5265] Dirac quantization approach of the Schwarzschild black hole leads to the exact solutions of the physical Hilbert space.

Theorem (Dixmier and Douady)

Algebra bundles whose fiber is the algebra of compact operators on Hilbert space are in one-to-one correspondence with elements of

$$[H] \in H^3(X, \mathbb{Z})$$

Principal bundles

$$\begin{array}{ccc} SU(n)/\mathbb{Z}_n & \rightarrow & P_H \\ & & \downarrow \\ & & X \end{array} \qquad H_{\mu\nu\lambda} = 0\ , \quad B_{\mu\nu} \neq 0\ ;$$

$$\begin{array}{ccc} \lim\limits_{n\to\infty} SU(n)/\mathbb{Z}_n & \rightarrow & P_H \\ & & \downarrow \\ & & X \end{array} \qquad H_{\mu\nu\lambda} \neq 0\ , \quad B_{\mu\nu} \neq 0$$

$$[H] \in H^3(X, \mathbb{Z})$$

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$$

Dp-branes as vector bundles

$$E_H = P_H \times M_n(\mathbb{C}) \ , \quad \text{где} \quad Aut(M_n(\mathbb{C})) = SU(n)/\mathbb{Z}_n$$

$$E_H = P_H \times \mathcal{K} \ , \quad \text{где} \quad Aut(\mathcal{K}) = \lim_{n\to\infty}\text{-}SU(n)/\mathbb{Z}_n \ ;$$

$(M_n(\mathbb{C}))$ - nxn matrix algebra

$\mathcal{K}$ - algebra of compact operators on ***H***

The space of sections of a vector bundle is the Azumaya or Rosenberg C* algebra, $C_0(X, E_H)$

$$K_j(X, [H]) = Ext(X \times \mathbb{R}^j, \ p_1^*[H]), \quad j \in \mathbb{N}$$

The set of unitarily equivalent classes of extensions of C * with the help of the algebra K modulo the splitting extensions

Nonlinear boundary value problems, Issue 13, 2003, p. 114-117.

Classification of Dp-branes with twisted K-groups

Theorem of Rosenberg: there exists the classifying element for $P\mathscr{U}'$ - bundle $Y \to \overline{X}$, $H^1(\overline{X}, P\mathscr{U}')$, classifying element $p: X \to \overline{X}$ in $H^1(\overline{X}, \mathbf{Z}_2)$ of $P\mathscr{U}$ - bundle $Y \to X$. Dixmier-Douady class defined by

$$\begin{array}{ccc} X & \xrightarrow{\delta} & BP\mathscr{U} \\ \downarrow{\scriptstyle p} & & \downarrow \\ \overline{X} & \xrightarrow{\phi} & BP\mathscr{U}' \end{array}$$

$$\delta \in [X, BP\mathscr{U}] \cong H^3(X, \mathbf{Z})$$

$$\begin{array}{ccc} P\mathscr{U} \to Y & & P\mathscr{U}' \to Y \\ \downarrow & \Longrightarrow & \downarrow \\ X & & \overline{X} \end{array}$$

Types of D-branes

- D-branes on compact spaces (Yu.Malyuta)
- D-branes on orbifold (this talk);
- D-branes on conifold (Klebanov and Strassler).

Conclusions

- With vector bundles where the fibers are C * -algebras of sections that are isomorphic to the algebra of compact operators on a Hilbert space, the principal bundles are associated. Within the framework of group extension $1 \to P\mathcal{U} \to P\mathcal{U}' \to Z_2 \to 1$ there exists a Dixmier-Dwady invariant in accordance with the pull-back diagram. Then one can obtain transitions between principal bundles.
- Since with Dixmier-Dwady invariant is asscociated the interaction of D- branes with the Neve-Schwartz B-field and the sections of associated vector bundles are the Hilbert spaces describing microstates of black holes, therefore, the transitions between the principal bundles are the phase transitions between black holes, classified by twisted K groups.