Group Theory Aspects of Non-Supersymmetric Heterotic Partition Functions

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in collaboration with

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[1] at this conference

Introduction

- SUSY breaking string models have received a lot of interest in the past decades
- In particular, there is the SO$(16) \times$ SO(16) heterotic string with $\mathcal{N} = 0$ in 10 dimensions
- Alternatively, one may break SUSY during compactification: e.g. Scherk–Schwarz, or particular orbifold geometries

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In general, heterotic non-SUSY string vacua suffer from a bunch of problems, such as a too large cosmological constant and instabilities

Cosmological constant/dilaton tadpole

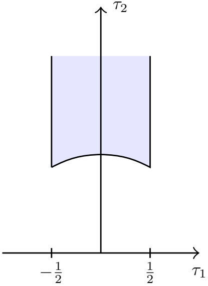
As one can see from general arguments, the non-SUSY heterotic partition function $\mathcal{Z}(\tau, \overline{\tau})$ is nonzero

Dienes '90

$$\Lambda \sim \int \frac{\mathsf{d}^2\tau}{\tau_2^2}\, \mathcal{Z}(\tau, \overline{\tau})$$

With the most prominent contribution coming from *off-shell tachyons*

Abel, Dienes, Mavroudi '15



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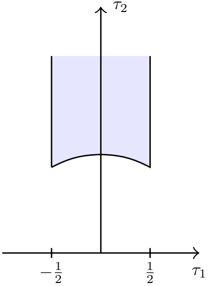

There are various proposals for this to vanish

- (space-time) Supersymmetry
- (generalized) Atkin-Lehner symmetry

Moore '87, Dienes

Heterotic Orbifolds

Orbifolds can be obtained in two subsequent steps

- define and mod out a lattice $\{e_\alpha\}_{\alpha=1,\ldots,D}$ to obtain a torus
- mod out a discrete isomorphism of the lattice (point group P)

For Abelian point groups

$$\mathrm{SO}(6) \supset D_{\mathbf{6}}(g) \;=\; \begin{pmatrix} e^{2\pi i v_1 J_{12}} & & \\ & e^{2\pi i v_2 J_{34}} & \\ & & e^{2\pi i v_3 J_{56}} \end{pmatrix} \;\rightarrow\; v_g = (v_1, v_2, v_3)$$

$\mathcal{N} = 1$ SUSY $\leftrightarrow \sum v_i = 0 \mod 2$

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Heterotic Orbifolds arise by an embedding of the geometric rotations and translations into the $E_8 \times E_8$ d.o.f.

The Heterotic Partition Function

The partition function can be organized

$$\mathcal{Z} = \sum_{g,h} \mathcal{Z}\begin{bmatrix} g \\ h \end{bmatrix} \quad \text{where } g, h \in \text{ space group}$$

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However, the unbroken SUSY transformations need *not* be the same for all (g, h) pairs

Local "SUSY enhancement": An $\mathcal{N} = 2$ Example

A $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with roto-translation

Fischer, Ratz, Torrado, Vaudrevange '12

$$v_\theta = \left(0, \frac{1}{2}, -\frac{1}{2}\right), \quad v_\omega = \left(\frac{1}{2}, -\frac{1}{2}, 0\right)$$

with generators $$g_\theta = \left(\theta \,\middle|\, \frac{1}{2} e_2\right), \quad g_\omega = (\omega \,|\, 0)$$

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Because of the roto-translation, $[g_\theta, g_\omega] \neq 0$, and hence

$$\mathcal{Z} \not\supset \mathcal{Z}\begin{bmatrix} g_\theta \\ g_\omega \end{bmatrix}$$ (the same holds for all $\mathrm{SL}(2,\mathbb{Z})$ images)

$\mathcal{N} \geq 2$ SUSY in each (g, h)-twisted sector,

but

$\mathcal{N} = 1$ in the intersection!

Q: Can one achieve the same for $\mathcal{N} = 0 \to \mathcal{N} \geq 1$?

Four dimensional spinor representation

On the space-time fermions, each $D_{\mathbf{6}}(g) \in \mathbf{6}$ of $\mathrm{SO}(6)$ corresponds to a $D_{\mathbf{4}}(g) \in \mathbf{4}$

Because $\mathrm{Spin}(6) \cong \mathrm{SU}(4)$

$\mathbf{4}$ needs to be in $\mathrm{SU}(4)$, i.e. $\det D_{\mathbf{4}}(g) = 1$ for all g

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Caveats

- Elements of the geometric space group have more than one embedding into spinor space $\rightarrow$ Witten twist
- Possible degeneracies lead to spinor embeddings that are non-isomorphic to the geometric point group

For our considerations: do not care too much about whether spinor embedding corresponds to geometric embedding (just look at all possible actions on spinor space)

Realizing $\mathcal{N} = 0$

Every element of the point group commutes with the identity, therefore the untwisted partition function is given by

$$\mathcal{Z}_{\text{untw.}} \;=\; \sum_{g\in P} \mathcal{Z}\begin{bmatrix}\mathbb{1}\\ g\end{bmatrix}$$

If we insist on SUSY breaking, then the projector of the untwisted sector must have trace 0

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or in terms of the representation content

4 does not contain a trivial singlet of P

Condition 2

Interplay: Modular orbits vs. projectors

Let us for the moment forget about the space group and consider the point group only

Then, all constructing elements g are just elements of a finite group, i.e. $\exists N$ s.t. $g^N = \mathbb{1}$

$$\mathcal{Z}\begin{bmatrix}\mathbb{1}\\ g\end{bmatrix} \xrightarrow{S} \mathcal{Z}\begin{bmatrix}g\\ \mathbb{1}\end{bmatrix} \xrightarrow{T} \mathcal{Z}\begin{bmatrix}g\\ g\end{bmatrix} \xrightarrow{T^{N-1}} \mathcal{Z}\begin{bmatrix}g\\ \mathbb{1}\end{bmatrix}$$

There may exist sectors that are not connected to these orbits, e.g. in Abelian $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifolds.

We always assume these sectors can be removed

Conditions for $\mathcal{N} \geq 1$ locally

We are effictively dealing with projectors of the type

$$\sum_{n=0}^{N-1} \mathcal{Z} \begin{bmatrix} g \\ g^n \end{bmatrix} \xrightarrow{\text{in group theory language}} \frac{1}{N} \sum_{n=0}^{N-1} \operatorname{tr} D_{\mathbf{4}}(g^n)$$

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Then our rationale becomes

$\mathcal{Z}\begin{bmatrix} \mathbb{1} \\ g \end{bmatrix}$ and entire modular orbit vanishes $\Leftrightarrow$ g preserves at least one Killing spinor $\Leftrightarrow$ **4** branches to trivial singlet $\mathbf{1}_0$ of $\mathbb{Z}_N^{(g)}$

A group-theoretical conjecture

Conjecture. For a given discrete group G, there does not exist a four dimensional representation with the properties that

(i) it has determinant 1,

(ii) it does *not* contain a *trivial singlet of G*,

(iii) to every $\mathbb{Z}_N$ subgroup of G it branches into at least one *trivial singlet of that* $\mathbb{Z}_N$ *subgroup*.

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It is crucial to insist on $\det = 1$ and dimension 4!

A chance for (generalizations of) Atkin–Lehner symmetry?

Idea: if $\mathcal{Z}(\tau,\overline{\tau}) \neq 0$, it might still integrate to zero

- Guaranteed if e.g. $\mathcal{Z}(-\frac{1}{N\tau}, -\frac{1}{N\overline{\tau}}) = -\mathcal{Z}(\tau,\overline{\tau})$ known as *Atkin–Lehner symmetry* Moore '87
- does not seem to work (at least not with four non-compact dimensions) TR Taylor '87, Balog and Tuite '87

but:

- We see that a large portion of the $\mathcal{Z}\begin{bmatrix}\mathbb{1}\\ g\end{bmatrix}$ vanishes even if $\mathcal{N}=0$ globally
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work in progress

Conclusions

- we have translated the vanishing condition for (g, h)-twisted sectors to a condition for the branching of the **4** to $\mathbb{Z}_N$ subgroups
- we have formulated (and tested) a general group-theoretic conjecture for a No-Go
- the partial results point towards alternative solutions to the cosmological constant problem in (heterotic) non-SUSY strings

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Thank You!