Protected axions in a clockwork gauge symmetry model

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based on arXiv:1804.01112
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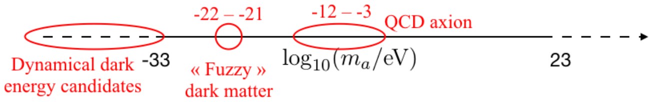
- **low masses are not automatically consistent with UV completions in a quantum theory of gravity**
- **some axion models require intermediate scale ($\sim 10^{9-11}$ GeV) or super-Planckian dynamics**

Mass range in axion models:



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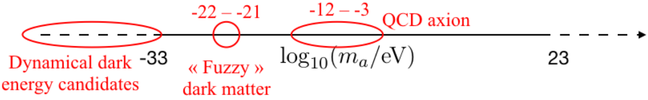

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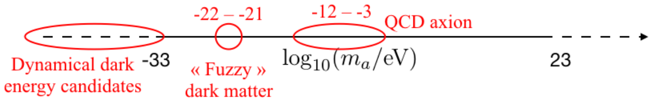

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However: **global symmetries expected to be broken by quantum gravity effects**

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Breaking of the shift symmetry: requires **non-local** suppressed effects

Axion decay constant range in axion models:

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f_a **naturally obtained from known scale?**

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Example of the scalar clockwork:

$$\mathcal{L} = -\sum_{k=0}^{N}(|\partial_\mu\phi_k|^2 + V(|\phi_k|^2)) - (\sum_{k=0}^{N-1}\epsilon_k\phi_k^q\phi_{k+1}^* + h.c.)$$

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$\hookrightarrow$ Clockwork Goldstone boson $a_{cl.} \sim \frac{\theta_0}{q^N} + ... + \frac{\theta_{N-1}}{q} + \theta_N$ and **effective decay constants** $f_{\text{eff}} \sim q^i f$

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Obtained from **5D with linear dilaton geometry**.
$\mathcal{L}_{5D} = g^{MN}\partial_M \phi \partial_N \phi(x^\mu, y)$ with $ds^2 = e^{4k|y|/3}(dx^2 + dy^2)$

This work:

Use a clockwork-inspired gauge group to protect an axion, and study its phenomenology

Outline

A gauge theory with a pGB

SM couplings

Application: FDM

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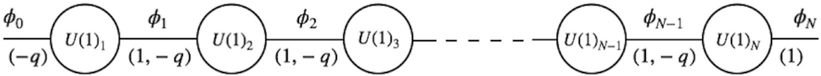
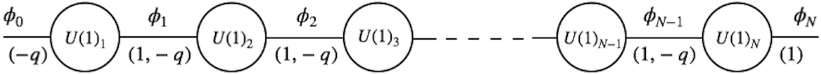


$$\mathcal{L} = -\frac{1}{4}\sum_{i=1}^{N} F_{\mu\nu,i}F_i^{\mu\nu} - \sum_{k=0}^{N} |D_\mu \phi_k|^2 - V\left(|\phi_0|^2, |\phi_1|^2, \ldots\right)$$

Ahmed & Dillon (2017), Coy Frigerio & Ibe (2017), Choi Im & Shin (2017)



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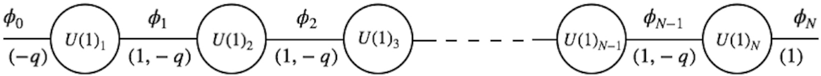

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In the spectrum for a complete spontaneous breaking, **Goldstone boson** a:

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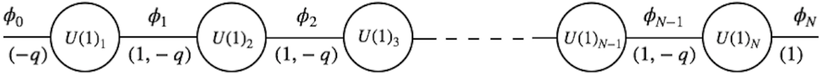

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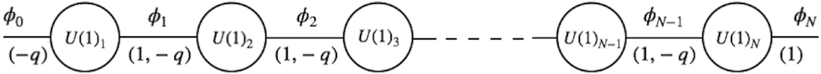

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How much approximate? Gauge invariant operators:

$$|\phi_k|^2 \text{ and } \phi_0\phi_1^q ... \phi_N^{q^N}$$

→ **exponential increase of the order of the breaking operators** with q and N

SM couplings

Couplings to SM fields:

$$\mathcal{L} \supset \frac{g_{a\gamma\gamma}}{f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{i g_{a,\mathrm{EDM}}}{f_a} a \overline{N} \gamma_{\mu\nu} \gamma^5 N F^{\mu\nu} + \frac{g_{aNN}}{f_a} \partial_\mu a \overline{N} \gamma^\mu \gamma^5 N + \frac{g_{aee}}{f_a} \partial_\mu a \bar{e} \gamma^\mu \gamma^5 e$$

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KSVZ model:

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Due to the number of additional fermions AND the high accidental global charges:

$$f_{\text{eff}} = \frac{f}{\sqrt{\sum_{\text{fermions}}(\text{charges})}} \sim \frac{f}{\sqrt{q^N q^N}}$$

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Example: coupling to the first SM generation

$$\mathcal{L} \supset -\frac{1}{M_P}\left(\overline{u_R} H\phi_i Y_u Q_L + \overline{d_R}(H\phi_i)^* Y_d Q_L + \overline{e_R}(H\phi_i)^* Y_e L_L\right) + h.c.$$

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Site-dependent coupling to the spins derived in minimal setup

$$\mathcal{L} \supset \boxed{\frac{g_{a\gamma\gamma}}{f_a} a F_{\mu\nu}\tilde{F}^{\mu\nu}} + \boxed{\frac{g_{aNN}}{f_a}\partial_\mu a \overline{N}\gamma^\mu\gamma^5 N} + \ldots$$

Axion field $a = \frac{\theta_0 + q\theta_1 + \ldots + q^N\theta_N}{\sqrt{1+q^2+\ldots+q^{2N}}}$

In the effective theory:

$$\boxed{\frac{\sqrt{1+q^2+\ldots+q^{2N}}}{f} aF\tilde{F}} \quad \boxed{\frac{q^i \partial_\mu a}{\sqrt{1+q^2+\ldots+q^{2N}}\, f}\overline{N}\gamma^\mu\gamma^5 N}$$

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Scales: f $\quad \frac{f_a}{g_{a\gamma\gamma}} \sim \frac{f}{q^N}$ $\quad \frac{f_a}{g_{aee}} \sim \frac{f}{q^{i-N}}$

Application: FDM

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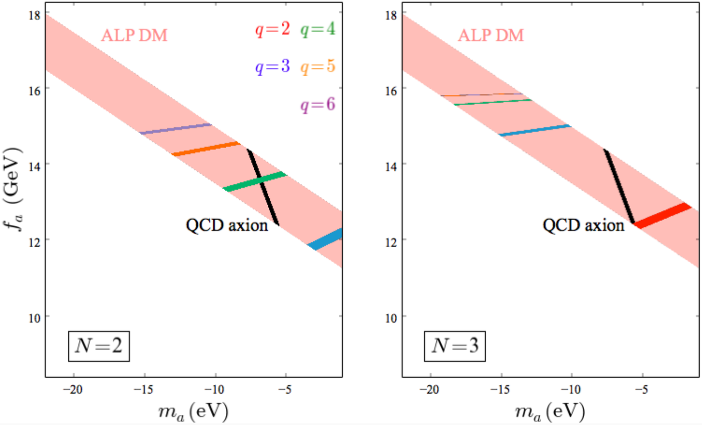
Focus here on **perturbative gravitational origin** and on **misalignment mechanism** (with pre-inflationnary breaking):

$$V = -\frac{\phi_0 \phi_1^q \ldots \phi_N^{q^N}}{M_P^{1+q+\ldots+q^N-4}} \supset -\left(\frac{f}{M_P}\right)^{1+q+\ldots q^N} M_P^4 \cos\left(\frac{a}{f_a}\right)$$

and

$$< a_{\text{init}} >= \text{random}$$

$\Omega_a h^2 = 0.12$ when:



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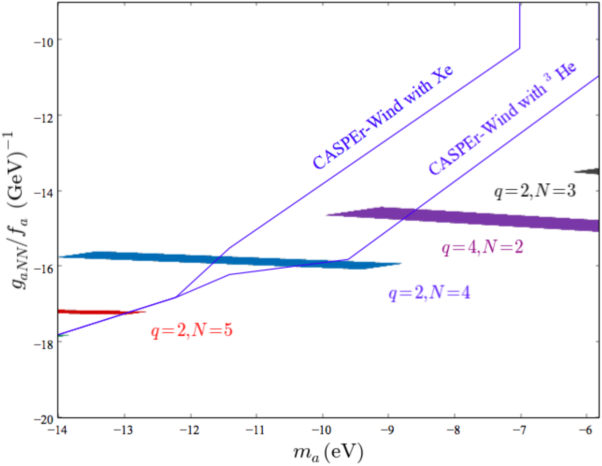

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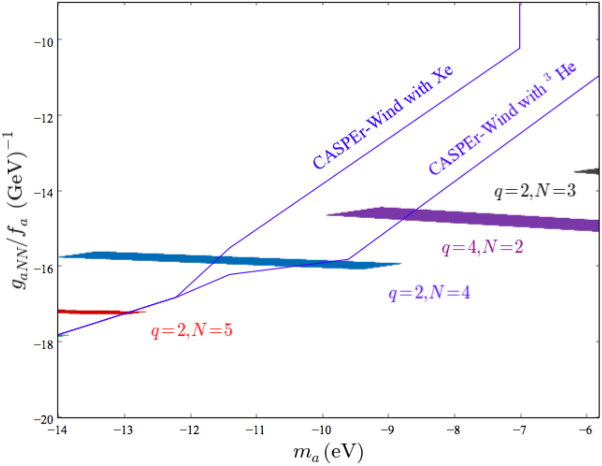
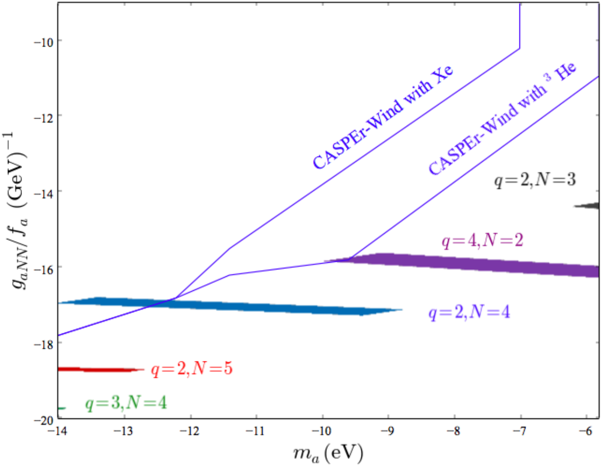


Detection of spin precession (with $\frac{\partial_\mu a}{f_a}\overline{N}\gamma^\mu\gamma^5 N$): Coupled at site 0



Conclusion

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In the minimal setup, the (unavoidable) **gravity contribution is sufficient to provide the correct DM density**, and spin-precession-based searches can detect such a particle.

Thank you!

Backups

Example of a Peccei-Quinn symmetry:

Peccei-Quinn symmetry:

Explains why the $\not{C}\not{P}$ "θ-term" $\mathcal{L}_{\rm QCD} \supset \frac{\theta_{\rm QCD}}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$ verifies $\theta_{\rm QCD} < 10^{-10}$

Postulates a global symmetry with a $SU(3)^2 \times U(1)_{PQ}$ anomaly $\rightarrow$ makes $\theta_{\rm QCD}$ dynamical (axion) and stabilizes it at $\theta_{\rm QCD} = 0$

PQ symmetry: Global symmetry with a $SU(3)^2 \times U(1)_{PQ}$ anomaly + axion$\rightarrow \theta_{\text{QCD}} = 0$

Specific realization: **KSVZ model** with
$\mathcal{L}_{PQ} \supset \phi \overline{Q_L} Q_R + h.c. - V(|\phi|^2)$, $\phi \xrightarrow{U(1)_{PQ}} e^{i\alpha}\phi$ and $\phi = \frac{f+r}{\sqrt{2}} e^{i\frac{a}{f}}$.
Then:

$$\text{QCD anom. + instantons} \rightarrow \mathcal{L} \supset m_\pi^2 f_\pi^2 \frac{\sqrt{m_u m_d}}{m_u+m_d} \cos\left(\frac{a}{f} - \theta_{\text{QCD}}\right)$$

Possible correction: $\mathcal{L}_{\not{PQ}} \supset \frac{\phi^n}{M_P^{n-4}} + h.c.$$\rightarrow$ **destabilizes** $\boldsymbol{\theta < 10^{-10}}$ **if** $\boldsymbol{n < 10}$ (if $f \gtrsim 10^9$ GeV). Indeed:

$$\frac{\phi^n}{M_P^{n-4}} \text{ term} \rightarrow \mathcal{L} \supset \left(\frac{f}{\sqrt{2}M_P}\right)^n M_P^4 \cos\left(\frac{na}{f}\right)$$

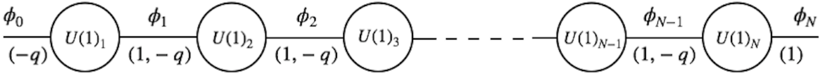
Protection of global symmetries: make them accidental.

- Ex: B, L in the renormalizable standard model lagrangian
- $U(1)_{PQ}$ protection: Barr and Seckel (1992)

Fields	ϕ_1	ϕ_2	$Q_L^{i=1\ldots q}$	$\widetilde{Q}_L^{i=1\ldots p}$	$Q_R^{i=1\ldots p+q}$
$SU(3)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$
$U(1)$	p	q	p	$-q$	0
$U(1)_{PQ}$	q	$-p$	q	p	0

where $\gcd(p,q) = 1$ and $p + q \geq 10$

$$\mathcal{L} \supset \underbrace{\phi_1 \overline{Q_L} Y Q_R + \phi_2^* \overline{\widetilde{Q}_L} \widetilde{Y} Q_R}_{\mathcal{L}_{PQ}} + \underbrace{\frac{\phi_1^q \phi_2^{*p}}{M_P^{p+q-4}}}_{\mathcal{L}_{\not{PQ}}} + h.c.$$



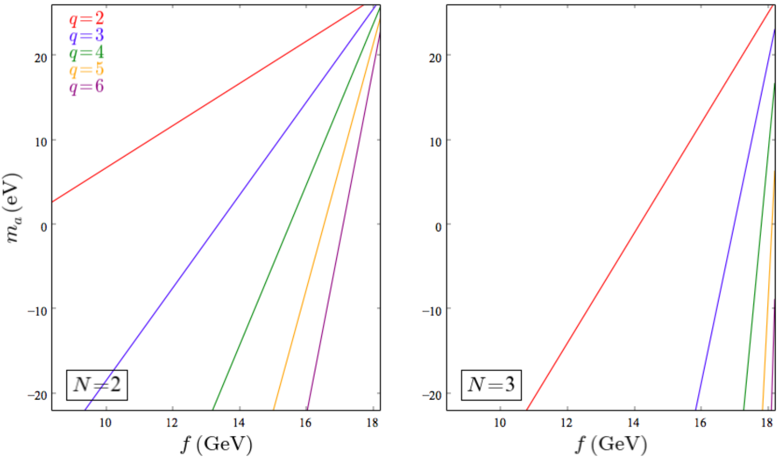
$$\mathcal{L} = -\frac{1}{4}\sum_{i=1}^{N} F_{\mu\nu,i}F_i^{\mu\nu} - \sum_{k=0}^{N} |D_\mu\phi_k|^2 - V(|\phi_0|^2, |\phi_1|^2, \ldots)$$

with one global accidental $U(1)$: $\phi_k \to e^{iq^k\alpha}\phi_k$

Gravitational breaking under control:

$$\mathcal{L} \supset \frac{\phi_0\phi_1^q\ldots\phi_N^{q^N}}{M_P^{1+\ldots-4}} \to m_a^{(\mathrm{grav})} = \left(\frac{f}{\sqrt{2}M_P}\right)^{\frac{q+\ldots+q^N-1}{2}} \sqrt{1+q^2+\ldots+q^{2N}}\,M_P$$

Mass suppression with **few additional gauge groups**:

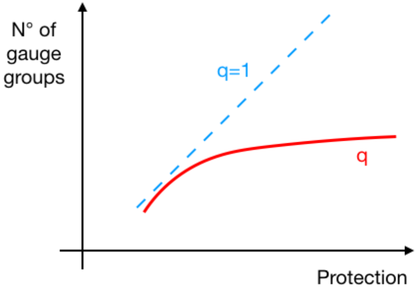


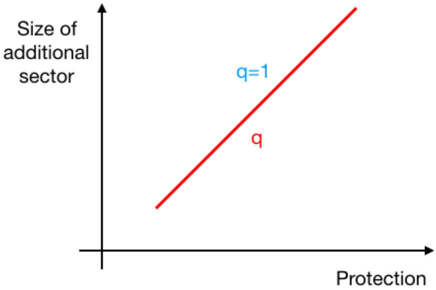
$$\mathcal{L} \supset \boxed{\frac{g_{a\gamma\gamma}}{f_a} a F_{\mu\nu}\tilde{F}^{\mu\nu}} + \frac{g_{aNN}}{f_a}\partial_\mu a \overline{N}\gamma^\mu\gamma^5 N + \ldots$$

Number of fermions $\sim q^N$: growing with protection quality.

General feature:

$$\mathcal{L} \supset -\sum_i (\mathcal{O}_i \overline{\psi_{i,L}}\psi_{i,R}) \xrightarrow{\text{triangle loops}} \frac{i}{32\pi^2}\log\left(\prod_i \mathcal{O}_i\right) F\tilde{F}$$





For a QCD axion:

Protection efficient if $m_a^{(\text{grav})} < 10^{-5}\left(m_a^{(\text{QCD})} \sim \frac{m_\pi f_\pi}{2f_a}\right)$

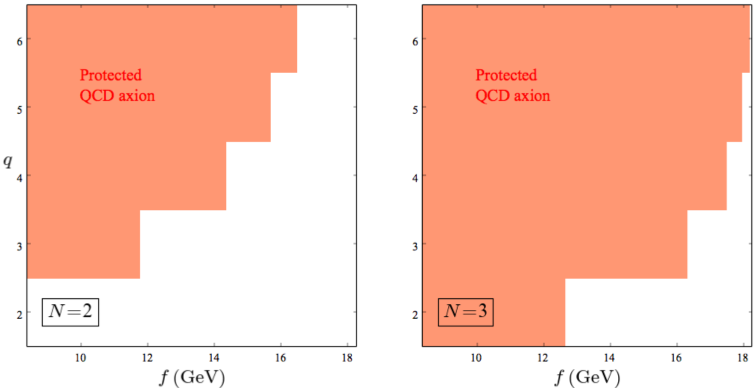
(with f_a defined by the axion-gluons coupling: $\mathcal{L} \supset \frac{a}{f_a} G\tilde{G}$)

In our setup:

$$f_a = \frac{f}{\sqrt{1 + q^2 + ... + q^{2N}}}$$

$\theta_{\rm QCD} < 10^{-10}$ if:

$$\left[m_a^{\rm (QCD)} \sim \frac{m_\pi f_\pi}{2 f_a}\right] > 10^5 \left[m_a^{\rm (grav)} \sim \left(\frac{f}{M_P}\right)^{\frac{q+\ldots+q^N-1}{2}} \frac{f}{f_a} M_P\right]$$



Protected QCD axion with $f \sim 10^{11}$ GeV and $\boldsymbol{q = 3, N = 2} \rightarrow$ 13 additional colored Dirac fermions (+13 additional singlet Dirac fermions)

No Landau pole for QCD below the Planck mass

Stability of the DM ALP's?

No anomaly: **no ALP-photon conversion** via usual $\mathcal{L} \supset \frac{a}{f_a} F\tilde{F}$

Instead: **derivative interactions** + tiny mass → **long lifetime**

Example: coupling to a heavy anomaly-free set of electrically charged fermions:

$$\mathcal{L} \supset y_1 \phi_i \overline{\psi_{R,1}} \psi_{L,1} + y_2 \phi_i \overline{\psi_{L,2}} \psi_{R,2} + h.c. \ .$$

$$\xrightarrow{\text{fermions integr.}} \mathcal{L}_{eff} \supset \frac{e^2}{48\pi^2 q^i f} \left(\frac{1}{m_1^2} - \frac{1}{m_2^2} \right) (\Box a F\tilde{F} - \frac{1}{2} \partial_\mu a F_{\nu\eta} \partial^\eta \tilde{F}^{\mu\nu})$$

Lifetimes for the FDM: $\sim 10^{300}$s