Multi-field effects
in a simple extension of $R^2$ Inflation

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Introduction

*Inflation*

- Horizon problem
- Flatness problem
- Origin of large scale structure

We can solve these problems with introducing accelerated expansion so-called Inflation and the predictions are in good agreement with CMB Observations
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*Inflation

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*Motivation of Multi-field

- Supergravity or Superstring theory may provide many scalar fields → Multi-field models

- In multi-field models, curvature perturbation is NOT conserved on super-horizon scale. → Any feature which single-field models don’t have?
Set up: A simple extension of Starobinsky model

van de Bruck and Paduraru (’15)

Jordan frame

\[ S_J = \int d^4 x \sqrt{-g} \left[ \frac{m_{pl}^2 R}{2} + \frac{\mu}{2} R^2 \right] + \int d^4 x \sqrt{-g} \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m_\chi^2 \chi^2 \right] \]

Motivation:

• Check the multi-field effects on asymptotic potential
• Essentially same with a class of models based on Supergravity  Ellis et al (’13-16)
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\]

**Einstein frame**

\[
S_E = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{m_{pl}^2 }{2} \tilde{R} - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} e^{-2\alpha \phi} (\partial \chi)^2 - \tilde{V} \right]
\]

\[
e^{2\alpha \phi} = 1 + \frac{2\mu R}{m_{pl}^2} \quad \alpha = \frac{1}{\sqrt{6m_{pl}}}
\]

\[
\tilde{V} = \frac{m_{pl}^4}{8\mu} (1 - e^{-2\alpha \phi})^2 + \frac{1}{2} m_\chi^2 e^{-4\alpha \phi} \chi^2
\]
Curvature perturbation

*Spectrums*

$$\langle \zeta_{k_1} \zeta_{k_2} \rangle = (2\pi)^3 \delta(k_1 + k_2) P_s(k_1)$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^3 \delta(k_1 + k_2 + k_3) B_s(k_1, k_2, k_3)$$

$$n_s = \frac{d \log P_s}{d \log k} \quad r \equiv \frac{P_t}{P_s}$$

$$f_{NL} \equiv \frac{B_s(k_1, k_2, k_3)}{P_s(k_1)P_s(k_2)} + \text{cyc. perm.}$$

In single-field cases...

$$\zeta \propto \left( \frac{k}{aH} \right)^2 \sim 0$$

conserved on super-horizon scale

Consistency relation:

in the squeezed limit

$$f_{NL} \sim (1 - n_s) \sim \mathcal{O}(10^{-2})$$

Lyth, Malik and Sasaki ('04)

Maldacena ('02)
Curvature perturbation

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\*Multi-field effects

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Lyth, Malik and Sasaki (’04)

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\[ f_{NL} \]

Lyth, Malik and Sasaki (’04)

Maldacena (’02)

isocurvature mode can transfer to adiabatic mode, \( \zeta \) evolves with time on super-horizon scale

\[ \rightarrow \text{We need to take into account multi-field effects on each quantities.} \]

\* \( f_{NL} \) can be enhanced by multi-field effects?

D.Wands et al(’00)
Superhorizon evolution of perturbations

\[ m_X/m_\phi = 0.1 \]

We developed numerical approach based on \( \delta N \)-formalism

If a trajectory turns/oscillates, each quantities may have steps, peaks or oscillations.

Evolutions on superhorizon scale!
$m_{\chi}/m_{\phi} = 1.0$

Field space search

$N_{\text{total}}$

$r$

$r < 0.11$

95% C.L.

$n_s$

$n_s = 0.968 \pm 0.006$

(68% C.L.)

$f_{NL}$

$f_{NL} = 0.8 \pm 5.0$

68% C.L.
Various mass-ratios

\[ m_\chi / m_\phi = 5.0 \]

\[ m_\chi / m_\phi = 1.0 \]

\[ m_\chi / m_\phi = 0.2 \]

We investigated the mass-ratios

\[ 10^{-3} \leq m_\chi / m_\phi \leq 10^3 \]
Multi-field model from string compactification

with Joe Conlon(Oxford), Kaz Kohri

Work in Progress
Summary

• There is a strong motivation to consider multi-field inflation models in string/supergravity inspired model-building.

• In the case of multi-field, we need to take into account transfer of isocurvature perturbation.

• In this work, we established the method to compute predictions in interacting multi-field models with non-canonical kinetic terms.

• $f_{NL}$ is as small as single field cases.

*If you are interested in multi-field analysis, I am happy to discuss with you!
BACK UP だ！！！
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\[ e^{2\alpha \phi} = 1 + \frac{2\mu R}{m_{pl}^2} \quad \alpha = \frac{1}{\sqrt{6} m_{pl}} \]

\[ \tilde{V} = \frac{m_{pl}^4}{8\mu} (1 - e^{-2\alpha \phi})^2 + \frac{1}{2} m^2 e^{-4\alpha \phi} \chi^2 \]

**conformal transformation:**

where

\[ \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad \Omega^2 = 1 + \frac{2\mu R}{m_{pl}^2} \]

**This model contains…**

- Two inflatons
- Interaction term
- Non-canonical kinetic term
δN-formalism

On super-horizon scale,

\[ \zeta \sim \delta N \equiv \mathcal{N}(t_*, t ; x) - N(t_*, t) \]

\[ = N_I \delta \phi^I + \frac{1}{2} N_{IJ} \delta \phi^I \delta \phi^J \]

I : derivatives wrt \( \phi^I \)

To compute each quantities, we only need background dynamics and \( N_I, \ N_{IJ} \)

*Non-canonical kinetic terms: Curved field space

\[ \mathcal{L}_{kin} = -\frac{1}{2} G_{IJ} \partial^\mu \phi^I \partial_\mu \phi^J \]

\( G_{IJ} \) can be interpreted as field space metric.
In curved(non-trivial) field space, covariant formalism is useful.

Points:

1. Contracting with field space metric
2. Derivatives should be replaced with Covariant derivatives

\[ \partial_I A_J \]

\[ \Downarrow \]

\[ \mathcal{D}_I A_J = \partial_I A_J - \Gamma^K_{IJ} A_K \]
$\delta N$-formalism

Power Spectrum:

$$P_s = \frac{H^2}{4\pi^2} N^I N_I, \quad P_t = \frac{H^*}{4\pi^2}$$

spectral tilt:

$$n_s = 1 - 2\epsilon_* - 2 \frac{1 + N_A \left( \frac{1}{3} R^{ABCD} \frac{V_B V_C}{V^2} - \frac{V^{;AD}}{V} \right) N_D}{N^I N_I}$$

tensor-scalar ratio:

$$r = \frac{P_t}{P_s} = \frac{1}{N^I N_I}$$

Non-Gaussianity:

$$f_{NL} = \frac{5}{6} \frac{N^I N^J D_I D_J N}{(N^K N_K)^2}$$