Kinetic decoupling and gauge invariance of the resonant annihilation in a vector dark matter model

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MD, Bohdan Grzadkowski, JHEP 1709 (2017) 159 [1705.10777]
Dark matter – motivation

Convincing evidence on various astrophysical and cosmological scales

Kinetic decoupling and gauge invariance of DM annihilation
Dark matter – motivation

Properties of dark matter:
- electrically neutral (non-luminous),
- non-relativistic (cold) (structure formation)
- stable or long-lived
- weakly interacting with ordinary matter

Dark matter interactions:
- annihilation – production in the early universe and indirect detection (FERMI-LAT, MAGIC, H.E.S.S, …)
- production – collider searches
- scattering on nucleons – direct detection (LUX, PANDA, XENON 1T)

Strong constraints

Small coupling regions are relevant

Gross, Lebedev, Mambrini, 2015
Breit-Wigner resonance

Breit-Wigner resonance \(2M_{DM} \approx M_R\)

enhanced annihilation \(\rightarrow\) suppressed coupling

- low sensitivity to direct detection
- velocity dependent cross-section \(\rightarrow\) possibility of enhanced indirect detection signals
- kinetic decoupling \(T_{DM} \neq T_{SM}\) ?
- large self-interaction cross-section ?
Boltzmann equation for DM phase space density

\[ f_{DM}(\vec{p}, t) L[f] = C[f] \]

\[ g \int L[f_{DM}] \frac{d^3p}{2\pi^3} \rightarrow \frac{dn}{dt} + 3Hn = -\langle \sigma v_{\text{rel}} \rangle (n^2 - n_{\text{EQ}}^2) \leftarrow g \int C[f_{DM}] \frac{d^3p}{2\pi^3} \]

DM yield

\[ Y = \frac{n}{s}, \quad x = \frac{M_{DM}}{T} \quad n - \text{number density}, \ s - \text{entropy density} \]

DM chemical decoupling

\[ \frac{dY}{dx} = -\alpha \frac{\langle \sigma v_{\text{rel}} \rangle}{x^2} (Y^2 - Y_{\text{EQ}}^2), \quad \alpha = \frac{s(M_{DM})}{H(M_{DM})} \]

Approximate solution

\[ Y_{\infty} \approx \frac{x_{CD}}{\alpha \langle \sigma v_{\text{rel}} \rangle} \]

Standard assumption: DM is kinetically coupled to SM during freeze-out, i.e. it has the same temperature as the SM thermal bath
Annihilation near the resonance

Non-relativistic annihilation cross-section – s-wave case

\[ \sigma v_{\text{rel}} = \sum_{f \neq i} \frac{64\pi \omega}{M_R^2} \frac{\eta_i \eta_f \beta_f}{(\delta + v_{\text{rel}}^2/4)^2 + \gamma^2} \]

Dimensionless parameters

\[ \eta_i/f = \frac{\Gamma B_{i/f}}{M_R \beta_{i/f}}, \quad \delta = \frac{4M_{DM}^2}{M_R^2} - 1, \quad \gamma = \frac{\Gamma_R}{M_R} \]

couplings position width

- cross section grows with falling temperature
- prolonged period of effective annihilation
- strong temperature dependence

\[ T_{DM} \neq T_{SM} \]

Kinetic decoupling and gauge invariance of DM annihilation

Mateusz Duch, Warsaw
Kinetic decoupling – simplified picture

Scatterings on the abundant relativistic SM states → thermal equilibrium

- relic abundance requires small coupling between DM and SM
- scattering process is not resonantly enhanced

Comparison of the Hubble rate to scattering rate

\[ H(T_{kd}) \sim \Gamma_{\text{scat}}(T_{kd}) \Rightarrow x_{kd} \left( \frac{\max[\delta, \gamma]^{3/2}}{10^{-6}} \right)^{\frac{1}{4}} \Rightarrow T_{KD} \sim T_{CD} \]

Kinetic and chemical decoupling temperatures are comparable

\[ T_{DM} = \begin{cases} T_{SM}, & \text{for } T \geq T_{KD} = T_{CD} \\ T_{SM}^2 / T_{KD}, & \text{for } T < T_{KD} = T_{CD} \end{cases} \]

non-relativistic expanding gas
Kinetic decoupling – detailed description

Second moment of Boltzmann equation
\[ \int p^2 L[f] = \int p^2 C[f] \]

Temperature parameter
\[ T_{DM} \propto \int p^2 f(p) d^3p \quad y = \frac{M_{DM} T_{DM}}{s^{2/3}} \]

Coupled set of Boltzmann equations
\[ \frac{dY}{dx} = -\frac{1 - \frac{2}{3} \frac{g'_{*s}}{g_{*s}}}{H x} s \left( Y^2 \langle \sigma v_{rel} \rangle x_{DM}(y) - Y_{EQ}^2 \langle \sigma v_{rel} \rangle x \right) \]
\[ \frac{dy}{dx} = -\frac{1 - \frac{2}{3} \frac{g'_{*s}}{g_{*s}}}{H x} \left[ 2M_{DM} c(T)(y - y_{EQ}) - sy \left( Y \left( \langle \sigma v_{rel} \rangle x_{DM} - \langle \sigma v_{rel} \rangle_2 | x_{DM} \right) - \frac{Y_{EQ}^2}{Y} \left( \langle \sigma v_{rel} \rangle x - \frac{y_{EQ}}{y} \langle \sigma v_{rel} \rangle_2 | x \right) \right] \]
Abelian vector dark matter

Additional complex scalar field $S$

- singlet of $U(1)_{Y} \times SU(2)_{L} \times SU(3)_{c}$, charged under $U(1)_{X}$

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{4} V_{\mu \nu} V^{\mu \nu} + (D_{\mu} S)^{*} D^{\mu} S + \tilde{V}(H, S)$$

$$V(H, S) = -\mu_{H}^2 |H|^2 + \lambda_{H}|H|^4 - \mu_{S}^2 |S|^2 + \lambda_{S}|S|^4 + \kappa |S|^2 |H|^2$$

Vacuum expectation values: $\langle H \rangle = \frac{\nu_{SM}}{\sqrt{2}}$, $\langle S \rangle = \frac{\nu_{x}}{\sqrt{2}}$

$U(1)_{X}$ vector gauge boson $V_{\mu}$

- Stability condition - no mixing of $U(1)_{X}$ with $U(1)_{Y}$

$Z_{2} : V_{\mu} \rightarrow -V_{\mu}$, $S \rightarrow S^{*}$, $S = \phi e^{i\sigma}$: $\phi \rightarrow \phi$, $\sigma \rightarrow -\sigma$

- Higgs mechanism in the hidden sector $M_{Z'} = g_{x}\nu_{x}$

Higgs couplings – mixing angle $\alpha$, $M_{h_{1}} = 125$ GeV

$$\mathcal{L} \supset \frac{h_{1} \cos \alpha + h_{2} \sin \alpha}{\nu} \left(2M_{W} W_{\mu}^{+} W_{\mu}^{-} + M_{Z}^{2} Z_{\mu} Z^{\mu} - \sum_{f} m_{f} \bar{f} f \right)$$
Resonance with a Higgs scalars

Small $\alpha$ required by relic abundance

\[ \langle \sigma \nu_{\text{rel}} \rangle \propto \sin \alpha \cos \alpha \]

Resonance with the SM-like Higgs

- $M_{Z'} \approx 125/2$ GeV
- decay channel $h_1 \rightarrow Z'Z'$, if open
  suppressed by $\sin^2 \alpha$ and by phase space

\[ \sqrt{1 - 4M_{Z'}^2/M_{h_1}^2} = \sqrt{\delta} \ll 1 \quad \Gamma_{h_1 \rightarrow Z'Z'} \ll \Gamma_{SM} \]

Resonance with the second Higgs

- $M_{Z'} \approx M_{h_2}/2$ GeV
- $h_2 \rightarrow SM SM$ suppressed by $\sin^2 \alpha$,
  $h_2 \rightarrow Z'Z'$ can dominate
- near threshold effects
Resummed propagator

\[ \Pi_{ij} = \frac{g_x^2 R_{2,i} R_{2,j}}{32 \pi^2 M_{Z'}^2} \left[ (s^2 - 4M_{Z'}^2 s + 12M_{Z'}^4) B_0(s, M_{Z'}^2, M_{Z'}^2) - (s^2 - m_i^2 m_j^2) B_0(s, \xi \cdot M_{Z'}^2, \xi \cdot M_{Z'}^2) \right] \]

Depends explicitly on the gauge-fixing parameter
May grow with \( s^2 \) breaking unitarity
Pinch technique

One loop corrections to SM-DM process

Tree-level Ward identities

\[
\begin{align*}
\bar{\nu} \gamma^\mu h_i Z' Z' + i M_{Z'} \bar{\nu} \gamma^\mu Z' G_Z' &= -g_s R_{2,i} \hat{\Pi}^{G_Z'}(r_1), \\
\bar{\nu} \gamma^\mu h_i Z' G_Z' + i M_{Z'} \bar{\nu} h_i G_Z' G_Z' &= -g_s \left[ \sum_j R_{2,j} \hat{\Pi}_{ji}(q^2) + R_{2,i} \hat{\Pi}^{G_Z'} G_Z'(r_2) \right], \\
\bar{\nu} \gamma^\mu h_i Z' Z' + M_{Z'}^2 \bar{\nu} h_i G_Z' G_Z' &= i g_s M_{Z'} \left[ \sum_j R_{2,j} \hat{\Pi}_{ji}(q^2) + R_{2,j} \left( \hat{\Pi}^{G_Z'} G_Z'(r_1) + \hat{\Pi}^{G_Z'} G_Z'(r_2) \right) \right]
\end{align*}
\]

Pinch self-energy

\[
\hat{\Pi}_{ij} = \frac{g_s^2 R_{2,i} R_{2,j}}{8\pi^2} \left[ \frac{(m_i m_j)^2}{4 M_{Z'}^2} + \frac{m_i^2 + m_j^2}{2} - (2s - 3 M_{Z'}^2) \right] B_0(s, M_{Z'}^2, M_{Z'}^2)
\]

Condition that guaranties good high-energy behaviour of the process with resummed propagator

\[
\lim_{s \to \infty} \frac{\bar{\nu} \gamma^\mu R_{2,i} R_{2,j} \bar{\nu} V h_i Z' Z'}{s} \left( q, r_1, r_2 \right) = \frac{1}{i g_s M_{Z'}} \sum_j R_{2,j} \hat{\Pi}_{ji}(q^2).
\]

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Bounds in the parameter space

- Mixing angle set by relic density constraint
- Maximal dark gauge coupling satisfying perturbative unitarity
- Enhancement of low-velocity cross-section → strong bounds from indirect searches
- Effects of early kinetic decoupling modify relic density by up to a factor of 2 in the allowed region
Summary

- Resonant annihilation of dark matter has strong temperature dependence and leads to an early kinetic decoupling.
- The evolution of dark matter density and temperature can be described with the set of coupled Boltzmann equations.
- In vector dark matter models, resonant annihilation requires gauge-independent treatment.