AXIONS, ANOMALOUS U(1)’S AND INSTANTONS

Quentin BONNEFOY and E.D., unpublished
Old subject, most (all ?) results known in the literature.

Many people (also in the audience) contributed to the subject:

J.E. Kim, H.P. Nilles, K. Choi, T.Banks, M.Dine, A. Ringwald, Z.Lalak, S. Pokorski, P.Svrcek, E.Witten...
Outline

1) Gaugino condensation and anomalous U(1)
2) The axions
3) Low-energy couplings of the light axion
4) Conclusions
1) Anomalous U(1) and gaugino condensation


Abelian gauge factors in string theory are often « anomalous ».

Gauge group

\[ G = G_{SM} \times G_h \times U(1)_X \]

Low-energy/massless spectrum has triangle gauge anomalies

\[ C_a = \frac{1}{4\pi^2} Tr(Q_X Q_a^2) \neq 0 \]

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But the string models are consistent due to the 4d version of the Green-Schwarz (GS) mechanism.

In what follows heterotic notation and universal GS mechanism. Type II/type I version similar and more flexible (non-universal).

Tree-level effective action (heterotic)

\[ \mathcal{L}_{S,V} = - \int d^4 \theta \ln (S + S^+ - \delta_{GS} V_X) \]

\[ + \int d^2 \theta \left[ \frac{S}{4} \left( \sum_a k_a \text{Tr} W_\alpha^a W_{a\alpha} + k_X \text{Tr} W_X^\alpha W_{X\alpha} \right) + \text{h.c.} \right] \]
is not gauge invariant and cancels the one-loop triangle gauge anomalies. Under a $U(1)_X$ gauge transformation, the axion in $S$ transform as

$$S \rightarrow S + \frac{i}{2} \delta_{GS} \alpha(x)$$

and anomalies are canceled provided

$$\delta_{GS} = \frac{C_a}{k_a} = \frac{C_X}{k_X} = \frac{C_g}{k_g}$$

where $C_g$ is the grav. anomaly. In the heterotic case

$$\delta_{GS} = \frac{1}{192\pi^2} \text{Tr} X$$
Suppose there is a gaugino condensation in a hidden sector, for concreteness SQCD with gauge group

\[ G_h = SU(N_c) \]  
and \( N_f < N_c \) quark flavors \( Q, \tilde{Q} \)

The mixed gauge anomaly \( U(1)_X [SU(N_c)]^2 \) is

\[ C_N = \frac{1}{4\pi^2} N_f (q + \tilde{q}) = k_N \delta_{GS}. \]

where \( q, \tilde{q} \) are \( U(1)_X \) charges of \( Q, \tilde{Q} \)

Degrees of freedom below the dynamical condensation scale \( \Lambda \) are the « mesons »

\[ M_i^i = Q_i \tilde{Q}_i \]
The dynamical scale $\Lambda$ is not gauge invariant

$$\Lambda = M_P e^{-8\pi^2 k_N S/(3N_c - N_f)}$$

However the effective action

$$W = (N_c - N_f) \frac{\Lambda \frac{3N_c - N_f}{N_c - N_f}}{(\text{det} M) \frac{1}{N_c - N_f}} + \left( \frac{\phi}{M_P} \right)^q + \tilde{q} m_i^\dagger M_i^i$$

Nonperturbative (Afleck-Dine-Seiberg) term

is gauge invariant precisely due to the GS conditions.
This implies that the gaugino condensation scale

\[ < \lambda \lambda > = \left( \Lambda^{3N_c-N_f} / \det M \right)^{1 \over N_c-N_f} \]

is also gauge invariant.

There is also a D-term potential

\[ V_D = \frac{g_X^2}{2} \left[ (q + \tilde{q}) \text{Tr} (M^+ M)^{1/2} - \phi^+ \phi + \xi^2 \right]^2 \]

with \( \xi^2 = \frac{\delta_{GS}}{s+\bar{s}} > 0 \)

Suppose one « integrate-out » the hidden sector fields*. One gets, adding also a constant $W_0$

\[ W_{eff} = W_0 + a M_P^3 \left( \frac{\Phi}{M_P} \right)^{N_f (q + \tilde{q})} N_c \epsilon - \frac{8 \pi^2 k N S}{N_c} \]

Hidden sector produced a « fractional » instantonic effect, which respect the gauge invariance. Such terms were computed explicitly, both from gauge theory (fractional) and stringy instantons in type II/I strings, with $S$ geometric moduli

*Assume in what follows that $S$ is stabilized.
2) The axions

There are three potential axions in the model:

\[ a_S, \ a_\Phi, \ a_M \]

where

\[ S = s + i a_S \]
\[ \Phi = V e^{i a_\Phi} \]
\[ M = M_0 \prod e^{i \sqrt{\frac{2}{N_f M_0}} a_M} \]

One of them is unphysical: the goldstone absorbed by the massive « anomalous » gauge field.
The two physical ones correspond to the gauge invariant combinations

\[ a_X \sim \frac{\delta G_S}{\sqrt{2s}} a_S + 2\sqrt{2V} a_\Phi - (q + \tilde{q}) \sqrt{2N_f M_0 a_M} \]

One of them is heavy, gets a mass from the hidden sector dynamics:

\[ a_h \sim \frac{1}{N_c - N_f} \left( 8\sqrt{2\pi^2 k_N s a_S} + N_c \sqrt{\frac{2}{N_f M_0}} a_M \right) + \frac{q + \tilde{q}}{\sqrt{2V}} a_\Phi \]
The second axion in massless in global SUSY and is a potential QCD axion or ALP.

The resulting PQ symmetry is accidental (...Svrcek,Witten)

In the limit \( V << 1 \) it is given by

\[
a_l \sim 8\sqrt{2} k_N sa_S - \frac{N_f (q + \bar{q})}{\sqrt{2} V} a_\Phi
\]

It corresponds precisely to the gauge-invariant combination appearing in the fractional instanton effect.

It has no component on the hadronic axion \( a_M \)

Conclusion: one can « integrate-out » mesons and work only with \( \Phi \) and \( S \)
One can introduce a small angle $\theta$, 

$$\tan \theta = \frac{4sV}{\delta GS}$$

such that

$$a_X = \cos \theta a_S + \sin \theta a_\phi$$

$$a_l = -\sin \theta a_S + \cos \theta a_\phi$$

In the unitary gauge $a_X = 0$ and therefore

$$a_S = -\sin \theta a_l, \quad a_\phi = \cos \theta a_l$$

Supergravity couplings generate an explicit breaking of the Peccei-Quinn symmetry and an axion potential.
\[ V \sim m_{3/2} M_P^3 \epsilon \frac{N_f (q+\bar{q})}{N_c} e^{-\frac{8 \pi^2 k N_s}{N_c}} \cos \left( \frac{2 \pi a_f}{f_l} \right) \]

where \( \epsilon = \frac{V}{M_P} << 1 \) and

\[ f_l \sim \frac{2 \sqrt{2} \pi N_c}{N_f (q+\bar{q})} V \]

For heterotic string, typically \( f_l \sim 10^{16} \) GeV

**Type II orientifolds:** S replaced by:

Kahler moduli \( T_i \), \( V^2 = \xi^2 \sim \sum_i \frac{\delta_i}{T_i + \bar{T}_i} \)

Twisted moduli \( M_\alpha \), \( V^2 = \xi^2 \sim \delta_\alpha M_\alpha \)
In these cases, it is possible to obtain

\[ f_l \sim V << M_P \]

- Other ways to naturally get \( f_l << M_P \): large xtra dims or quiver models; see parallel talk of Quentin (with S. Pokorski)

The light axion can solve the strong CP problem if

\[ m_{3/2} \langle \lambda \lambda \rangle < 10^{-5} \Lambda_{QCD}^4 \]

where

\[ \langle \lambda \lambda \rangle = M_P^3 \left( \frac{V}{M_P} \right)^{N_f (q+\bar{q})} N_c e^{-\frac{8\pi^2 k N S}{N_c}} \]

Ex: \[ \langle \lambda \lambda \rangle = 1 \text{ GeV}^3 \quad , \quad m_{3/2} = 10 \text{ KeV} \]

Stringy instantons can lead easier to smaller axion masses
3) Axion low-energy couplings

After integrating-out all fermions, the gauge coupling should be manifestly **gauge invariant**

\[ f_a = k_a S - C_a \ln \frac{\Phi}{M_P} \]

Axion coupling to gluons completely determined

\[ \frac{C_3}{V} a_\Phi G \tilde{G} \quad \Rightarrow \quad \frac{k_3 \delta G S}{V} a_l G \tilde{G} \]

Axion couplings to fermions proportional to their charges

\[ U(1)_X \]
\[
\frac{q_i}{V} \partial_m a_l \bar{\Psi}_i \gamma^m \gamma_5 \Psi_i
\]

- Phenomenologically most interesting case is for **Froggatt-Nielsen type flavor models** with anomalous \( U(1)_X \)

\[ \Phi = \text{flavon}, \quad V = 0.1 - 0.01 \ M_P \]

In this case, charges \( q_i \) are related to fermion masses: first generation fermions have the largest charges/couplings: **flavorful axion models**.

In this case, axion decay constant is **larger** than the standard « axion window »

\[
4 \times 10^8 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}.
\]

However upper bound not as solid as the lowest bound.
For $V << 10^{-2}$ these are not flavor models and it is increasingly difficult to charge SM fields under $U(1)_X$ (Yukawas)

Anomaly cancelation in this case require other (KSVZ-like) heavy colored fermions, which generate the couplings to gauge fields.
Conclusions

- Effective string models with anomalous U(1) have natural candidates for light axions.

- Gauge instantons/gaugino condensation or stringy instantons + SUGRA generate small axion masses.

- GUT scale axion decay constants go together with axiflavor models: axion couplings correlated to fermion masses and couplings

- Intermediate scale axion decays possible, correlated with small values of the FI terms after moduli stabilisation.
Thank you