Protected axions in a clockwork gauge symmetry model

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based on arXiv:1804.01112
in collaboration with E. Dudas and S. Pokorski

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StringPheno 2018
Warsaw, July 4th 2018
Axions: pseudo Nambu-Goldstone bosons (pNGB)
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Two examples of limitations in axion model building:
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Two examples of limitations in axion model building:

- low masses are not automatically consistent with UV completions in a quantum theory of gravity
- some axion models require intermediate scale ($\sim 10^{9-11}$ GeV) or super-Planckian dynamics
Mass range in axion models:

- Dynamical dark energy candidates
- « Fuzzy » dark matter
- $\log_{10}(m_a/eV)$
- QCD axion

Mass range:
- Small masses: need for a controlled explicit breaking of the axionic symmetry
- However: global symmetries expected to be broken by quantum gravity effects
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Example: 5d abelian gauge theory compactified on $S_1/\mathbb{Z}_2$, with Dirichlet boundary conditions for $A_\mu \rightarrow$ leaves in the spectrum a massless scalar (zero mode of $A_5$) with a shift symmetry

**Breaking** of the shift symmetry: requires **non-local** suppressed effects
Axion decay constant range in axion models:

$$\mathcal{L} \supset \frac{g_{a\gamma\gamma}}{f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu} + \ldots$$
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Quintessence/inflation/relaxion: $f_a \gtrsim M_P$
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\( f_a \) naturally obtained from known scale?
Clockwork models:
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Example of the scalar clockwork:

\[ \mathcal{L} = - \sum_{k=0}^{N} (|\partial_{\mu} \phi_k|^2 + V(|\phi_k|^2)) - (\sum_{k=0}^{N-1} \epsilon_k \phi_k^q \phi_{k+1}^* + h.c.) \]
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\[ \leftrightarrow \quad \text{Clockwork Goldstone boson } a_{cl.} \sim \frac{\theta_0}{q^N} + \ldots + \frac{\theta_{N-1}}{q} + \theta_N \]

and effective decay constants \( f_{\text{eff}} \sim q^i f \)
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Obtained from 5D with linear dilaton geometry.

\( \mathcal{L}_{5D} = g^{MN} \partial_M \phi \partial_N \phi(x^\mu, y) \) with \( ds^2 = e^{4k|y|/3}(dx^2 + dy^2) \)
This work:

Use a clockwork-inspired gauge group to protect an axion, and study its phenomenology
Outline

A gauge theory with a pGB

SM couplings

Application: FDM

Conclusion
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A gauge theory: 4D UV-completion of a deconstructed abelian gauge theory on a linear dilaton background
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\[
\mathcal{L} = -\frac{1}{4} \sum_{i=1}^{N} F_{\mu\nu,i} F_{i}^{\mu\nu} - \sum_{k=0}^{N} |D_{\mu} \phi_{k}|^{2} - V(\phi_{0}^{2}, \phi_{1}^{2}, \ldots)
\]

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with one global accidental $U(1)$: $\phi_k \rightarrow e^{iq^k \alpha} \phi_k$
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In the spectrum for a complete spontaneous breaking, Goldstone boson $a$:

\[
a \sim \frac{1}{q^N f_0} \theta_0 + \frac{1}{q^{N-1} f_1} \theta_1 + \ldots + \frac{1}{f_N} \theta_N
\]
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In the spectrum for a complete spontaneous breaking, **Goldstone boson** \( \alpha \)

\[ a \sim \frac{1}{q^{N}f_{0}} \theta_{0} + \frac{1}{q^{N-1}f_{1}} \theta_{1} + \ldots \rightarrow \text{Site-dependent couplings?} \]
\[ \mathcal{L} = -\frac{1}{4} \sum_{i=1}^{N} F_{\mu \nu, i} F_{i}^{\mu \nu} - \sum_{k=0}^{N} |D_{\mu} \phi_{k}|^{2} - V(|\phi_{0}|^{2}, |\phi_{1}|^{2}, ...) \]

with one global accidental \( U(1) \): \( \phi_{k} \rightarrow e^{iqk\alpha} \phi_{k} \)

How much approximate? Gauge invariant operators:

\[ |\phi_{k}|^{2} \text{ and } \phi_{0} \phi_{1} ... \phi_{N}^{q} \]

\( \rightarrow \) exponential increase of the order of the breaking operators with \( q \) and \( N \)
SM couplings
Couplings to SM fields:

\[ \mathcal{L} \supset \frac{g_{a\gamma\gamma}}{f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{ig_{a,\text{EDM}}}{f_a} a N \gamma_{\mu\nu} \gamma^5 N F^{\mu\nu} + \frac{g_{aNN}}{f_a} \partial_{\mu} a \overline{N} \gamma^\mu \gamma^5 N + \frac{g_{ae\bar{e}}}{f_a} \partial_{\mu} a \overline{e} \gamma^\mu \gamma^5 e \]
\[ \mathcal{L} \supset \frac{g_{a\gamma\gamma}}{f_a} aF_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g_{aNN}}{f_a} \partial_\mu a\overline{N} \gamma^\mu \gamma^5 N + \ldots \]
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KSVZ model:

\[ \mathcal{L} \supset \phi \overline{Q}_L Q_R + h.c. \xrightarrow{Q \text{ triangle loop}} \frac{a}{f} F \tilde{F} \]
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Gauge-anomalous now. **Need more fermions:**

\[ \mathcal{L} \supset \phi_0 \bar{Q}_{L,0} Q_{R,0} + h.c. \]
A gauge theory with a pGB SM couplings Application: FDM

\[ \mathcal{L} \supset \frac{g_a \gamma}{f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g_a N N}{f_a} \partial_\mu a \bar{N} \gamma^\mu \gamma^5 N + \ldots \]

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\[ \xrightarrow{U(1)_1 \text{ anom.}} \mathcal{L} \supset \phi_0 \bar{Q}_{L,0} Q_{R,0} + \sum_{i=1}^{q} \phi_1 \bar{Q}^i_{L,1} Q^i_{R,1} + h.c. \]
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\[ \mathcal{L} \supset \phi Q_L Q_R + h.c. \quad Q \text{ triangle loop} \quad \frac{a}{f} F \tilde{F} \]

\[ U(1)_1 \text{ anom.} \quad \mathcal{L} \supset \phi_0 Q_{L,0} Q_{R,0} + \sum_{i=1}^{q} \phi_1 Q_{L,1}^i Q_{R,1}^i + h.c. \]

\[ U(1)_2 \text{ anom.} \quad \mathcal{L} \supset \phi_0 Q_{L,0} Q_{R,0} + \sum_{i=1}^{q} \phi_1 Q_{L,1}^i Q_{R,1}^i + \sum_{i=1}^{q^2} \phi_2 Q_{L,2}^i Q_{R,2}^i + h.c. \]

\[ U(1)_3 \text{ anom.} \quad \mathcal{L} \supset ... \]
A gauge theory with a pGB SM couplings

Application: FDM

Conclusion

\[
\mathcal{L} \supset \frac{g_a \gamma_\gamma}{f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g_a N N}{f_a} \partial_\mu a \bar{N} \gamma^\mu \gamma^5 N + ... 
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\]

\[
\xrightarrow{U(1)_2 \text{ anom.}} \ldots 
\]

\[
\xrightarrow{\text{triangle loops}} \sqrt{1+q^2+\ldots+q^{2N}} \frac{f}{f} a F \tilde{F} 
\]
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\[ f_{\text{eff}} \sim \frac{f}{q^N} \]
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Due to the number of additional fermions AND the high accidental global charges:

\[ f_{\text{eff}} = \frac{f}{\sqrt{\sum_{\text{fermions}} \text{charges}}} \sim \frac{f}{\sqrt{q^N q^N}} \]
A gauge theory with a pGB

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A gauge theory with a pGB SM couplings Application: FDM Conclusion

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Example: coupling to the first SM generation

\[ \mathcal{L} \supset -\frac{1}{M_P} \left( \bar{u}_R H \phi_i Y_u Q_L + \bar{d}_R (H \phi_i)^* Y_d Q_L + \bar{e}_R (H \phi_i)^* Y_e L_L \right) + h.c. \]
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\[ \text{chiral redef.} \quad \mathcal{L} \supset \frac{-i q^i \partial_\mu a}{2 \sqrt{1 + \ldots + q^{2N} f}} \left( \bar{u} \gamma_5 \gamma^\mu u + \bar{d} \gamma_5 \gamma^\mu d + \bar{e} \gamma_5 \gamma^\mu e \right) \]
\[ \mathcal{L} \supset \frac{g_a}{f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g_a N N}{f_a} \partial_\mu a \bar{N} \gamma^\mu \gamma^5 N + \ldots \]

Example: coupling to the first SM generation

\[ \mathcal{L} \supset - \frac{1}{M_P} \left( u_R H \phi_i Y_u Q_L + d_R (H \phi_i)^* Y_d Q_L + e_R (H \phi_i)^* Y_e L_L \right) + h.c. \]

chiral redef. \quad \mathcal{L} \supset \frac{-i q^i \partial_\mu a}{2 \sqrt{1 + \ldots + q^{2N} f}} (\bar{u} \gamma_5 \gamma^\mu u + \bar{d} \gamma_5 \gamma^\mu d + \bar{e} \gamma_5 \gamma^\mu e) \]

**Site-dependent coupling to the spins** derived in minimal setup
Axion field \( a = \frac{\theta_0 + q\theta_1 + \ldots + q^N\theta_N}{\sqrt{1+q^2+\ldots+q^{2N}}} \)

In the effective theory:

\[
\mathcal{L} \supset \frac{g_{a\gamma\gamma}}{f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g_{aN}}{f_a} \partial_\mu a \overline{N} \gamma^\mu \gamma^5 N + \ldots
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\]

Scales:

\[
\begin{align*}
&f \quad \frac{f_a}{g_{a\gamma\gamma}} \sim \frac{f}{q^N} \\
&\frac{f_a}{g_{aee}} \sim \frac{f}{q^{i-N}}
\end{align*}
\]
Application: FDM
For an ALP dark matter candidate:
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Masses can be as low as $m \sim 10^{-22}$ eV ("fuzzy" dark matter candidate)

Can be perturbative or non-perturbative
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Can be **perturbative or non-perturbative**

Focus here on **perturbative gravitational origin** and on **misalignment mechanism** (with pre-inflationary breaking):

\[
V = -\frac{\phi_0 \phi_1^q \ldots \phi_N^q}{M_P^{1+q+\ldots+q^N-4}} \supset -\left(\frac{f}{M_P}\right)^{1+q+\ldots+q^N} M_P^4 \cos\left(\frac{a}{f_a}\right)
\]

and

\[
\langle a_{\text{init}} \rangle = \text{random}
\]
\[ \Omega_a h^2 = 0.12 \text{ when:} \]

\begin{align*}
\text{ALP DM} & \quad q=2, q=4 \\
& \quad q=3, q=5 \\
& \quad q=6 \\
\text{QCD axion} & \\
\end{align*}

N=2

N=3
Detection of spin precession (with $\frac{\partial \mu \alpha}{f_a} \overline{N} \gamma^\mu \gamma^5 N$):
Detection of spin precession (with $\frac{\partial_{\mu} a}{f_a} \overline{N} \gamma^\mu \gamma^5 N$):
Detection of spin precession (with $\frac{\partial_{\mu} a}{f_a} \overline{N} \gamma_{\mu} \gamma^5 N$): Coupled at site N
Detection of spin precession (with $\frac{\partial a}{f_\alpha} \overline{N} \gamma^\mu \gamma^5 N$): Coupled at site 0
Conclusion
We considered a pGB protected against (gravitational) breaking effects. Its mass is easily very small, even with few additional gauge groups.
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It can have all usual axion couplings, associated to scales which display the clockwork charges of the scalars. While axion-spin couplings are generated in minimal setups, anomalous couplings to gauge fields require an additional fermion sector.
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It can have all usual axion couplings, associated to scales which display the clockwork charges of the scalars. While axion-spin couplings are generated in minimal setups, anomalous couplings to gauge fields require an additional fermion sector.

In the minimal setup, the (unavoidable) gravity contribution is sufficient to provide the correct DM density, and spin-precession-based searches can detect such a particle.
Thank you!
Backups
Example of a Peccei-Quinn symmetry:

**Peccei-Quinn symmetry:**

Explains why the $\mathbb{CP}^n$ "$\theta$-term" $\mathcal{L}_{\text{QCD}} \supset \frac{\theta_{\text{QCD}}}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$ verifies $\theta_{\text{QCD}} < 10^{-10}$

Postulates a global symmetry with a $SU(3)^2 \times U(1)_{PQ}$ anomaly → makes $\theta_{\text{QCD}}$ dynamical (axion) and stabilizes it at $\theta_{\text{QCD}} = 0$
PQ symmetry: Global symmetry with a $SU(3)^2 \times U(1)_{PQ}$ anomaly + axion $\rightarrow \theta_{QCD} = 0$

Specific realization: **KSVZ model** with

$$\mathcal{L}_{PQ} \supset \phi \overline{Q}_L Q_R + h.c. - V(|\phi|^2), \phi \xrightarrow{U(1)_{PQ}} e^{i\alpha} \phi \text{ and } \phi = \frac{f+r}{\sqrt{2}} e^{i\frac{a}{f}}.$$  

Then:

$$\text{QCD anom. + instantons } \rightarrow \mathcal{L} \supset m_{\pi}^2 f_{\pi}^2 \overline{\sigma m_u m_d} \cos\left(\frac{a}{f} - \theta_{QCD}\right)$$

Possible correction: $\mathcal{L}_{PQ} \supset \frac{\phi^n}{M_P^{n-4}} + h.c. \rightarrow \text{destabilizes} \theta < 10^{-10}$ if $n < 10$ (if $f \gtrsim 10^9$ GeV). Indeed:

$$\frac{\phi^n}{M_P^{n-4}} \text{ term } \rightarrow \mathcal{L} \supset \left(\frac{f}{\sqrt{2} M_P}\right)^n M_P^4 \cos\left(\frac{na}{f}\right)$$
Protection of global symmetries: make them accidental.

- Ex: B, L in the renormalizable standard model lagrangian
- $U(1)_{PQ}$ protection: Barr and Seckel (1992)

<table>
<thead>
<tr>
<th>Fields</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$Q^i_L = 1\ldots q$</th>
<th>$\tilde{Q}^i_L = 1\ldots p$</th>
<th>$Q^i_R = 1\ldots p+q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU(3)$</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$U(1)$</td>
<td>$p$</td>
<td>$q$</td>
<td>$p$</td>
<td>$-q$</td>
<td>0</td>
</tr>
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<td>$U(1)_{PQ}$</td>
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<td>$q$</td>
<td>$p$</td>
<td>0</td>
</tr>
</tbody>
</table>

where $\gcd(p, q) = 1$ and $p + q \geq 10$

$$
\mathcal{L} \supset \phi_1 Q_L Y Q_R + \phi_2^* \tilde{Q}_L \tilde{Y} Q_R + \frac{\phi_1 \phi_2^p}{M_P^{p+q-4}} + h.c.
$$

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\[ \mathcal{L} = -\frac{1}{4} \sum_{i=1}^{N} F_{\mu\nu,i} F_{i}^{\mu\nu} - \sum_{k=0}^{N} |D_{\mu} \phi_{k}|^{2} - V(|\phi_{0}|^{2}, |\phi_{1}|^{2}, ...) \]

with one global accidental $U(1)$: $\phi_{k} \rightarrow e^{iq^{k} \alpha} \phi_{k}$

Gravitational breaking under control:

\[ \mathcal{L} \supset \frac{\phi_{0}\phi_{1}^{q}...\phi_{N}^{q^{N}}}{M_{P}^{1+...+4}} \rightarrow m_{a}^{(grav)} = \left( \frac{f}{\sqrt{2} M_{P}} \right)^{\frac{q+...+q^{N}-1}{2}} \sqrt{1+q^{2}+...+q^{2N}} M_{P} \]
Mass suppression with **few additional gauge groups**:
A gauge theory with a pGB SM couplings

Application: FDM

Conclusion

\[ \mathcal{L} \supset \frac{g_{\alpha \gamma \gamma}}{f_a} a F_{\mu \nu} \tilde{F}^{\mu \nu} + \frac{g_{a NN}}{f_a} \partial_\mu a \overline{N} \gamma^\mu \gamma^5 N + \ldots \]

Number of fermions \( \sim q^N \): growing with protection quality.

**General feature:**

\[ \mathcal{L} \supset - \sum_i (\mathcal{O}_i \overline{\psi}_{i,L} \psi_{i,R}) \xrightarrow{\text{triangle loops}} \frac{i}{32 \pi^2} \log \left( \prod_i \mathcal{O}_i \right) F \tilde{F} \]
For a QCD axion:

Protection efficient if $m_{a}^{(\text{grav})} < 10^{-5} \left( m_{a}^{(\text{QCD})} \sim \frac{m_{\pi}f_{\pi}}{2f_{a}} \right)$

(with $f_{a}$ defined by the axion-gluons coupling: $\mathcal{L} \supset \frac{a}{f_{a}} G \tilde{G}$)

In our setup:

$$f_{a} = \frac{f}{\sqrt{1 + q^{2} + \ldots + q^{2N}}}$$
\[ \theta_{\text{QCD}} < 10^{-10} \text{ if:} \]
\[
\begin{bmatrix}
  m_a^{(\text{QCD})} \\
  m_a^{(\text{grav})}
\end{bmatrix} \sim \frac{m_\pi f_\pi}{2f_a} > 10^5 \begin{bmatrix}
  m_a^{(\text{grav})} \\
  (\frac{f}{M_P})^{q+\ldots+q^N-1} \frac{f}{f_a M_P}
\end{bmatrix}
\]
Protected QCD axion with $f \sim 10^{11}$ GeV and $q = 3, N = 2 \rightarrow 13$ additional colored Dirac fermions (+13 additional singlet Dirac fermions)

No Landau pole for QCD below the Planck mass
Stability of the DM ALP’s?

No anomaly: **no ALP-photon conversion** via usual
\[ \mathcal{L} \supset \frac{a}{f_a} F \tilde{F} \]

Instead: **derivative interactions** + tiny mass → long lifetime

Example: coupling to a heavy anomaly-free set of electrically charged fermions:
\[ \mathcal{L} \supset y_1 \phi_i \psi_{R,1} \psi_{L,1} + y_2 \phi_i \psi_{L,2} \psi_{R,2} + h.c. . \]

\[
\begin{align*}
\text{fermions integr.} & \quad \mathcal{L}_{\text{eff}} \supset \frac{e^2}{48\pi^2 q_i f} \left( \frac{1}{m_1^2} - \frac{1}{m_2^2} \right) \left( \Box a F \tilde{F} - \frac{1}{2} \partial_\mu a F_{\nu\eta} \partial^\eta \tilde{F}^{\mu\nu} \right)
\end{align*}
\]

Lifetimes for the FDM: \(~ 10^{300} \) s