Why some string theorists care about complexity?

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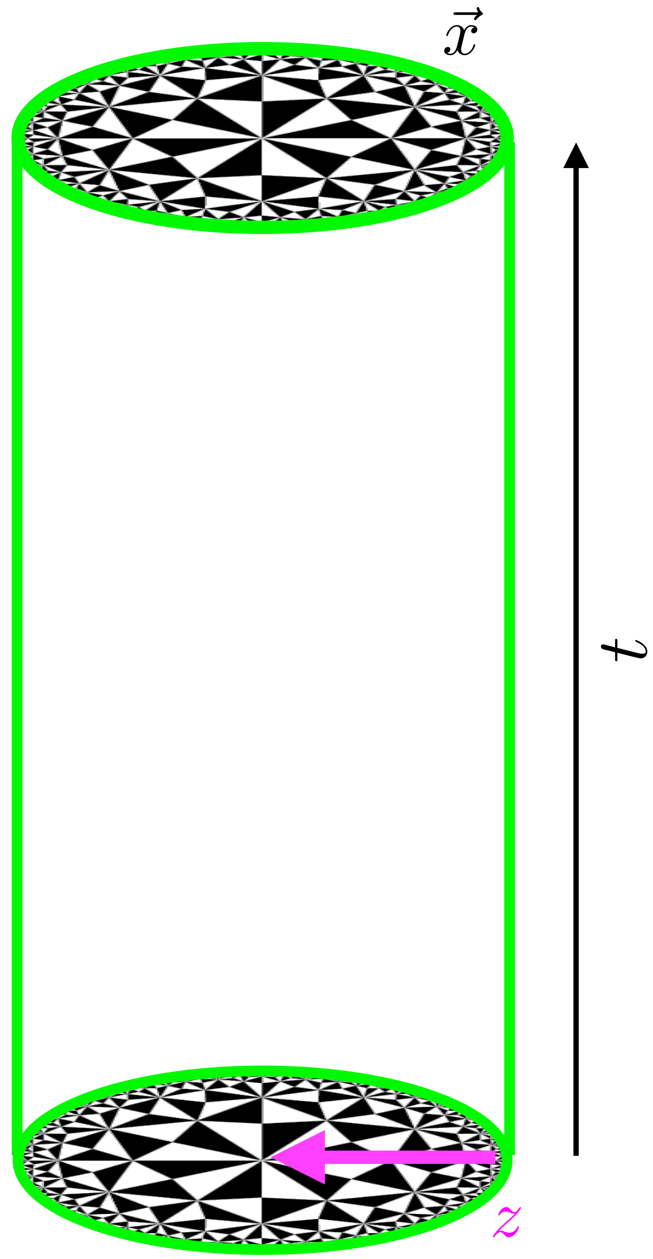
1707. 08582 with Chapman, Marrochio & Pastawski

1807.xxxxx with Chapman, Eisert, Hackl, Jefferson, Marrochio, Myers & Pastawski

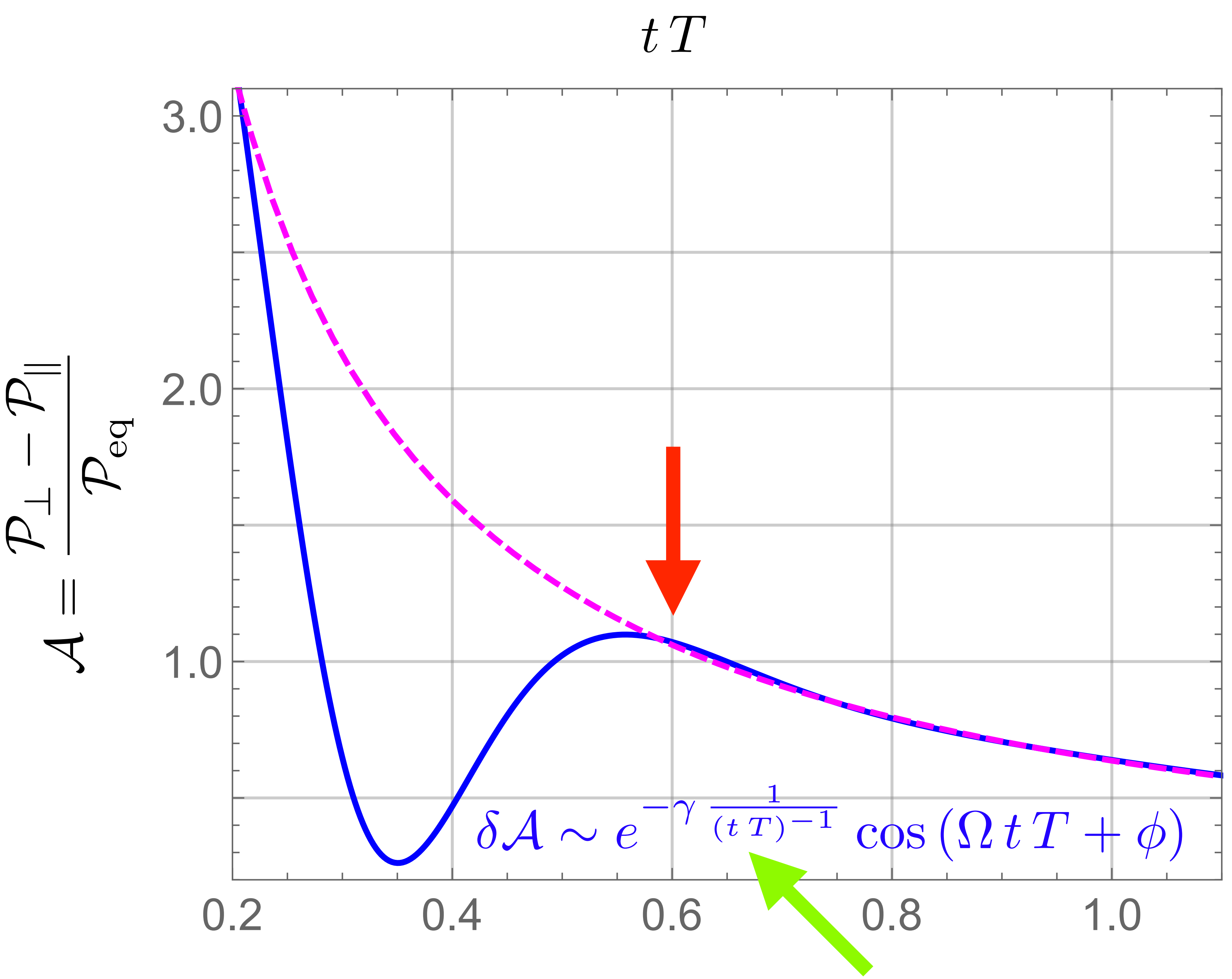
I will be talking about things around holography



$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left(dz^2 - dt^2 + d\vec{x}^2\right)$$

Aside: another kind of string phenomenology

1610.02023 lecture notes on "Holography, Hydrodynamization and Heavy-Ion Collisions"

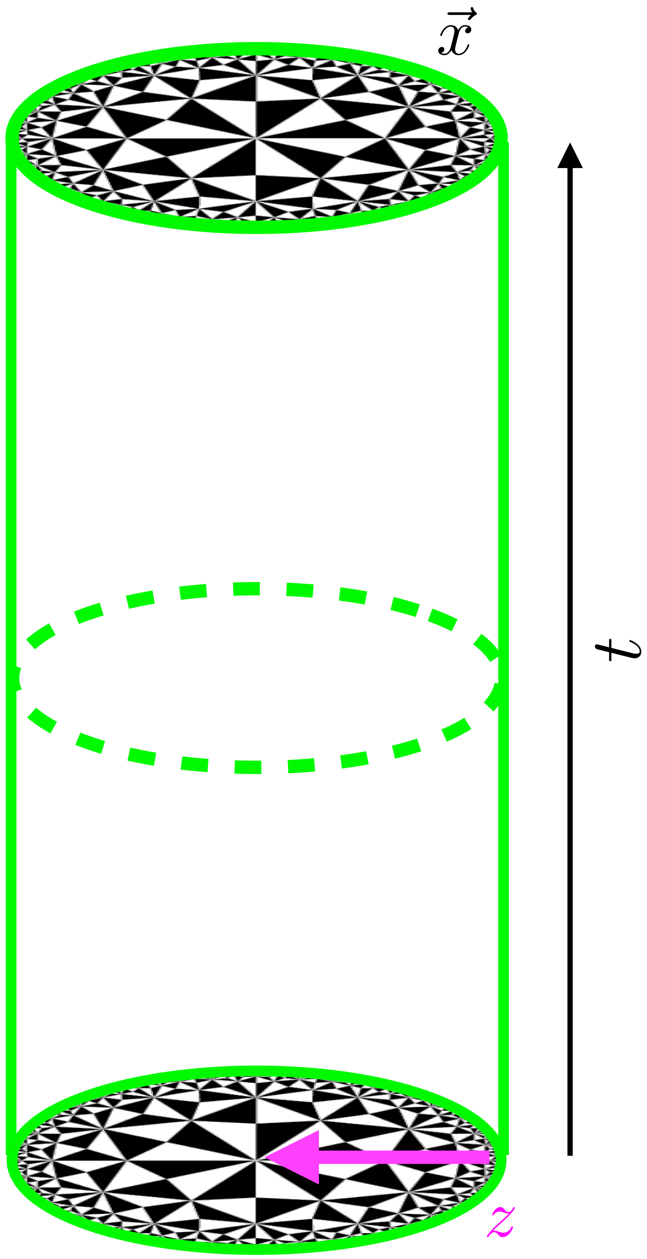


N=4 SYM (holography)

viscous hydro ($\frac{\eta}{s} = \frac{1}{4\pi}$):

$$8\,\frac{\eta}{s}\,(t\,T)^{-1}$$

Back to the main part of the talk



$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left(dz^2 - dt^2 + d\vec{x}^2\right)$$

How to decode the bulk geometry from ρ_{hQFT}?

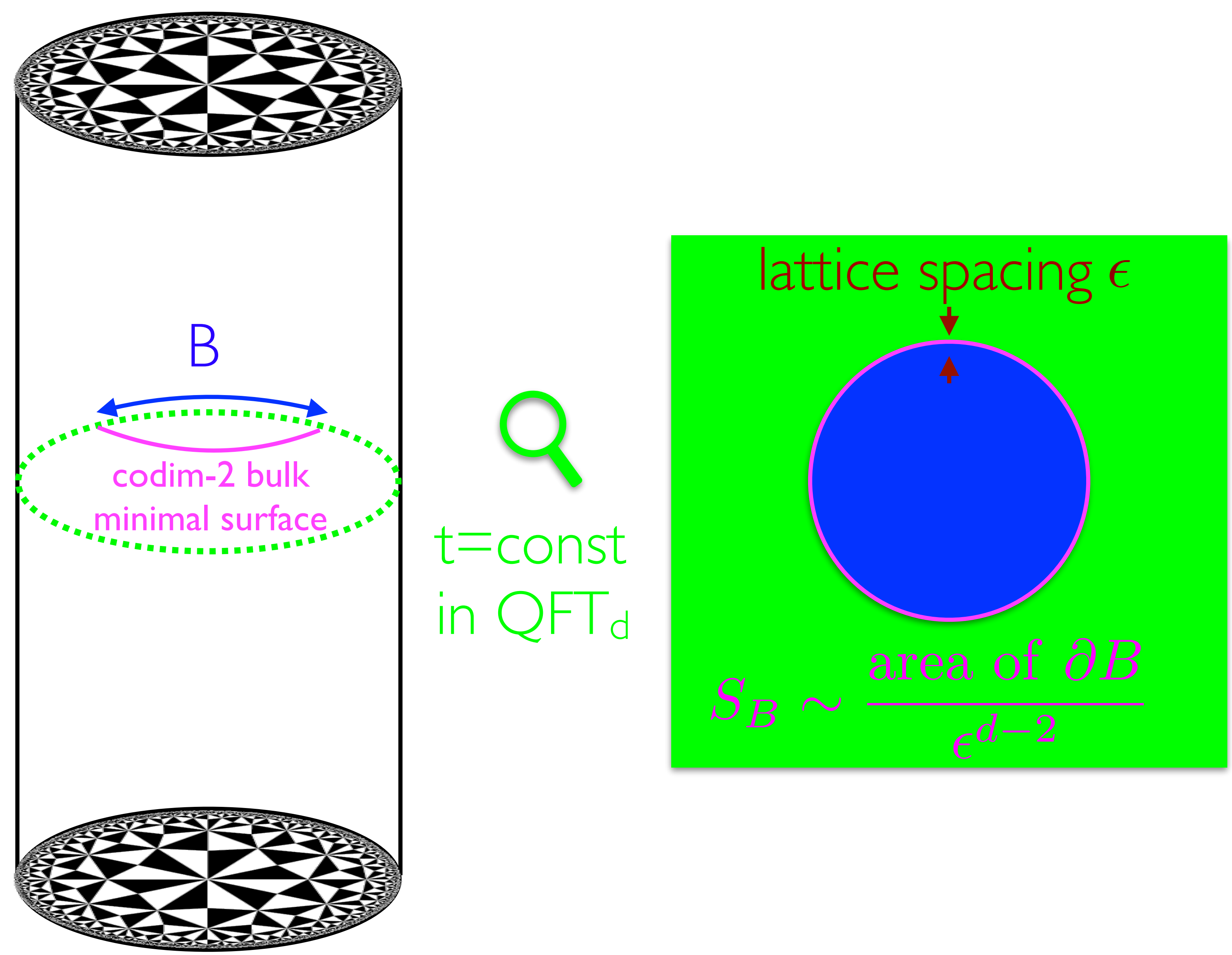
quant-info $\cap$ cond-mat $\longrightarrow$ hep-th

Entanglement / $|\uparrow\rangle_B|\downarrow\rangle_{\bar{B}} - |\downarrow\rangle_B|\uparrow\rangle_{\bar{B}}$ vs $|\uparrow\rangle|\downarrow\rangle$ / - key prop. of quantum-many body sys.

A powerful way to quantify it: entanglement entropy $S_B = -\mathrm{tr}\,(\rho_B \log \rho_B)$

In holography: $S_B = \dfrac{\text{bulk area}}{4\,G_N}$

Ryu & Takayanagi hep-th/0603001



“Entanglement is not enough”

Susskind et al. 2014-2018

AdS$_{d+2}$ / CFT$_{d+1}$:

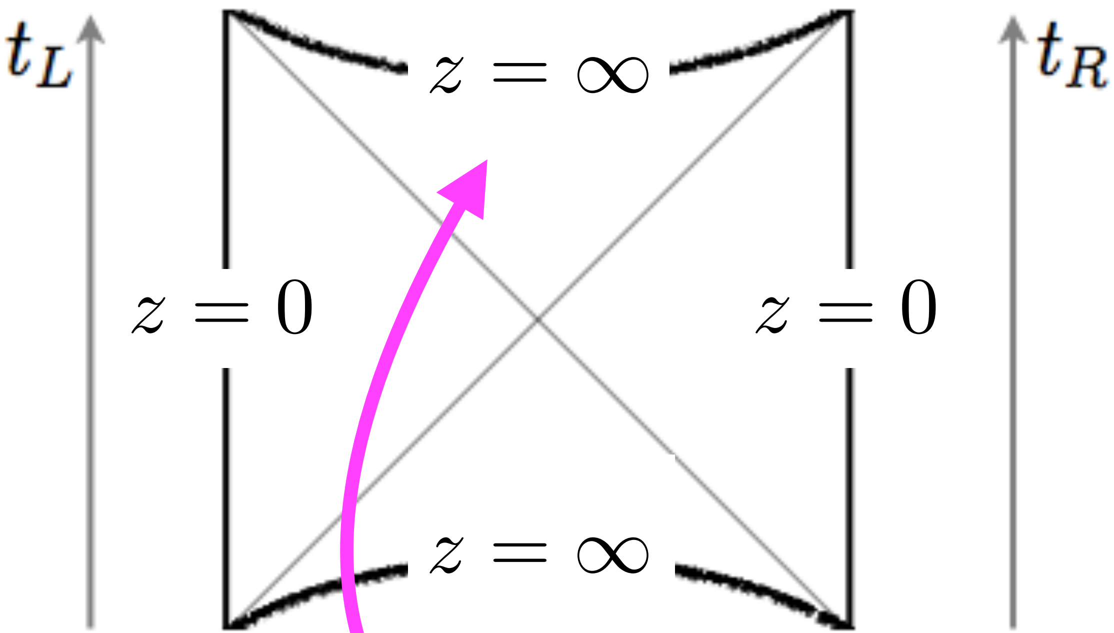


Figure adapted from 1509.07876

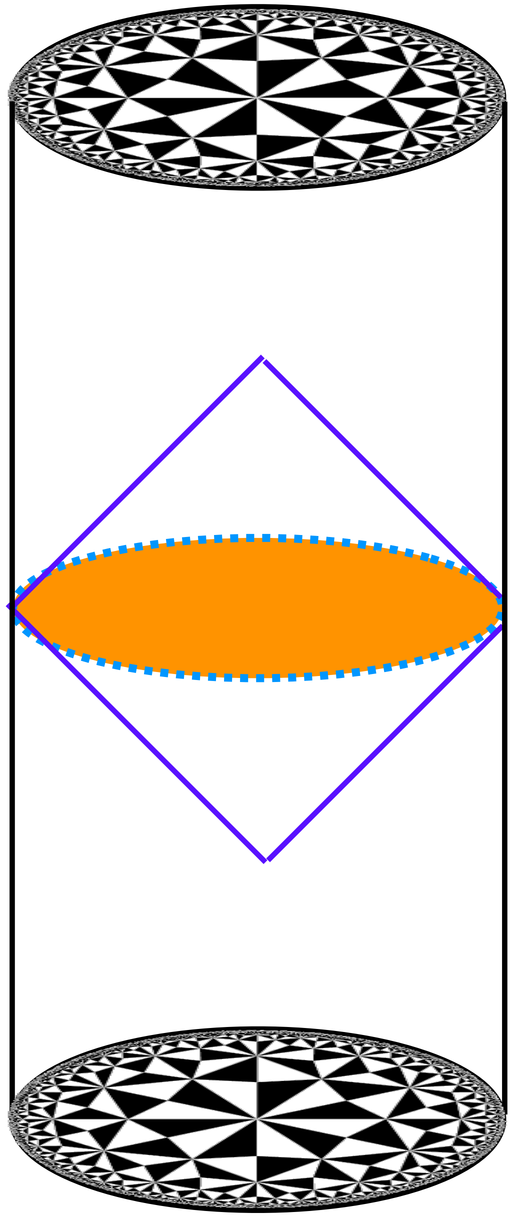
$$|\mathrm{TFD}\rangle \sim \sum_E e^{-\beta E/2} e^{-i E (t_L + t_R)} |E\rangle_L |E\rangle_R$$

In the eternal AdS-Schwarzschild black hole Penrose diagram there are regions in the interior not penetrated by any Ryu-Takayanagi surface!

There are, however, other geometric probes of these regions

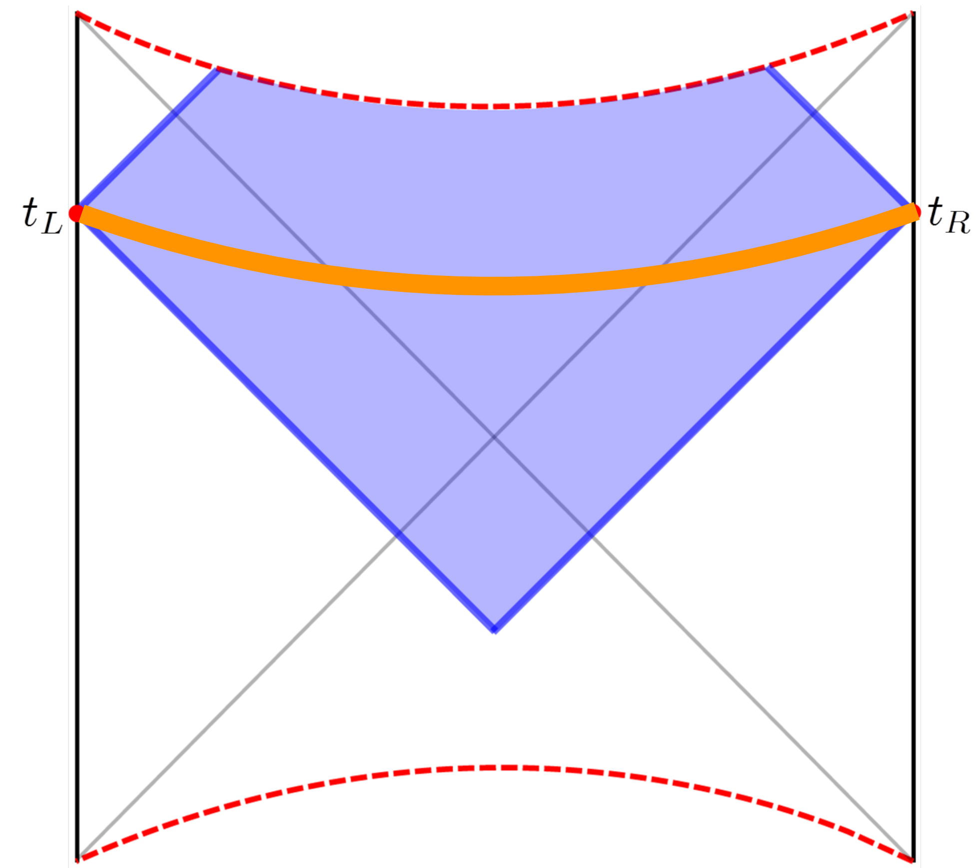
What are we interested in reproducing?

see Myers et al. 2016-2018



$\mathcal{C}_V \sim$ Volume of codim-1 max volume bulk slice

$\mathcal{C}_A \sim$ Action in codim-0 bulk region with null bdries



I

$$\mathcal{C}_V[\mathrm{AdS}_{d+1}] \sim \frac{\text{vol occupied by hCFT}_\mathrm{d}}{\epsilon^{d-1}}$$

$$\mathcal{C}_A[\mathrm{AdS}_{d+1}] \sim \frac{\text{vol occupied by hCFT}_\mathrm{d}}{\epsilon^{d-1}} \log\frac{\epsilon}{\alpha}$$

II

$$\mathcal{C}_{A/V}[\mathrm{AdS}-\mathrm{Schw}_{d+1}]\Big|_{t_L+t_R=0} - 2\,\mathcal{C}_{A/V}[\mathrm{AdS}_{d+1}] \sim S_\beta$$

III

$$\partial_{t_L+t_R}\mathcal{C}_{A/V}[\mathrm{AdS}-\mathrm{Schw}_{d+1}] \sim \mathrm{const}$$

$\mathcal{C}_{A/V}$ stands for complexity?

4) How to count gates and deal with UV divergences?

3) How to make sense now of the approximation?

$$\text{Complexity}\,\mathcal{C}\text{: min. number of elem. unitary operations } \delta U \text{ s.t. } |T\rangle \approx \overbrace{\delta U \ldots \delta U}^{\# \equiv \mathcal{C}} |\uparrow \ldots \uparrow\rangle$$

2) What can now act as a set of elementary unitary operations (gates)?

1) What can be a simple reference state in continuum?

< 2017: entanglement entropy in a QFT ✔ vs. complexity in a QFT ✘

5) We want an approach that is computable ⟶ Gaussian States and free QFTs$_{d+1}$

I. Vacuum

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see also 1707.08570 by Jefferson, Myers

Holography = strong coupling QFTs. We do free QFTs. Universality to the rescue?

Now target / reference state is GS of $\int \mathrm{d}^{d-1}x \left\{\pi^2 + (\partial_x \phi)^2 + m_{1/2}^2 \phi^2\right\}$

We put the theory on the lattice to UV regulate it

$\phi_1,\ \pi_1 \quad \phi_2,\ \pi_2 \quad \dots \quad \phi_N,\ \pi_N$

Gates: $\delta U = e^{i\phi_1 \pi_3\, \delta s}$ etc $\longrightarrow$ $\mathrm{SP}(2N,\mathbb{R})$ group.

To calculate complexity, we will define a metric on* $\mathrm{SP}(2N,\mathbb{R})$ and calculate geodesics

Many choices, but soluble ones $\xrightarrow[\text{limit}]{\text{cont.}}$ $\mathcal{C} \sim \sqrt{\mathrm{vol} \int_{|k|\leq\Lambda} \mathrm{d}^{d-1}k \left(\log \frac{m_1^2 + k^2}{m_2^2 + k^2}\right)^2}$ ✗

What compares ✓ with $\mathcal{C}_{V/A}$ is $\mathcal{C} \sim \mathrm{vol} \int_{|k|\leq\Lambda} \mathrm{d}^{d-1}k \left|\log \frac{k}{m_2}\right|$

$|R\rangle$ GS of $\int \mathrm{d}^{d-1}x \left\{\pi^2 + m_2^2 \phi^2\right\}$

L^1 norm

II. Formation of TFD

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For the TFD state, we have additional gates such as $\delta U = e^{i\,\phi_1^L\,\phi_3^R}$

However, there are choices one can make such that

$$\mathcal{C}_{|TFD(t_L+t_R=0)\rangle} \sim \underbrace{\text{vol}\int_{k\le\beta^{-1}} d^{d-1}k\,(\dots)}_{S_\beta} + 2\times \underbrace{\text{vol}\int_{k\le\Lambda} d^{d-1}k\,\left|\log\frac{k}{m_2}\right|}_{\mathcal{C}_{|0\rangle}}$$

As a result we get sth very similar to

I

$$\mathcal{C}_V[\text{AdS}_{d+1}] \sim \frac{\text{vol occupied by hCFT}_\text{d}}{\epsilon^{d-1}}$$

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III. Time-dependence of TFD

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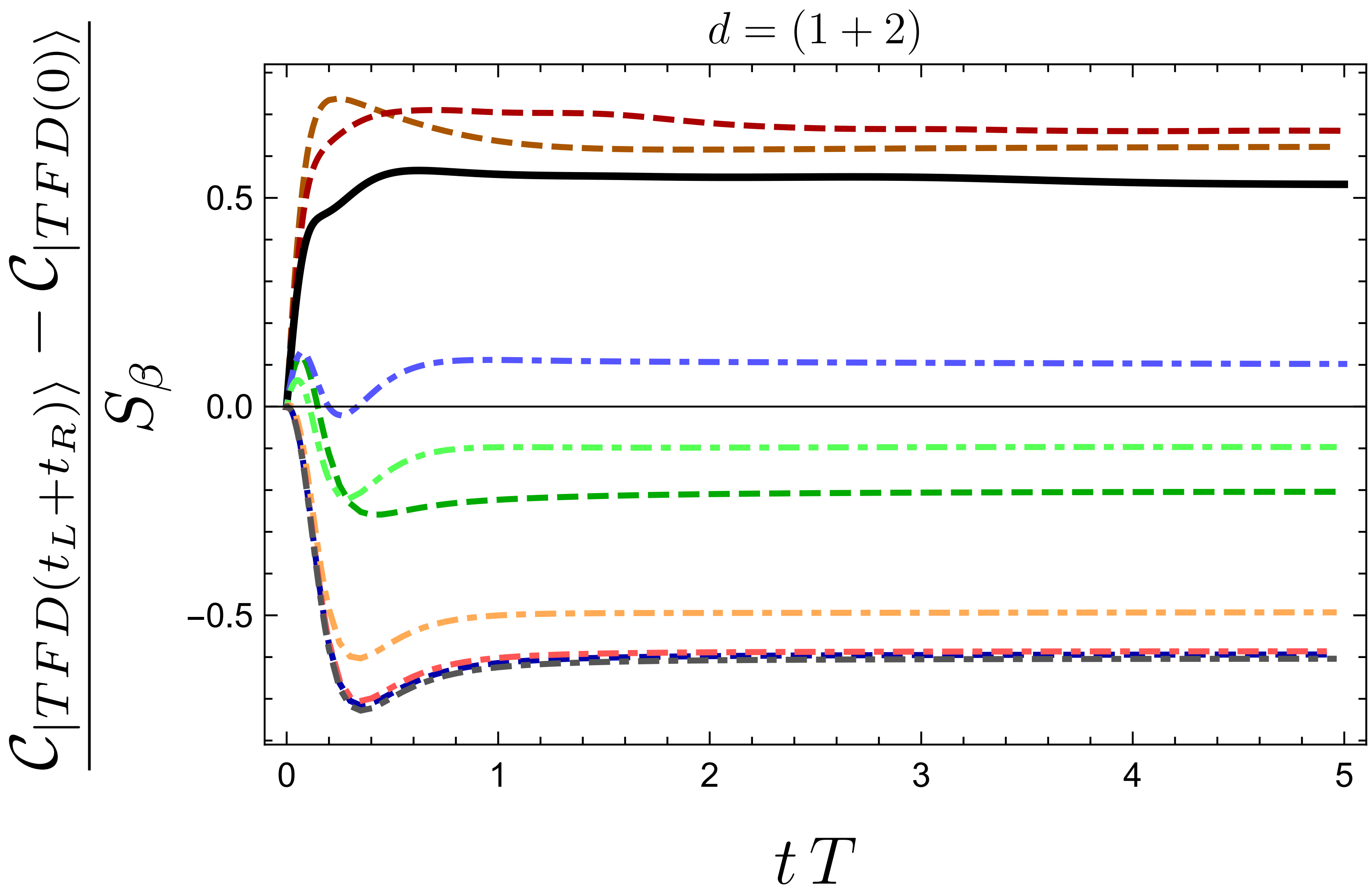

Complexity saturates since it is a sum of oscillatory funcs (free QFT!) that dephase

Not surprisingly, this is in stark contrast with holography:

$$\boxed{\begin{gathered}\partial_{t_L+t_R}\mathcal{C}_{A/V}[\mathrm{AdS}-\mathrm{Schw}_{d+1}] \\ \sim \mathrm{const}\end{gathered}}$$

III

Outlook

1707.08582 with Chapman, Marrochio & Pastawski
see also **1707.08570** by Jefferson, Myers
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The big picture: what is bulk in the hQFT language?

Here, focus on bulk volumes / actions and their conjectured relation to complexity

Poorly understood $\longrightarrow$ key idea: work in free QFT and count on some universalities

✓ I

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✓ II

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✗ III

$$\partial_{t_L+t_R}\mathcal{C}_{A/V}[\mathrm{AdS}-\mathrm{Schw}_{d+1}] \sim \mathrm{const}$$

Beautiful parallel with thermodynamics vs. η/s