Geometrical (in)stability during inflation?

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Motivation

Isocurvature mass

Canonical heavy field

Canonical inflaton

Massless fields

Discussion
Motivation

String compactifications feature many scalars:

- $h^{2,1}$ complex structure moduli
- $h^{1,1}$ Kahler moduli
- axio-dilaton
- brane position moduli

Typically $\mathcal{O}(10) - \mathcal{O}(100)$ scalars

Non trivial structures in 4D Lagrangian.

$$\mathcal{L} = K_{IJ} \partial \Phi^I \partial \bar{\Phi}^J - V$$

- Complex scalars
- Curved Kahler metric
- F+D term potential
Observations seem to favour effective **single field models**

\[ f_{nl} \leq \mathcal{O}(10) \quad \beta_{iso} \ll 1 \]

Several string inflation models, most single field

**Model building problem**

Theory

**Cosmological Pert. Theory**

Observations
\[ S = \int \sqrt{-g} d^4 x \left( \frac{1}{2} R - \frac{1}{2} G_{ab} g^{\mu \nu} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi) \right) \]

Expand \( \phi_i = \phi_i^0 + \delta \phi_i^0 \) and \( g_{\mu \nu} = g_{\mu \nu}^0 + \delta g_{\mu \nu} \)

Background dynamics: Friedmann + KG eqs

Gauge invariant perturbations: \( Q^a = \delta \phi^a + \frac{\dot{\phi}^a}{H} \psi \)

\[
\frac{D^2 Q^a}{dt^2} + 3H \frac{DQ^a}{dt} - \frac{\nabla^2}{a^2} Q^a + C^{a}_{\ b} Q^b = 0
\]
Isocurvature Mass

On superhorizon scales

\[ \ddot{Q}_N + 3H \dot{Q}_N + m_{\text{eff}}^2 Q_N = 0 \]

Isocurvature mass:

\[ m_{\text{eff}}^2 \equiv \frac{V_{NN}}{H^2} + 3\eta_{\perp}^2 + \epsilon \mathbb{R} \]

\[ V_{NN} = N^a N^b \nabla_b V_a \quad \eta_{\perp} = \frac{N^a V_a}{H|\dot{\phi}|} \]

Usually for **hierarchical mass spectra**:

\[ \frac{m_{\text{eff}}^2}{H^2} \approx \frac{m_H^2}{H^2} \gg 1 \]

\[ \eta_{\perp} \approx 0 \quad \text{decoupling} \]
Isocurvature Mass

Reduces to single field model:

\[ \begin{align*}
Q_T & \text{ constant curvature} \\
Q_N & \text{ decaying isocurvature for } k \gg aH
\end{align*} \]

- \( P_k \propto \frac{H^2}{\epsilon} k^{n_s-1} \)
- \( n_s = 1 - 2\epsilon_H - \eta_H \)
- \( \epsilon_H = -\frac{\dot{H}}{H^2} \)
- \( \eta_H = -\frac{\epsilon_H}{\epsilon_H H} \)
- \( r = 16 \epsilon_H \)

\[ \frac{m_{eff}^2}{H^2} = \frac{V_{NN}}{H^2} + 3\eta_\perp^2 + \epsilon R \]

Curvature can be negative

\[ 0 > m_{eff}^2 \neq m_H^2 \gg H^2 \]

Background well behaved, perturbations not so much.
Negative field space curvature can turn isocurvature tachyonic

Uncontrolled growth of isocurvature perturbations:
- validity of the perturbative treatment
- backreaction on the background
- sudden end of inflation?

**Minimal model:**

\[
\mathcal{L} / \sqrt{|g|} = \frac{1}{2} G_{IJ} \partial \phi^I \partial \phi^J - V(\phi)
\]
Geometrical Instability

Assumptions:

\[ G_{IJ} = \begin{pmatrix} 1 & 0 \\ 0 & f^2(\phi^1) \end{pmatrix} \]

\[ V(\phi) = V(\phi^1) + V(\phi^2) \]

\[ \mathbb{R} = -2 \frac{f_{11}}{f} \]

necessary condition: \[ \frac{f_{11}}{f} \gg 0 \]

\[ \Box \phi^2 = - \frac{V_2}{f^2} - 2 \frac{f_1}{f} \phi^1 \dot{\phi}^2 \]

\[ \Box \phi^1 = -V_1 + f f_1(\dot{\phi}^2)^2 \]

Consider:

1. heavy spectator field
2. massless scalars
\[ \square \chi = -V_\chi + f f_\chi (\dot{\phi})^2 \]

1. Trivial solution
2. Geodesic motion \( \eta_\perp = 0 \)

\[ \ddot{\chi} = \chi = V_\chi = f f_\chi (\dot{\phi})^2 = 0, \]

\[ \ddot{\chi}, 3H \dot{\chi} \ll V_\chi = f f_\chi (\dot{\phi})^2 \neq 0 \]

\[ \frac{m_{\text{eff}}^2}{H^2} = \frac{V_{11}}{H^2} - 2\epsilon \frac{f_{11}}{f} \]
What does this mean?

A  premature end of inflation

B  kick the heavy mode

FIG. 1: Modified predictions for the spectral index and tensor-to-scalar ratio in our prototypical example based on the Starobinsky potential, for the scale that crosses the Hubble radius 55 e-folds before the end of inflation. The shaded area corresponds to the Planck 95% C.L. constraints [1].
Canonical Heavy Field

2 Sidetracked solution

Non-geodesic motion \( \eta_\perp = \frac{f \chi \dot{\phi}^2}{H} \)

\[
\frac{m_{\text{eff}}^2}{H^2} = \frac{V_{\chi \chi}}{H^2} + \left[ 6 \left( \frac{f \chi}{f} \right)^2 - 2 \frac{f_{\chi \chi}}{f} \right] \epsilon \]

Sidetracked inflation: effective single field inflation \[\text{[Garcia-Saenz et al. 2018]}\]
modified speed of sound/dispersion relation

Probe stability of 1 and 2 by adding small perturbation \( \chi = \bar{\chi} + \delta \chi \)

\[
m_{\text{eff}}^2 < 0 \iff 1 \text{ is repulsive}
2 \text{ is the attractor of the system}
\]

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Example:

\[ f^2(\chi) = 1 + 2 \frac{\chi^2}{M^2} \]

\[ \mathcal{R} = -\frac{4M^2}{(M^2 + 2\chi^2)^2} \approx -\frac{4}{M^2} \]

Trivial trajectory: unstable + never reached dynamically

Attractor sol.
Non-Canonical Heavy Field

\[ \ddot{\chi} + 3H\dot{\chi} = -\frac{V_\chi}{f^2} - 2\frac{f_\phi}{f} \dot{\chi}\dot{\phi} \]

geodesic motion \[ \eta_\perp = 0 \]

\[ \frac{m_{eff}^2}{H^2} = \frac{V_{\chi\chi}}{H^2 f^2} + \frac{f_\phi}{f} \frac{V_\phi}{H^2} - 2\frac{f_{\phi\phi}}{f} \epsilon , \]

defining \[ \frac{f_\phi}{f} \frac{V_\phi}{H^2} = -3 \frac{f_1}{f} \frac{\dot{\phi}}{H} = -3 \frac{d \log f}{dN} \equiv -3 g(N) \]

\[ \frac{m_{eff}^2}{H^2} = \frac{V_{\chi\chi}}{H^2 f^2} - 3g - g^2 + g\epsilon - \frac{dg}{dN} \]
Non-Canonical Heavy Field

\[
\frac{m_{\text{eff}}^2}{H^2} = \frac{V_{XX}}{H^2 f^2} - 3g - g^2 + g\epsilon - \frac{dg}{dN}
\]

\[g > 0\]
- \(f\) grows
- \(\chi\) gets lighter
- 2 field model

\[g < 0\]
- \(f\) decreases
- \(\chi\) gets heavier
- \(m_{\text{eff}}^2 > 0\)

No instability
Massless Fields

Massless $\chi$ flat direction $V = V(\phi)$

1 Canonical $\chi$

$3H\dot{\chi} = ff\chi(\dot{\phi})^2 \quad \eta_\perp \neq 0$

$$\frac{m_{eff}^2}{H^2} = 2 \left( \left( \frac{f\chi}{f} \right)^2 - \frac{f\chi\chi}{f} \right) \epsilon + \mathcal{O}(\epsilon^2)$$

e.g. $f = f_0 e^{(\chi/M)^p}$

- $p > 1$ unstable
- $p = 1 \quad m_{eff}^2 \sim 0$
- $0 < p < 1$ stable

[Achucarro et al. 2016]
Massless Fields

2 Non-canonical $\chi$

\[ \ddot{\chi} + 3H\dot{\chi} \left( 1 + \frac{2}{3} \frac{f_\phi \dot{\phi}}{f/H} \right) = 0 \]

\[ \ddot{\chi} = \dot{\chi} = 0 \]

\[ \frac{m_{\text{eff}}^2}{H^2} = -3 \frac{f_\phi}{f} \frac{\dot{\phi}}{H} - 2 \frac{f_{\phi\phi}}{f} \epsilon \]

e.g.:

\[ \mathcal{L} = \frac{(\partial \phi)^2}{2} + e^{\pm 2\phi/M} \frac{(\partial \chi)^2}{2} \rightarrow \frac{m_{\text{eff}}^2}{H^2} = \mp \sigma(\dot{\phi}) \frac{3}{M} \sqrt{2\epsilon} - \frac{2}{M^2} \epsilon \]

Can be unstable

Explicit string inflation models with instability
No instability for heavy scalars

Kinetically coupled massless fields can be subject to instability

Non-generic feature: model and parameter dependent

Phenomenological applications?

Non-perturbative methods are necessary to analyse the system
- Stochastic inflation
- Numerical relativity
THANK YOU