Geometrical (in)stability during inflation ?

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Summary

- Motivation
- Isocurvature mass
- Canonical heavy field
- Canonical inflaton
- Massless fields
- Discussion

Motivation

<u>String compactifications feature many scalars:</u>

- $h^{2,1}$ complex structure moduli
- $h^{1,1}$ Kahler moduli
- axio-dilaton
- brane position moduli

Typically $\mathcal{O}(10) - \mathcal{O}(100)$ scalars

Non trivial structures in 4D Lagrangian. $\mathcal{L} = K_{I\bar{J}} \partial \Phi^I \partial \bar{\Phi}^{\bar{J}} - V$

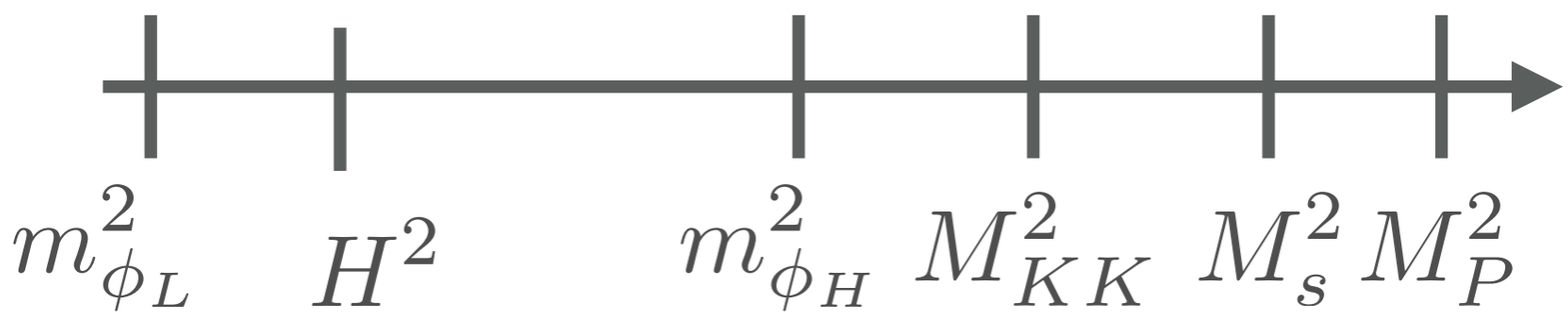
- Complex scalars
- Curved Kahler metric
- F+D term potential

Motivation

Several string inflation models, most single field

Observations seem to favour effective **single field models**

$$f_{nl} \leq \mathcal{O}(10) \qquad\qquad \beta_{iso} \ll 1$$



Model building problem

Theory ⟷ Cosmological Pert. Theory ⟷ Observations

Perturbation Theory

$$S = \int \sqrt{-g} d^4x \left(\frac{1}{2}R - \frac{1}{2}G_{ab} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi) \right)$$

Expand $\phi_i = \phi_i^0 + \delta\phi_i^0$ and $g_{\mu\nu} = g_{\mu\nu}^0 + \delta g_{\mu\nu}$

[Sasaki&Stewart 1995]
[Langlois&Renaux-Petel 2008]
[Achucarro et al. 2010]

Background dynamics: Friedmann + KG eqs

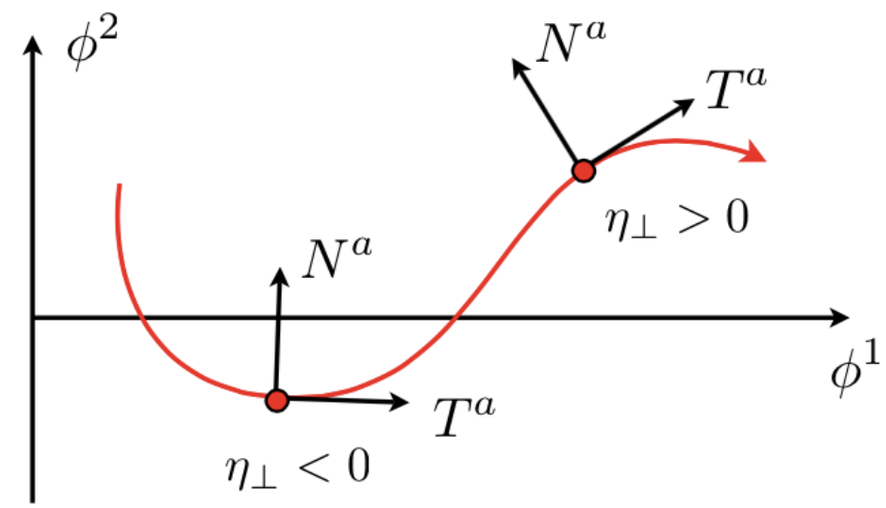
Gauge invariant perturbations: $Q^a = \delta\phi^a + \frac{\dot{\phi}^a}{H}\psi$

$$\frac{D^2 Q^a}{dt^2} + 3H \frac{DQ^a}{dt} - \frac{\nabla^2}{a^2} Q^a + C^a{}_b Q^b = 0$$

Isocurvature Mass

On superhorizon scales

$$\ddot{Q}_N + 3H\dot{Q}_N + m_{eff}^2 Q_N = 0$$



[Achucarro et al. 2010]

Isocurvature mass:

$$\frac{m_{eff}^2}{H^2} \equiv \frac{V_{NN}}{H^2} + 3\eta_\perp^2 + \epsilon\mathbb{R}$$

$$V_{NN} = N^a N^b \nabla_b V_a \qquad \eta_\perp = \frac{N^a V_a}{H|\dot{\phi}|}$$

Usually for **hierarchical mass spectra**: $\frac{m_{eff}^2}{H^2} \approx \frac{m_H^2}{H^2} \gg 1$

$\eta_\perp \approx 0$ decoupling

Isocurvature Mass

Reduces to single field model: $\begin{cases} Q_T & \text{constant curvature} \\ Q_N & \text{decaying isocurvature} \end{cases}$ for $k \gg aH$

- $P_k \propto \frac{H^2}{\epsilon} k^{n_s - 1}$
- $n_s = 1 - 2\epsilon_H - \eta_H$ $\qquad \epsilon_H = -\frac{\dot{H}}{H^2} \qquad \eta_H = -\frac{\dot{\epsilon_H}}{\epsilon_H H}$
- $r = 16\,\epsilon_H$

$$\frac{m_{eff}^2}{H^2} \equiv \frac{V_{NN}}{H^2} + 3\eta_\perp^2 + \epsilon\mathbb{R}$$

Curvature can be negative

$$0 > m_{eff}^2 \neq m_H^2 \gg H^2$$

Background well behaved, perturbations not so much.

Geometrical Instability

Negative field space curvature can turn isocurvature **tachyonic**

Geometric instability

[Renaux-Petel&Turzyński, 2015]

Uncontrolled growth of isocurvature perturbations:

- validity of the perturbative treatment
- backreaction on the background
- sudden end of inflation?

Minimal model:

$$\mathcal{L}/\sqrt{|g|} = \frac{1}{2} G_{IJ} \partial\phi^I \partial\phi^J - V(\phi)$$

Geometrical Instability

Assumptions:

$$G_{IJ} = \begin{pmatrix} 1 & 0 \\ 0 & f^2(\phi^1) \end{pmatrix}$$

$$V(\phi) = V(\phi^1) + V(\phi^2)$$

$$\mathbb{R} = -2\frac{f_{11}}{f}$$

necessary condition: $\frac{f_{11}}{f} \gg 0$

$$\Box\phi^2 = -\frac{V_2}{f^2} - 2\frac{f_1}{f}\dot{\phi}^1\dot{\phi}^2$$

$$\Box\phi^1 = -V_1 + f f_1(\dot{\phi}^2)^2$$

Consider:

1. heavy spectator field
2. massless scalars

Canonical Heavy Field

$$\Box\chi = -V_\chi + f f_\chi (\dot\phi)^2$$

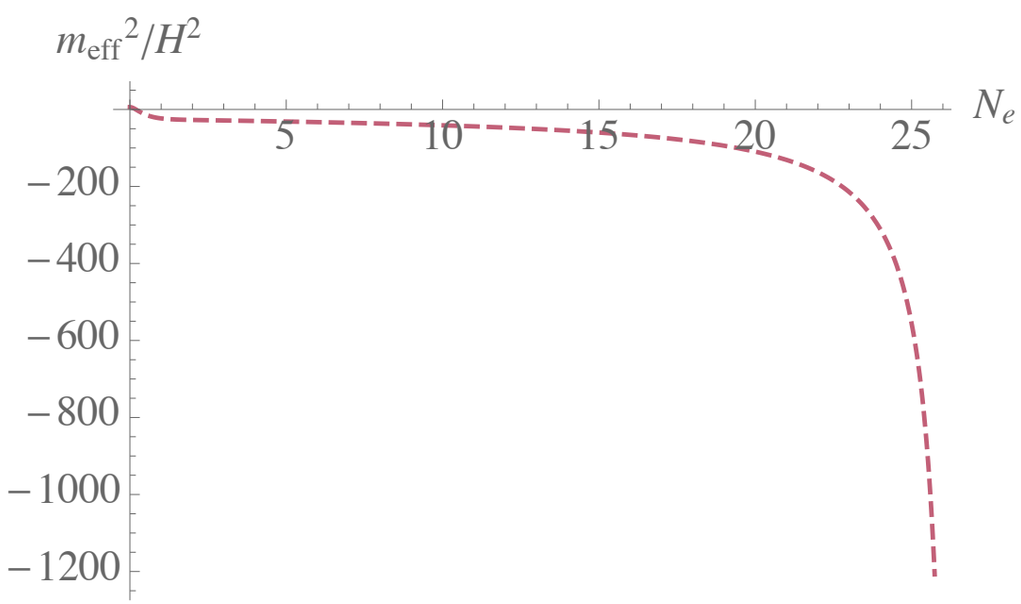
1 $\ddot\chi = \dot\chi = V_\chi = f f_\chi (\dot\phi)^2 = 0\ ,$

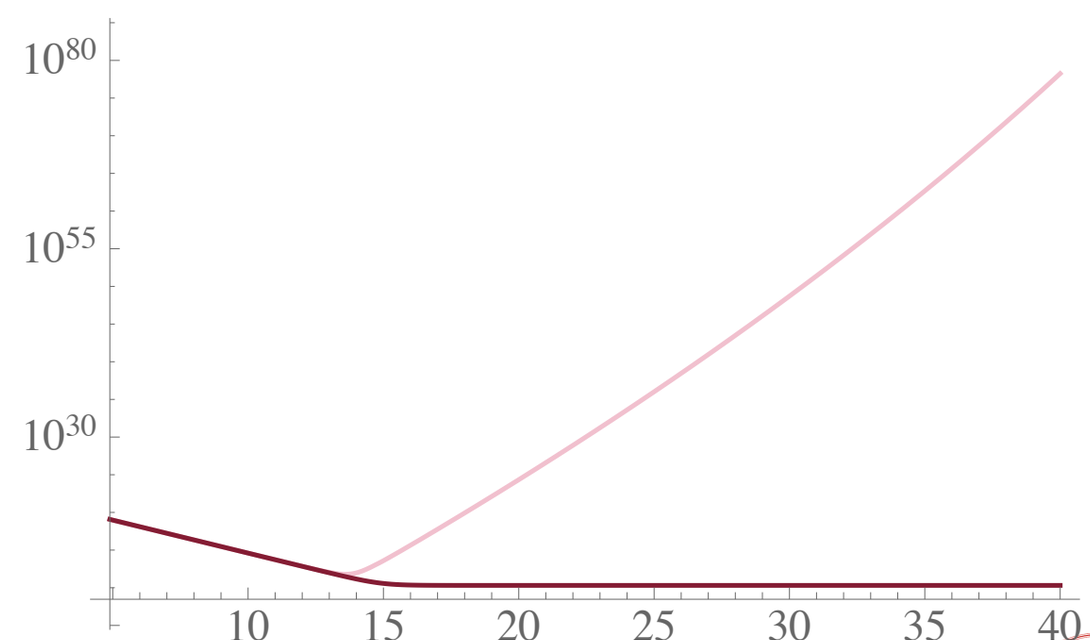
2 $\ddot\chi\ ,\ 3H\dot\chi \ll V_\chi = f f_\chi (\dot\phi)^2 \neq 0$

1 <u>Trivial solution</u>

geodesic motion $\eta_\perp = 0$

$$\frac{m_{eff}^2}{H^2} = \frac{V_{11}}{H^2} - 2\epsilon\frac{f_{11}}{f}$$





Canonical Heavy Field

What does this mean?

[Renaux-Petel&Turzyński, 2015]

A premature end of inflation

B kick the heavy mode

} modify predictions

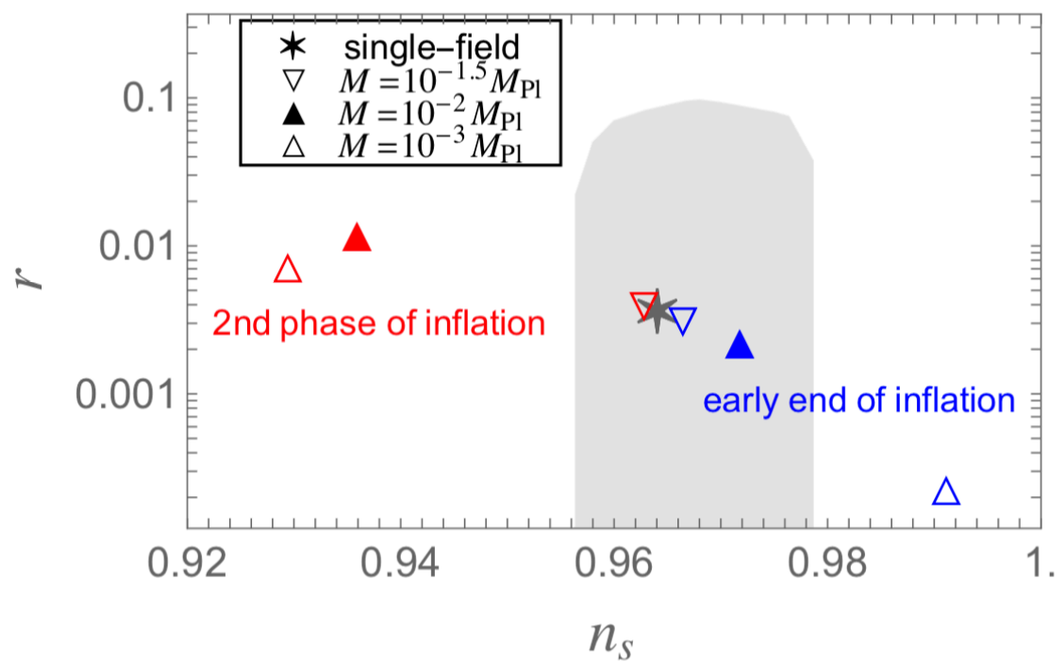


FIG. 1: Modified predictions for the spectral index and tensor-to-scalar ratio in our prototypical example based on the Starobinsky potential, for the scale that crosses the Hubble radius 55 e-folds before the end of inflation. The shaded area corresponds to the *Planck* 95% C.L. constraints [1].

Canonical Heavy Field

2 Sidetracked solution

Non-geodesic motion $\eta_\perp = \dfrac{f_\chi \dot{\phi}^2}{H}$

$$\frac{m_{eff}^2}{H^2} = \frac{V_{\chi\chi}}{H^2} + \left[6\left(\frac{f_\chi}{f}\right)^2 - 2\frac{f_{\chi\chi}}{f}\right]\epsilon \longrightarrow \boxed{\textbf{No instability} \quad 8\left(\frac{f_\chi}{f}\right)^2 \epsilon}$$

Sidetracked inflation: effective single field inflation [Garcia-Saenz et al. 2018]

modified speed of sound/dispersion relation

Probe stability of 1 and 2 by adding small perturbation $\chi = \bar{\chi} + \delta\chi$

$m_{eff}^2 < 0 \Leftrightarrow$ 1 is repulsive

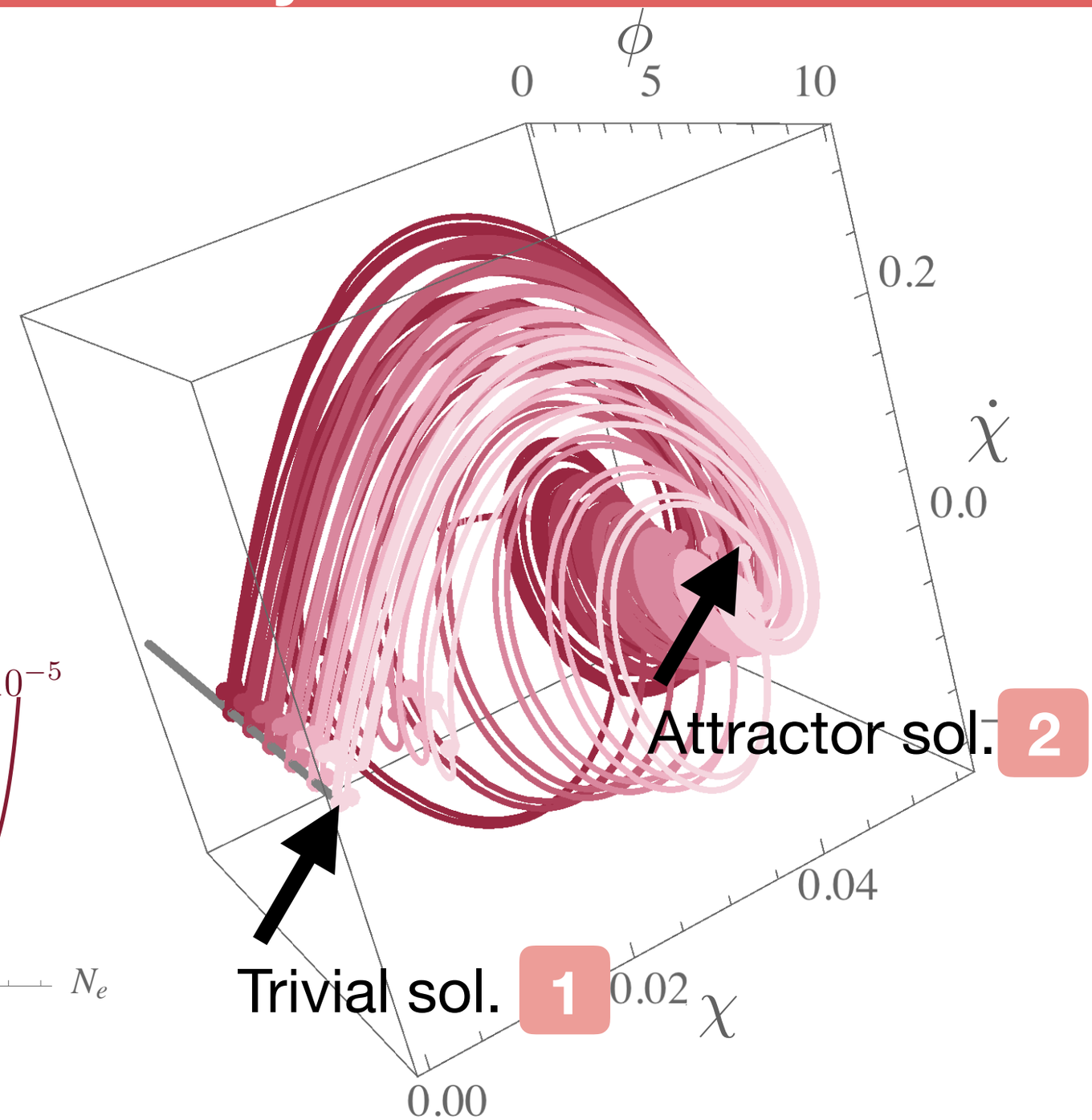
2 is the attractor of the system

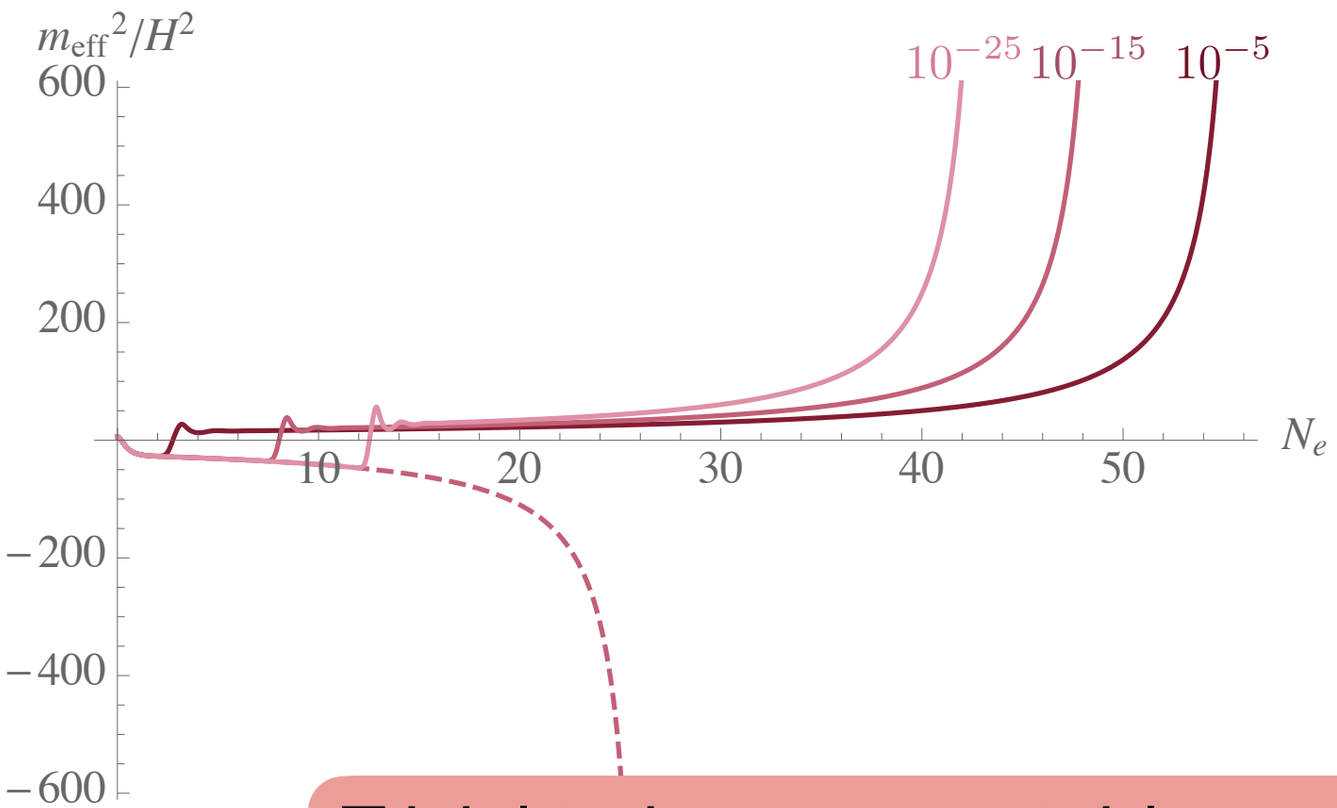
Canonical Heavy Field

Example:

$$f^2(\chi) = 1 + 2\frac{\chi^2}{M^2}$$

$$\mathbb{R} = -\frac{4M^2}{(M^2+2\chi^2)^2} \approx -\frac{4}{M^2}$$





Trivial trajectory: unstable + never reached dynamically

Non-Canonical Heavy Field

$$\ddot{\chi} + 3H\dot{\chi} = -\frac{V_\chi}{f^2} - 2\frac{f_\phi}{f}\dot{\chi}\dot{\phi}$$

$$\boxed{V_\chi = \dot{\chi} = 0}$$

geodesic motion $\eta_\perp = 0$

$$\frac{m_{eff}^2}{H^2} = \frac{V_{\chi\chi}}{H^2 f^2} + \frac{f_\phi}{f}\frac{V_\phi}{H^2} - 2\frac{f_{\phi\phi}}{f}\epsilon \ ,$$

defining $$\frac{f_\phi}{f}\frac{V_\phi}{H^2} = -3\frac{f_1}{f}\frac{\dot{\phi}}{H} = -3\frac{d\log f}{dN} \equiv -3g(N)$$

$$\boxed{\frac{m_{eff}^2}{H^2} = \frac{V_{\chi\chi}}{H^2 f^2} - 3g - g^2 + g\epsilon - \frac{dg}{dN}}$$

Non-Canonical Heavy Field

$$\frac{m_{eff}^2}{H^2} = \frac{V_{\chi\chi}}{H^2 f^2} - 3g - g^2 + g\epsilon - \frac{dg}{dN}$$

$g > 0$

f grows

χ gets lighter

2 field model

$g < 0$

f decreases

χ gets heavier

$m_{eff}^2 > 0$

No instability

Massless Fields

Massless χ flat direction $V = V(\phi)$

1 Canonical χ $3H\dot{\chi} = f f_\chi (\dot{\phi})^2$ $\eta_\perp \neq 0$

$$\frac{m_{eff}^2}{H^2} = 2\left(\left(\frac{f_\chi}{f}\right)^2 - \frac{f_{\chi\chi}}{f}\right)\epsilon + \mathcal{O}(\epsilon^2)$$

e.g. $f = f_0 e^{(\chi/M)^p}$

- $p > 1$ unstable
- $p = 1$ $m_{eff}^2 \sim 0$
- $0 < p < 1$ stable

[Achucarro et al. 2016]

Massless Fields

2 Non-canonical χ

$$\ddot{\chi} + 3H\dot{\chi}\left(1 + \frac{2}{3}\frac{f_\phi}{f}\frac{\dot{\phi}}{H}\right) = 0$$

$$\ddot{\chi} = \dot{\chi} = 0$$

$$\frac{m_{eff}^2}{H^2} = -3\frac{f_\phi}{f}\frac{\dot{\phi}}{H} - 2\frac{f_{\phi\phi}}{f}\epsilon$$

e.g.:

$$\mathcal{L} = \frac{(\partial\phi)^2}{2} + e^{\pm 2\phi/M}\frac{(\partial\chi)^2}{2} \rightarrow \frac{m_{eff}^2}{H^2} = \mp\sigma(\dot{\phi})\frac{3}{M}\sqrt{2\epsilon} - \frac{2}{M^2}\epsilon$$

Can be unstable

Explicit string inflation models with instability

Summary

- No instability for heavy scalars
- Kinetically coupled massless fields can be subject to instability
- Non-generic feature: model and parameter dependent
- Phenomenological applications?
- Non-perturbative methods are necessary to analyse the system
  - Stochastic inflation
  - Numerical relativity

THANK YOU