U(1) Symmetries with Higher Charge Singlets in F-Theory and Type IIB

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Plan

- Introduction and motivation
- F-theory in the perturbative type IIB limit
- Morrison-Park at weak coupling
- IIB version of a charge 3 model
- Charge 4 model
- Conclusions and Prospects
Introduction and motivation

• In contrast to $SU(5)$ F-theory GUTs, other MSSM-like realisations in F-th. DO exhibit a weak coupling limit without significant obstructions.
  
  Lin, Weigand’14’16; Cvetič, Klevers, DM, Oehlmann, Reuter’15;...

• ...we want to explore the weak coupling limit of F-th $U(1)$ models with different singlet spectra:
  
  ◦ $q = 1, 2$
    
    Morrison, Park’12;...

  ◦ $q$ up to 3: non-UFD fibers
    
    Klevers, DM, Oehlmann, Piragua, Reuter’14; Klevers, Taylor’16; Morrison, Park’16; Raghuram’17

  ◦ $q = 4$: UnHiggsing a $U(1) \times U(1)$ model
    
    Cvetič, Klevers, Piragua, Taylor’15; Raghuram’17
Starting with the Weierstrass form

\[ y^2 = x^3 + fxz^4 + gz^6, \]

make the coord. change \( s = x - b_2 z^2 / 3 \). Then the WSF reads

\[ y^2 = s^3 + b_2 s^2 z^2 + 2b_4 s z^4 + b_6 z^6, \]

with

\[ b_4 = f + \frac{b_2^2}{3}, \quad b_6 = g - \frac{2}{27} b_2^3 + \frac{2}{3} b_2 b_4. \]

setting the scalings \( b_2 \rightarrow b_2, b_4 \rightarrow \epsilon b_4, b_6 \rightarrow \epsilon^2 b_6. \)

- For \( \epsilon = 0 \) the fiber degenerates globally. Better study a resolved case

\[ y^2 = s^3 \lambda + \tilde{b}_2 s^2 z^2 + 2\tilde{b}_4 t s z^4 + \tilde{b}_6 t^2 z^6 \]

with \( s \mapsto s\lambda, y \mapsto y\lambda \) and \( \epsilon \mapsto t\lambda. \)
F-theory in the perturbative type IIB limit

- At $\lambda = t = 0$

  $$X_3 : \xi^2 = b_2 ;$$

  with $\xi = y/sz$. $b_2 = 0$ will be the invariant locus of $\sigma : \xi \to -\xi$ (orientifold locus)

- In the limit $\lambda \to 0$, the equation

  $$W_E : y^2 = \tilde{b}_2 s^2 z^2 + 2\tilde{b}_4 t s z^4 + \tilde{b}_6 t^2 z^6$$

  defines a $\mathbb{P}^1$ fibration degenerating whenever:

  $$\Delta_E = \tilde{b}_4^2 - \tilde{b}_2 \tilde{b}_6 = 0 .$$

- Discriminant loci permit to read out the gauge symmetries surviving the weak coupling.
Morrison-Park at weak coupling

Consider the following WSF

\[ y^2 = x^3 + \left(c_1 c_3 - b^2 c_0 - \frac{c_2^2}{3}\right) x + c_0 c_3^2 - \frac{1}{3} c_1 c_2 c_3 + \frac{2}{27} c_2^3 - \frac{2}{3} b^2 c_0 c_2 + \frac{b^2 c_1^2}{4}. \]

Setting the variable \( x = x - \frac{1}{3} b_2 \) (in the patch \( z = 1 \)),

\[ y^2 = x^3 + c_2 x^2 + (c_1 c_3 - b^2 c_0) x + c_0 c_3^2 - b^2 c_0 c_2 + \frac{b^2 c_1^2}{4}, \]

with the scalings

\[ b \to \epsilon^0 b, \quad c_0 \to \epsilon^2 c_0, \quad c_1 \to \epsilon^1 c_1, \quad c_2 \to \epsilon^0 c_2, \quad c_3 \to \epsilon^0 c_3. \]

The weak discriminant reads,

\[ \Delta_E = (c_3^2 - c_2 b^2) \left(\frac{c_1^2}{4} - c_2 c_0\right). \]

w/ the double cover CY \( \xi^2 = c_2 \).
Morrison-Park at weak coupling

In the CY, the discriminant splits as

$$\Delta_E = (c_3 - \xi b) (c_3 + \xi b) \left( \frac{c_1^2}{4} - \xi^2 c_0 \right)$$

- A single brane $Z$ at $c_3 - \xi b = 0$ and its image $\tilde{Z}$ at $c_3 + \xi b = 0$, give rise to a $U(1)$ symmetry.
- A Whitney brane $W$ at the invariant locus

$$\frac{c_1^2}{4} - \xi^2 c_0 = 0$$

- Two charged siglets: $q = 1$ at $Z \cap W$, $q = 2$ at $Z \cap \tilde{Z}$.
- $U(1)$ remains GS massless as $Z_\perp = Z - \tilde{Z} = 0$
U(1) models with charge 3 singlets

\[ p_{F_3} = s_1 u^3 e_1^2 + s_2 u^2 v e_1^2 + s_3 u v^2 e_1^2 + s_4 v^3 e_1^2 + s_5 u^2 w e_1 + s_6 u v w e_1 + s_7 v^2 w e_1 + s_8 u w^2 + s_9 v w^2 \]

The fiber \( p_{F_3} = 0 \) can be mapped to a WSF, with \( f(s_i) \) and \( g(s_i) \). The WCL can be obtained in the regime

\[ s_1 \to \epsilon^1 s_1, \quad s_5 \to \epsilon^1 s_5, \quad s_8 \to \epsilon^1 s_8, \quad s_i \to \epsilon^0 s_i \quad (i \neq 1, 5, 8). \]

CY Equation + Weak discriminant \( X_3 : \xi^2 = \frac{s_6^2}{4} - s_2 s_9 \)

\[ \Delta_E = -\frac{1}{4} s_9 \left( -s_3 s_6 s_7 + s_2 s_7^2 + s_3^2 s_9 + s_4 \left( s_6^2 - 4 s_2 s_9 \right) \right) \]

\[ \times \left( s_2^2 s_8^2 + s_2 \left( -s_5 s_6 s_8 + s_5^2 s_9 - 2 s_1 s_8 s_9 \right) + s_1 \left( s_6^2 s_8 - s_5 s_6 s_9 + s_1 s_9^2 \right) \right). \]
U(1) models with charge 3 singlets

- We have two D7 branes $X$, $Y$ and their images ($\tilde{X}$, $\tilde{Y}$) plus a Whitney brane $W$.
- $Y_- = -2X_-$ implies that the orientifold odd axion coupling leaves a massless U(1) combination.

$$Q = 2Q_X + Q_Y.$$ 

- We find a perfect match between vertical fluxes in F-th. and gauge fluxes in type IIB. Chiralities march perfectly.

<table>
<thead>
<tr>
<th>Type IIB</th>
<th>$X\tilde{X}$</th>
<th>$X\tilde{Y}$</th>
<th>$WX$</th>
<th>$\tilde{Y}Y$</th>
<th>$XY$</th>
<th>$YW$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mathbf{1}_{(2,0)}$</td>
<td>$\mathbf{1}_{(1,1)}$</td>
<td>$\mathbf{1}_{(-1,0)}$</td>
<td>$\mathbf{1}_{(0,-2)}$</td>
<td>$\mathbf{1}_{(1,-1)}$</td>
<td>$\mathbf{1}_{(0,1)}$</td>
</tr>
<tr>
<td>F-theory</td>
<td>-</td>
<td>$\mathbf{1}_3$</td>
<td>$\mathbf{1}_{-2}$</td>
<td></td>
<td></td>
<td>$\mathbf{1}_1$</td>
</tr>
</tbody>
</table>
Detour: Model with $\mathbb{Z}_3$ discrete symmetry

\[ \Delta_E = -\frac{1}{4} \left( -s_3 s_6 s_7 + s_2 s_7^2 + s_3^2 s_9 + s_4 (s_6^2 - 4s_2 s_9) \right) \]
\[ \times \left[ -s_{10} s_2^3 + s_{10} (s_1 s_6^3 - s_2 s_6 (s_5 s_6 + 3s_1 s_9) + s_2^3 (s_6 s_8 + 2s_5 s_9)) \right. \]
\[ \left. + s_9 (s_2^2 s_8^2 + s_2 (-s_5 s_6 s_8 + s_5^2 s_9 - 2s_1 s_8 s_9) + s_1 (s_6^2 s_8 - s_5 s_6 s_9 + s_1 s_5^2)) \right] . \]

- U(1) divisor has an odd part $Z_\mathrm{odd} = -3X_\mathrm{odd}$, hence it becomes GS massive.
- D1 instanton charges under the U(1) are 0 mod 3. The $\mathbb{Z}_3$ symmetry is manifest in the instanton sector. Similar argument holds for D3’s.
Detour: Model with $\mathbb{Z}_3$ discrete symmetry

• Ideal for the discretely charged singlet in F-theory is generated for 50 non transversally intersecting polynomials. That makes it hard to compute its chirality.

• $\mathbb{Z}_3$ model has two WC limits:

\[
s_1 \rightarrow \epsilon^1 s_1, \quad s_5 \rightarrow \epsilon^1 s_5, \quad s_8 \rightarrow \epsilon^1 s_8, \quad s_{10} \rightarrow \epsilon^1 s_{10},
\]

and

\[
s_1 \rightarrow \epsilon^1 s_1, \quad s_2 \rightarrow \epsilon^1 s_2, \quad s_3 \rightarrow \epsilon^1 s_3, \quad s_4 \rightarrow \epsilon^1 s_4,
\]

chiralities can be computed in both WC limits.

\[
\chi(1)_F = \chi(1)_{w.c.} + a_1 S_9 \bar{K}_B(2\bar{K}_B - S_9) = \chi(1)'_{w.c.} + a_2 S_7 \bar{K}_B(2\bar{K}_B - S_7),
\]

• one obtains

\[
\chi(1)_F = \Lambda(S_7 - 2S_9)(2S_7 - S_9)(-3\bar{K}_B + S_7 + S_9).
\]

for $G_4 = \frac{\Lambda}{9} \left(3S_3(3S_3 + 3\bar{K}_B - 2S_7 - 2S_9) - 2S_7^2 + 5S_7S_9 - 2S_9^2\right)$
Charge 4 model

\[ U(1) \text{ with } q = 3, \text{ } z \text{ coord. of the section} \]

\[ \hat{z} = s_7 s_8^2 - s_6 s_8 s_9 + s_5 s_9^2, \]

is singular for \( s_8 = s_9 = 0 \). Models with higher charges have a non UFD structure.

Charge 4 model: Higgs a \( U(1) \times U(1) \) with a vev field \( q = (1, -1) \) to \( U(1) \). Model also contains \( q = (2, 2) \), so one gets a charge 4 singlet.

Fiber Eq.

\[ p = u(s_1 u^2 + s_2 uv + s_3 v^2 + s_5 uw + s_6 vw + s_8 w^2) + (a_1 v + b_1 w)(d_0 v^2 + d_1 vw + d_2 w^2) = 0 \]

WC limit,

\[ s_1 \to \epsilon s_1, \quad s_2 \to \epsilon s_2, \quad s_3 \to \epsilon s_3, \quad d_0 \to \epsilon d_0, \]
Charge 4 model

CY threefold eq.

\[ \xi^2 = b_2 = \frac{s_6^2}{4} - a_1 d_1 s_5 , \]

Weak discriminant

\[ \Delta_E = -\frac{1}{16} a_1 \Delta_2 \Delta_3 , \]

all factors split. No invariant brane!

Two U(1)s become GS massive by couplings to two orientifold odd axions.

F-th. U(1) charge:

\[ Q = \frac{1}{2} (5Q_X - Q_Y + 3Q_Z) , \]

\[
\begin{array}{cccccccccc}
1_{(2,0,0)} & 1_{(0,2,0)} & 1_{(0,0,2)} & 1_{(1,-1,0)} & 1_{(1,1,0)} & 1_{(1,0,-1)} & 1_{(1,0,1)} & 1_{(0,-1,-1)} & 1_{(0,-1,1)} \\
XX & Y\bar{Y} & Z\bar{Z} & XY & X\bar{Y} & XZ & X\bar{Z} & YZ & Y\bar{Z} \\
- & -1 & 3 & 3 & 2 & 1 & 4 & -1 & 2
\end{array}
\]
Conclusions

- All F-th U(1) models built so far generically have a perturbative IIB version.
- Beyond charge 4?
- What are the brane distribution patterns in IIB that have a proper F-theory uplift?
- How does this extend to 6D? Restrictions on the number of odd axions (odd divisors).
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Dzięki!