Aspects of F-theory from a Total Space Approach

James Gray – Virginia Tech

Base on work with:

Lara Anderson and Brian Hammack
arXiv:1805.05497
Lara Anderson, Antonella Grassi and Paul-Konstantin Oehlmann
arXiv:1801.08658
Lara Anderson, Xin Gao and Seung-Joo Lee
arXiv:1708.07907

Earlier work with:

Alexander Haupt and Andre Lukas
arXiv:1303.1832, arXiv:1405.2073
Lara Anderson, Xin Gao and Seung-Joo Lee
arXiv:1608.07554, 1608.07555

- Often in studying F-theory compactifications we start with an elliptic fibration (often described by a Weierstrass model) over some specified base.
- Here, we are going to instead start with the total space of a Calabi-Yau and work towards fibrations and F-theory physics from that starting point.
- It is useful to follow such a procedure as it enables us to spot some features that may be not easy to see in the more conventional approach.
  - I will give two examples of such features that we have worked on in the last 12 months.

Complete Intersection Calabi-Yau (CICYs)

- A family of CICYs is described by a configuration matrix:

$$[\mathbf{n}|\mathbf{q}] \equiv \left[\begin{array}{c|ccc} n_1 & q_1^1 & \dots & q_K^1 \\ \vdots & \vdots & \ddots & \vdots \\ n_m & q_1^m & \dots & q_K^m \end{array}\right]$$

with m rows and K+1 columns.

- Ambient space is $\mathbb{P}^{n_1} \times \ldots \times \mathbb{P}^{n_m}$
- Remaining columns give degree of defining relations:

Calabi-Yau condition:

$$\sum_{\alpha=1}^{K} q_\alpha^r = n_r + 1$$

D-fold condition:

$$\sum_r n_r - K \stackrel{!}{=} D$$

- Three-Folds:
  - Hübsch, Commun.Math.Phys. 108 (1987) 291
  - Green et al, Commun.Math.Phys. 109 (1987) 99
  - Candelas et al, Nucl.Phys. B 298 (1988) 493
  - Candelas et al, Nucl.Phys. B 306 (1988) 113
- Data Set classified: 7890 configuration matrices in the set.
- Four-Folds:
  - Brunner et al, Nucl.Phys. B498 (1997) 156-174
  - JG et al, JHEP 1307 (2013) 070
  - JG et al, JHEP 1409 (2014) 093
- Data set classified: 921,497 configuration matrices in the set.

Properties of CICYs: Torus Fibrations

- Consider configuration matrices which can be put in the form:

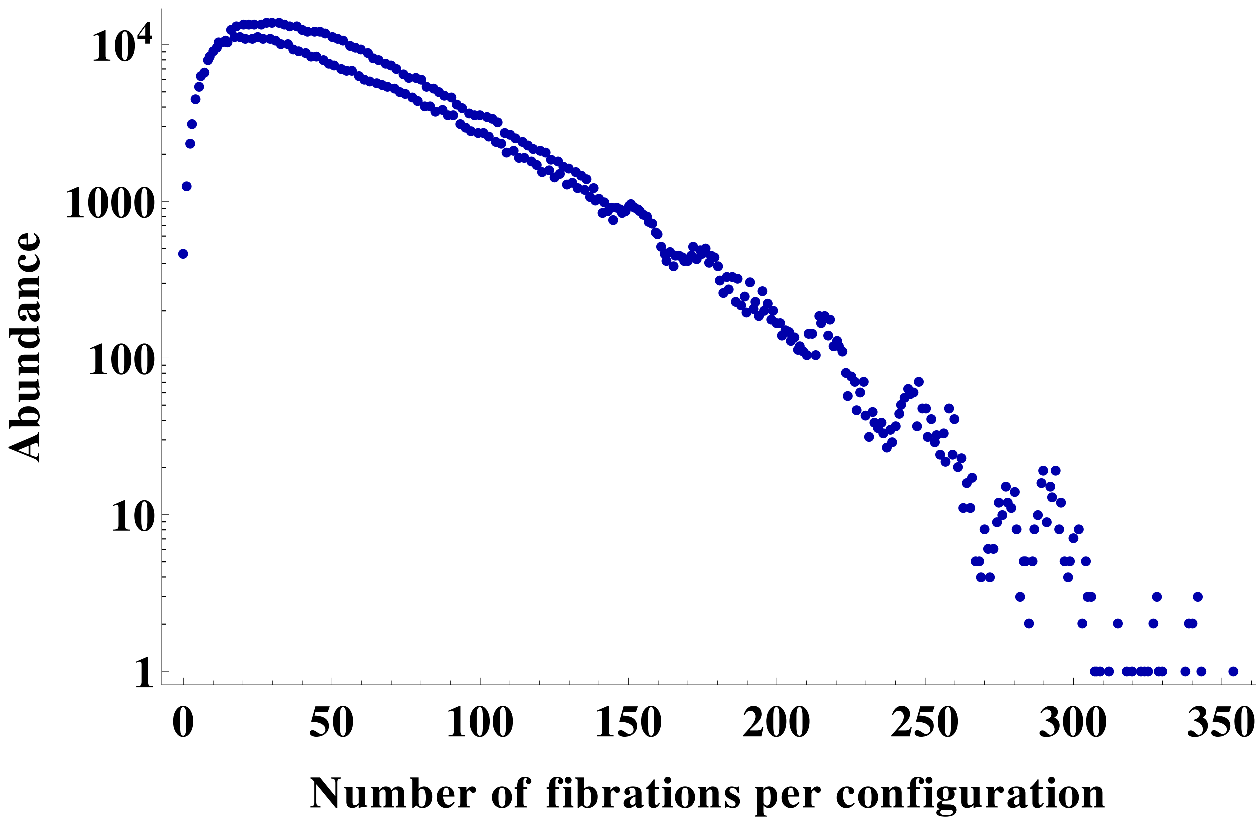
$$\left[\begin{array}{c|c} \mathcal{A}_1 & \mathcal{F} \end{array}\right] = T^2$$

$$\text{Base: } \left[\begin{array}{c|c} \mathcal{A}_2 & \mathcal{B} \end{array}\right] \qquad \left[\begin{array}{c|cc} \mathcal{A}_1 & 0 & \mathcal{F} \\ \mathcal{A}_2 & \mathcal{B} & \mathcal{T} \end{array}\right]$$

- This is an torus fibred four-fold
- In our list of 921,497 matrices, 921,020 have such a fibration structure.

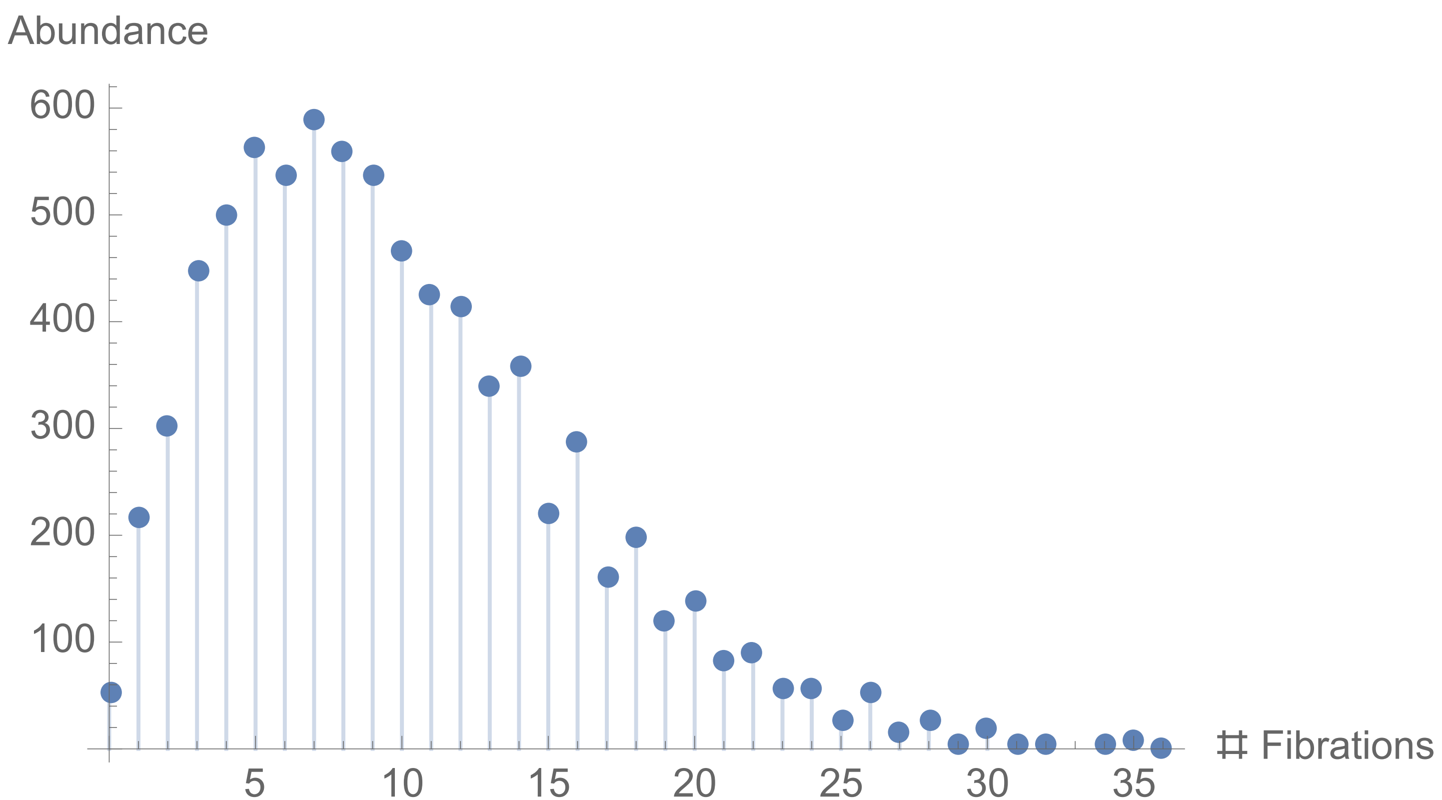
A given manifold/configuration matrix may admit many obvious torus fibrations…

See also S. Johnson and W. Taylor arXiv:1406.0514 and arXiv:1605.08052 and Y.-C. Huang and W. Taylor arXiv:1805.05907



- Total of 50,114,908 different torus fibrations.
- Average of 54.4 fibrations per manifold.

•Threefolds:



- Total of 77,744 different torus fibrations in data set.
- Average of 9.85 fibrations per manifold…

Can we go beyond these obvious fibrations?

- Conjecture by Kollár (rough description):

  A Calabi-Yau threefold is genus one fibered if and only if there exists a divisor D *such that*

$$D \cdot C \geq 0 \quad \textit{for every algebraic curve } C$$

$$D^3 = 0$$

$$D^2 \neq 0$$

(and similarly in higher dimensional cases)

- *Proven in threefold case by Oguiso, Wilson.*

- The question is, do we have good computational control over all of the elements of $h^{1,1}$?
- In favorable cases we do. For example in the case,

$$X = \left[\begin{array}{c|c} \mathbb{P}^2 & 3 \\ \mathbb{P}^2 & 3 \end{array} \right]$$

all divisor classes descend from divisor classes in the ambient space.

- In non-favorable cases we don't. For example

$$X' = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^2 & 3 & 0 \\ \mathbb{P}^2 & 0 & 3 \end{array} \right]$$

has $h^{1,1} = 19$ but $h^{1,1}$ of the ambient space is only 3.

- Of 7890 CICY threefolds in the original list, only 4874 are favorable.

- We can obtain new configuration matrices describing the same manifolds by the process of contraction/splitting:

$$\left[\begin{array}{c|ccccc} n & 1 & 1 & \dots & 1 & 0 \\ \mathbf{n} & \mathbf{u}_1 & \mathbf{u}_2 & \dots & \mathbf{u}_{n+1} & \mathbf{q} \end{array}\right] \longleftrightarrow \left[\begin{array}{c|cc} \mathbf{n} & \sum_{a=1}^{n+1} \mathbf{u}_a & \mathbf{q} \end{array}\right]$$

Euler number doesn't change $\Leftrightarrow$ manifolds same

  - Use this to increase the size of the ambient space affording the configuration a better chance of being favorable

- By splitting we have obtained favorable descriptions of all but 7842 of the 7890 CICYS.

- We can then compute data such as intersection numbers, line bundle cohomology etc completely in these cases.

What about the remaining 48?

- It turns out that these can all be written as hypersurfaces in direct products of del Pezzo surfaces.
- For example:

$$X_3 = \left[\begin{array}{c|cccc} \mathbb{P}^1 & 1 & 0 & 0 & 1 \\ \mathbb{P}^2 & 2 & 0 & 0 & 1 \\ \mathbb{P}^4 & 0 & 2 & 2 & 1 \end{array}\right]$$

  can be written as the anti-canonical hypersurface inside

$$dP_4 = \left[\begin{array}{c|c} \mathbb{P}^1 & 1 \\ \mathbb{P}^2 & 2 \end{array}\right] \quad \text{times} \quad dP_5 = \left[\begin{array}{c|cc} \mathbb{P}^4 & 2 & 2 \end{array}\right]$$

- Enough is known about the divisors of del Pezzo's that we can then find a favorable description of these spaces too.

Thus we find a favorable description of all CICYs.

- With these descriptions we can compute almost all of the information we need to investigation the fibrations of all CICYs. There are, however, some subtleties associated to the Kahler cone structure.
- For the 4874 “Kahler favorable” cases which are favorable in products of projective spaces, and for which the Kahler cone is the naive one induced from the ambient space, obvious fibrations and Kollár fibrations coincide.

However, in general there can be many more Kollár fibrations than obvious ones.

- A good example is the Schoen manifold – which admits an infinite number of genus one fibrations!

(See also Grassi,Morrison; Aspinwall, Gross; Oguiso; Piateckii-Shapiro, Shafarevich).

Fibrations and quotients

- One can create a new (non-simply connected) Calabi-Yau by quotienting a CICY by a freely acting symmetry.
- Example: Take the bi-cubic:

$$X = \left[\begin{array}{c|c} \mathbb{P}^2 & 3 \\ \mathbb{P}^2 & 3 \end{array} \right]$$

- With homogeneous coordinates:

$$x_{a,i} \qquad\qquad a = 1, 2 \quad i = 0, 1, 2$$

- And quotient by the following $\mathbb{Z}_3$ group action:

$$g : x_{a,j} \to \omega^j x_{a,j}$$

- It is clear in this case that the quotienting preserves the fibration.

Work with Lara Anderson and Brian Hammack:

See Braun, JHEP 1104 (2011) 005 for a partial classification of symmetries

- Of the 1632 symmetry-CICY pairs (for manifolds with fibration), 1552 of them preserve *some* fibration (95%).
- Of 20700 fibration/symmetry pairs, 17161 preserved.

Symmetry	Fibs preserved	Fibs not preserved	%preserved
$\mathbb{Z}_2$	8812	464	95%
$\mathbb{Z}_3$	175	201	46.5%
$\mathbb{Z}_4$	120	244	33.0%
$\mathbb{Z}_5$	0	30	0.0%
$\mathbb{Z}_6$	62	438	12.4%
$\mathbb{Z}_2 \times \mathbb{Z}_2$	7711	1488	83.8%
$\mathbb{Z}_2 \times \mathbb{Z}_4$	105	200	34.4%
$\mathbb{Z}_3 \times \mathbb{Z}_3$	176	0	100%

- There are several larger symmetries that appear (including non-Abelian symmetries), none of which preserve any fibrations:

$$\mathbb{Z}_8\ ,\ \mathbb{Z}_{10}\ ,\ \mathbb{Z}_{12}\ ,\ Q_8\ ,\ \mathbb{Z}_2 \times Q_8\ ,\ \mathbb{Z}_3 \rtimes \mathbb{Z}_4\ ,$$
$$\mathbb{Z}_8 \times \mathbb{Z}_2\ ,\ \mathbb{Z}_4 \rtimes \mathbb{Z}_4\ ,\ \mathbb{Z}_8 \rtimes \mathbb{Z}_2\ ,\ \mathbb{Z}_4 \times \mathbb{Z}_4\ ,$$
$$\mathbb{Z}_{10} \times \mathbb{Z}_2$$

- In any case where the fibration is preserved, the base of the quotiented fibration is divided by same group as total space.
- Classifications of the bases etc was provided in the paper.

Multiple fibrations and Duality

- We can use these multiple nested fibration structures to derive some lots of interesting dualities in F-theory and heterotic. For example:
  - Start with two different fibrations of the same Calabi-Yau. This will correspond to two F-theory models that share an M-theory limit.
  - Start with two different fibrations of the same Calabi-Yau in a heterotic compactification. These will have seemingly different F-theory duals which actually give the same physics.
  - And so on…

see arXiv:1608.07555

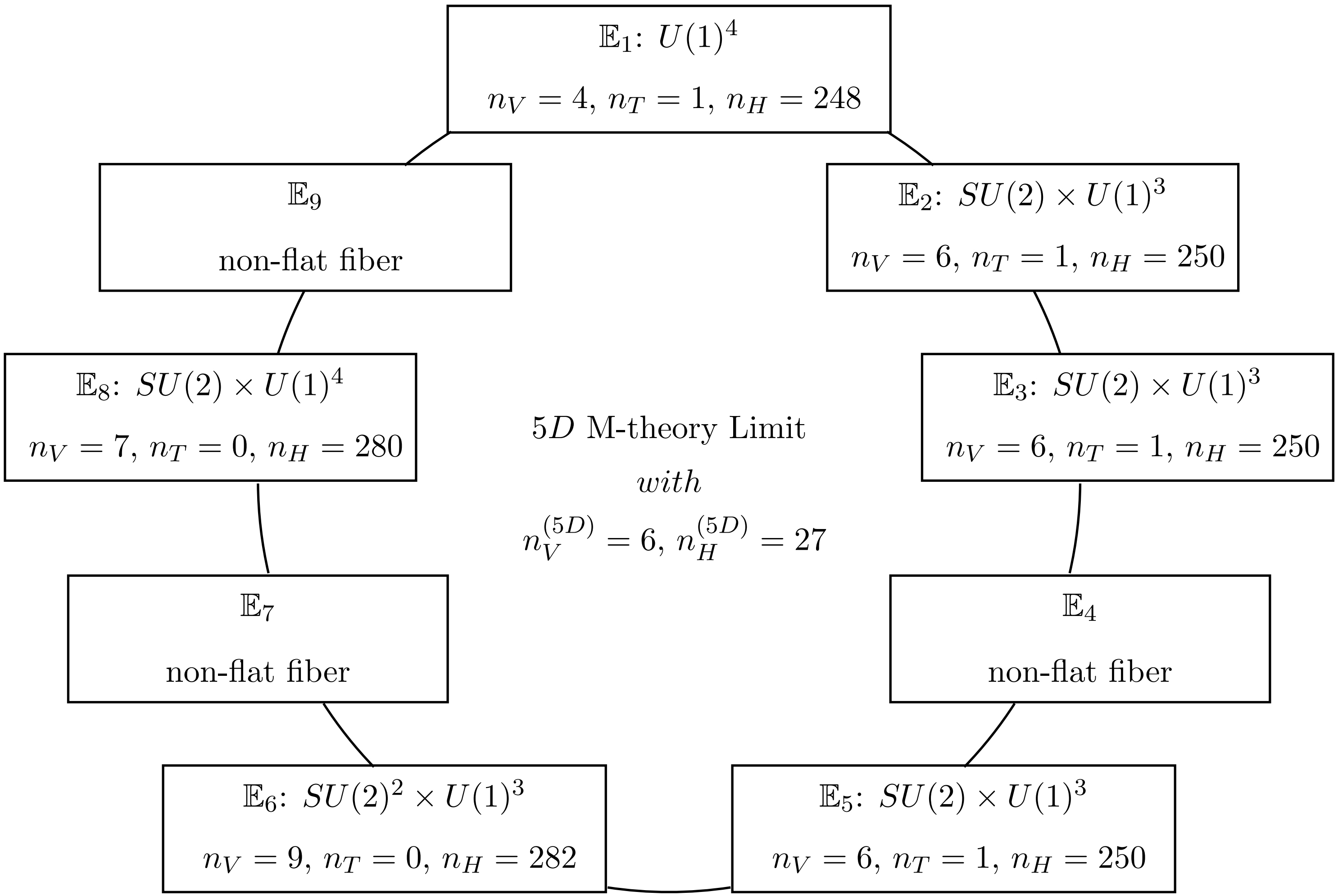


Figure 7: *F-theory models in 6D with the same 5D M-theory limit where* $n_V^{(5D)} = 6$ *and* $n_H^{(5D)} = 27$.

- Why are we interested in these dualities in string phenomenology?:
  - They give us new approaches to studying poorly understood phenomena such as certain superpotential effects in F-theory compactifications to four dimensions.
  - Understanding these dualities could potentially allow for more effective scans for models – avoiding highly non-obvious, and rather extensive, redundancies.

A second example: discrete charged `SCFT sectors'

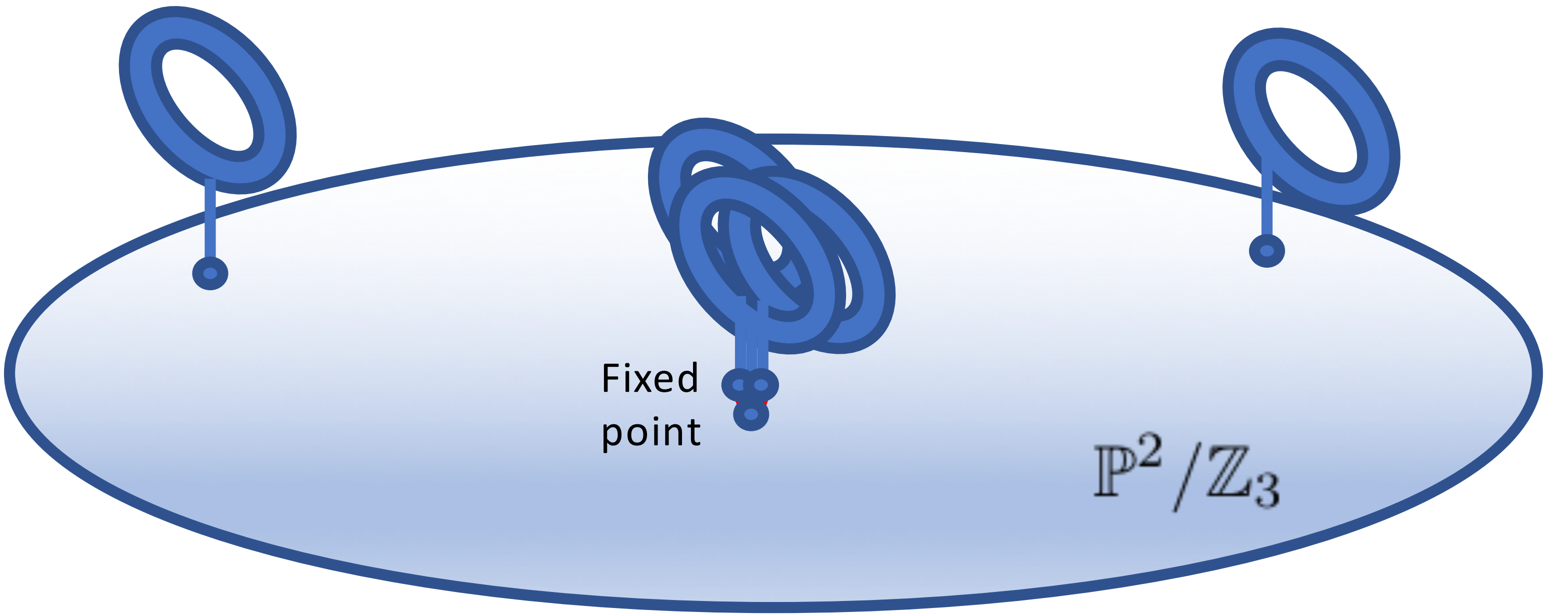
- Let us return to the quotient geometries. Consider as an example:

$$\left[\begin{array}{c|c} \mathbb{P}^2 & 3 \\ \mathbb{P}^2 & 3 \end{array}\right] \Big/ \mathbb{Z}_3$$

- The base of this fibration is $\mathbb{P}^2/\mathbb{Z}_3$. The action is not free and thus we have orbifold singularities in the base.

→ Gives rise to SCFTs at those points as in Del Zotto et al, arXiv:1412.6526

- But the overall threefold in our case is smooth – how is this possible?



- We have multiple fibers over the fixed points in the base.
- Therefore we have a multi-section rather than a section.
- Therefore we have a discrete symmetry in the lower dimensional effective theory.

- We were able to show that the SCFTs located at the fixed points are charged under this discrete symmetry.
  - To see: go to tensor branch and identify discretely charged matter fields over the blow up divisors.
- Note that this is rather different behavior to the generic Weierstrass model over the base $\mathbb{P}^2/\mathbb{Z}_3$ (which has a section rather than a multi-section).
- You could get this by tuning an appropriate U(1) into the Weierstrass model then breaking it with a vev… but this can be hard to see without the resolved, total space, geometry to guide you.