Non Abelian T-dualities in Gauged Linear Sigma Models

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Motivation

T-Duality is a $R \to \alpha'/R$ symmetry of string theory which exchanges winding modes with momentum modes.

Mirror symmetry: Exchanges Kähler deformations with complex deformations on dual manifolds. It is connected to T-duality [Morrison, Plesser, 95][Strominger, Yau, Zaslow, 96].

The Gauged Linear Sigma Models (GLSMs) [Witten, 93] in 2D with $(2,2)$ SUSY are a powerful tool to describe strings propagating in a Kähler manifold. Under the renormalization group GLSMs have infrared points with conformal invariance: strings NLSM.

[Hori, Vafa, 00] developed a proof of mirror symmetry for supersymmetric sigma models on Kähler manifolds, based on studying Abelian T-dual models of the GLSMs.

Motivation

We study non-Abelian T-duality [Buscher, 88][de la Ossa, Quevedo, 93][Giveon, Rocek, 94] in GLSMs.

We aim to explore the connections between GLSM non Abelian T-dualities and symmetries between target manifolds of string theory. For example to explore mirror symmetry on determinantal CY varieties [Jockers,Kumar,Lapan,Morrison,Romo, 12].

Non-abelian GLSMs [Hori,Tong,07] provide more general ambient spaces than toric varities. CY manifolds in these spaces are related to determinantal, Pfaffian and more general non-complete intersection varieties. [Căldăraru,Sharpe,Knapp, 17] studied Grassmanian embeddings and Pfaffian CYs related by dualities [Hori,13]. Those are [Seiberg,94] dualities in GLSMs.

It is estimated that there is a far bigger class of non-complete intersections CYs [Tonoli, 01] than of complete intersections CYs [Kreuzer, Skarke,00].

Summary

We construct the Abelian T-duality on a 2D $\mathcal{N} = 2$ U(1) GLSM as a gauging of a global U(1) symmetry and the addition of Lagrange multipliers. This procedure in contrast to [Hori, Vafa, 00] allows a generalization to the non-Abelian case.

We describe the non-Abelian T-duality on the 2D $\mathcal{N} = 2$ U(1) GLSM by gauging global non-Abelian symmetries.

We obtain the e.o.m. to describe the dual model. We solve them for the SU(2) group for a choice of vector superfield obtaining the dual model with a generated twisted superpotential, and the dual vacuum manifold of the GLSM.

Non perturbative corrections: Making an ansatz for the instantons, the effective potential for the U(1) gauge field on the original theory matches the one of the dual theory.

$(2,2)$ Gauged Linear Sigma Models in 2d

The reduction to 2d of $\mathcal{N}=1$ 4d supersymmetric $U(1)$ gauge field theory gives rise to a $\mathcal{N}=2$ (2,2) 2d theory.

The $\mathcal{N}=1$ superfield language is employed to describe chiral superfields (csf) $\bar{D}_{\dot{\alpha}}\Phi=0$, antichiral superfields (acsf) $D_{\alpha}\bar{\Phi}=0$ and vector superfields (vsf) $V^{\dagger}=V$.

In the duality twisted chiral superfields (twisted-csf) arise. The twisted-csf satisfy $D_{+}X=\bar{D}_{-}X=0$ and the twisted-acsf satisfy $\bar{D}_{+}\bar{X}=D_{-}\bar{X}=0$.

The Lagrangian of a GLSM with gauge group $U(1)$ with vsf V_0 and N csfs Φ_i with charge Q_i can be written as [Hori, Vafa, 00]

$$L_0=\int d^4\theta\left(\sum_{i=1}^{N}\bar{\Phi}_i e^{2Q_iV_0}\Phi_i-\frac{1}{2e^2}\bar{\Sigma}_0\Sigma_0\right)-\frac{1}{2}\int d^2\tilde{\theta}t\Sigma_0+c.c., \qquad (1)$$

where $t=r-i\theta$. The parameters are the $U(1)$ gauge coupling e, the Fayet-Iliopoulos(FI) term r and the Theta angle θ.

$(2,2)$ Gauged linear sigma models in 2d

The twisted field strength for V_0 is given by $\Sigma_0 = \frac{1}{2}\bar{D}_+ D_- V_0$.

Generically there are $N-1$ global $U(1)$ symmetries.

We implement T-Duality of a $(2,2)$ GLSM with gauge group $U(1)$ with vsf V_0 and 2 csfs $\Phi_{1,2}$ with charges Q_1 and Q_2.

There is 1 global $U(1)$: the phase rotations of Φ_1 and Φ_2 modulo the $U(1)$ gauge transformations. This is gauged to obtain the vector field V and we add a Lagrange multiplier.
The T-dualization is implemented with respect to Φ_1 as

$$\begin{aligned} L_1 &= \int d^4\theta \left(\bar{\Phi}_1 e^{2Q_0 V_0 + 2QV} \Phi_1 + \bar{\Phi}_2 e^{2Q_{0,2} V_0} \Phi_2 + \Psi\Sigma + \bar{\Psi}\bar{\Sigma} - \frac{1}{2e^2}\bar{\Sigma}_0 \Sigma_0 \right) \quad (2) \\ &+ \frac{1}{2}\left(-\int d^2\tilde{\theta} t \Sigma_0 + c.c. \right). \end{aligned}$$

Abelian T-duality in the GLSM

Integrating the Lagrange multipliers Ψ and $\bar{\Psi}$ one obtains the original model.

Dual model: We integrate V to obtain $\bar{\Phi}_1 e^{2Q_0 V_0 + 2QV} \Phi_1 = (\Lambda + \bar{\Lambda}) /(2Q)$, with $\Lambda = \bar{D}_+ D_- \Psi$. The dual Lagrangian reads

$$
\begin{aligned}
L_1 &= \int d^4\theta \left(-\frac{\Lambda + \bar{\Lambda}}{2Q} \ln\left(\frac{\Lambda + \bar{\Lambda}}{2Q}\right) + \bar{\Phi}_2 e^{2Q_{0,2} V_0} \Phi_2 \right) \\
&+ \frac{1}{2}\left(\int d^2\tilde{\theta} (\Lambda Q_0/Q - t)\Sigma_0 + \int d^2\bar{\tilde{\theta}} (\bar{\Lambda} Q_0/Q - \bar{t})\bar{\Sigma}_0 \right), \\
&- \int d^4\theta \left(\frac{1}{2e^2} \bar{\Sigma}_0 \Sigma_0 \right).
\end{aligned} \tag{3}
$$

Mirror symmetry: The csf Φ_1 is exchanged by the twisted csf Λ [Hori, Vafa, 00].

Non Abelian T-duality

The action can have non-Abelian global symmetries. We gauge these symmetries to describe non Abelian T-dual models.

Local non-Abelian transformations for superfields are required. Consider a non Abelian group with generators T_A, a chiral superfield Φ transforms as $\Phi' = e^{i\Lambda}\Phi$, $\Lambda = \Lambda^A T_A$, $\bar{D}_{\dot{\alpha}}\Lambda = 0$, with Λ a csf in the adjoint of the group.

A vsf transforms as $e^{V'} = e^{i\bar{\Lambda}} e^V e^{-i\Lambda}$, $V = V^A T_A$. The twisted chiral gauge field strength $\Sigma = \frac{1}{2}\{\bar{\mathcal{D}}_+, \mathcal{D}_-\}$ and its conjugate transform as

$$\Sigma \rightarrow e^{i\Lambda}\Sigma e^{-i\Lambda},\ \bar{\Sigma} \rightarrow e^{i\bar{\Lambda}}\bar{\Sigma}e^{-i\bar{\Lambda}}. \tag{4}$$

Non Abelian T-duality

Consider a GLSM with Abelian gauge group $U(1)$, N chiral superfields $\Phi_{k,i}$ with charges Q_k, $\sum_k n_k = N$, $i = 1, ..., n_k$ where n_k is the number of superfields with charge n_k.

There is a non-Abelian global symmetry G that can be gauged to obtain the vsf V. By adding a Lagrange multiplier Ψ one gets a new action.

$$\begin{aligned} L_2 &= \int d^4\theta \left(\sum_k \bar{\Phi}_{k,i} (e^{2Q_k V_0 + V})_{ij} \Phi_{k,j} + \mathrm{tr}(\Psi \Sigma) + \mathrm{tr}(\bar{\Psi}\bar{\Sigma}) \right). \\ &- \int d^4\theta \frac{1}{2e^2} \bar{\Sigma}_0 \Sigma_0 + \frac{1}{2} \left(- \int d^2\tilde{\theta} t \Sigma_0 + c.c. \right). \end{aligned} \tag{5}$$

Integrating out Ψ the original action is obtained.

For $i, j = 1, 2$ and $Q_1 = Q_2$ it gives as SUSY vacua the $\mathbb{CP}^1$ variety.

Non Abelian T-duality

Integrating out V the dual action is obtained. One gets the equation of motion

$$0 = (D_+\bar{D}_-\bar{\Psi}_a + D_-\bar{D}_+\Psi_a + \{\chi, D_+\Psi_a\} + \{\bar{\chi}, \bar{D}_+\bar{\Psi}_a\} + (\bar{\Phi}e^{2QV_0}e^{V}T_a\Phi))\Delta V_a, \quad (6)$$

with $\chi = e^{-V}D_-e^{V}$ and $\Delta V = e^{-V}\delta e^{V}$ is gauge invariant.

Let us work now for the SU(2) global symmetry case: This requires two chiral sf with equal charge under the U(1) gauge symmetry:

$$\begin{aligned} L_{model} &= \int d^4\theta \left(\bar{\Phi}_i (e^{2QV_0+V})_{ij}\Phi_j + \mathrm{tr}(\Psi\Sigma) + \mathrm{tr}(\bar{\Psi}\bar{\Sigma})\right) \quad (7) \\ &- \int d^4\theta \frac{1}{2e^2}\bar{\Sigma}_0\Sigma_0 - \frac{1}{2}\left(\int d^2\tilde{\theta}\, t\Sigma_0 + c.c.\right), \end{aligned}$$

with $i, j = 1, 2$, and the chiral and anti-chiral superfields constitute $SU(2)$ doublets.

GLSM with and SU(2) global symmetry

Integrating out V one gets:

$$e^{2QV_0}\bar{\Phi}_i e^{V}_{ij}\Phi_j = \sqrt{(X_1+\bar{X}_1)^2+(X_2+\bar{X}_2)^2+(X_3+\bar{X}_3)^2}. \qquad (8)$$

We take an Abelian direction in $SU(2)$ for the vsf $V = n_a\sigma_a$ with $n_a = const$, $\sum_a n_a^2 = 1$. The twisted-csf and the twisted gauge field strength have components

$$\begin{aligned} X_i &= x_i + \sqrt{2}\theta^+\bar{\chi}_+ + \sqrt{2}\bar{\theta}^-\chi_- + 2\theta^+\bar{\theta}^- G_i + ..., \qquad (9)\\ \Sigma_0 &= \sigma_0 + i\sqrt{2}\theta^+\bar{\lambda}_+ - i\sqrt{2}\bar{\theta}^-\lambda_- + 2\theta^+\bar{\theta}^-(D - iF_{01}) + ... \end{aligned}$$

In this case $\mathrm{tr}(\Psi\Sigma) = \frac{1}{2}X_a n_a|V|$. Integrating the vector superfield we have

$$|V| = 2QV_0 + \ln 2|\Phi_1|^2 + \ln(\mathcal{K}(X_i, \bar{X}_i, n_j)). \qquad (10)$$

Considering a gauging direction $V = |V| n_a \sigma_a$

The dual Lagrangian may be written as

$$\begin{aligned} L_{dual} &= \int d^4\theta \sqrt{(X_1+\bar{X}_1)^2+(X_2+\bar{X}_2)^2+(X_3+\bar{X}_3)^2} + \\ &+ \frac{1}{2}\int d^4\theta X_a n_a \ln(\mathcal{K}(X_i,\bar{X}_i,n_j)) + c.c. \\ &+ 2Q\int d\bar{\theta}^- d\theta^+ \left(X_a n_a - \frac{t}{4}\right)\Sigma_0 + 2Q\int d\bar{\theta}^+ d\theta^- \left(\bar{X}_a n_a - \frac{\bar{t}}{4}\right)\bar{\Sigma}_0 \\ &- \int d^4\theta \frac{1}{2e^2}\bar{\Sigma}_0\Sigma_0. \end{aligned} \tag{11}$$

We integrate the auxiliary fields to obtain the scalar potential.

Considering also the Lagrangian contribution term $\frac{2Q}{4}\int d\bar{\theta}^- d\theta^+ \left(X_a \bar{D}_+ n_a\right) D_- V_0 + c.c.$ (for non constant n_a) the dual variety is given by $\sum_a n_a^2 = 1, \sum_a x_a n_a = \frac{t}{2Q} + B,\ x_2+\bar{x}_2 = x_3+\bar{x}_3 = 0.$

Considering a gauging direction $V = |V| n_a \sigma_a$

In the dual model one can obtain an effective superpotential for the gauge field V_0.

The total twisted superpotential taking and Ansatz for the instanton contributions reads [Hori, Vafa,00]

$$\widetilde{W} = 2QX_a n_a \Sigma_0 - t\Sigma_0 + 2\mu e^{-X_a n_a}. \tag{12}$$

By integrating out X_a one gets the effective superpotential for the gauge field strength Σ_0:

$$W_{eff}(\Sigma_0) = -2Q\Sigma_0 \ln\left(\frac{Q\Sigma_0}{\mu}\right) + 2Q\Sigma_0 - t\Sigma_0. \tag{13}$$

This matches the result in the original model considering one-loop effects [Witten,93].

Conclusions

We have implemented non-Abelian T-duality for (2,2) 2D GLSMs, obtaining the general e.o.m after integrating the gauged field.

We analyzed an SU(2) example and a family of Abelian-duals embedded in the non-Abelian T-duality. We extended the solutions to a general direction inside the group, studying the geometry of the dual models.

We took steps toward the inclusion of non-perturbative effects: An Ansatz reproduces correctly the effective potential for the U(1) gauge field V_0.

Future work:

Study the connection between the non-Abelian T-duality in the GLSM to Mirror Symmetry.
Formulate the T-duality for a non-Abelian gauge group. Study explicit examples of dualities for CY varieties, possibly determinantal.

Thanks!