AXIONS, ANOMALOUS U(1)'S AND INSTANTONS

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Old subject, most (all ?) results known in the literature.

Many people (also in the audience) contributed to the subject:

J.E. Kim, H.P. Nilles, K. Choi, T.Banks, M.Dine, A. Ringwald, Z.Lalak,
S. Pokorski, P.Svrcek, E.Witten…

Outline

1) Gaugino condensation and anomalous U(1)
2) The axions
3) Low-energy couplings of the light axion
4) Conclusions

1) Anomalous U(1) and gaugino condensation

(**P. BINÉTRUY (**1955-2017), E.D., 1996; Arkani-Hamed, Dine,Martin, 1998,…)

Abelian gauge factors in string theory are often « anomalous ».

Gauge group $$G = G_{SM} \times G_h \times U(1)_X$$

Low-energy/massless spectrum has triangle gauge anomalies

$$C_a = \frac{1}{4\pi^2} Tr(Q_X Q_a^2) \neq 0$$

But the string models are consistent due to the 4d version of the Green-Schwarz (GS) mechanism.
In what follows heterotic notation and universal GS mechanism. Type II/type I version similar and more flexible (non-universal).

Tree-level effective action (heterotic)

$$\mathcal{L}_{S,V} = -\int d^4\theta \ln(S + S^+ - \delta_{GS} V_X) + \int d^2\theta \left[\frac{S}{4} (\sum_a k_a \mathrm{Tr} W_a^\alpha W_{a\alpha} + k_X \mathrm{Tr} W_X^\alpha W_{X\alpha}) + \mathrm{h.c.} \right]$$

is not gauge invariant and cancels the one-loop triangle gauge anomalies. Under a $U(1)_X$ gauge transformation, the axion in S transform as

$$S \to S + \frac{i}{2}\delta_{GS}\alpha(x)$$

and anomalies are canceled provided

$$\delta_{GS} = \frac{C_a}{k_a} = \frac{C_X}{k_X} = \frac{C_g}{k_g}$$

where C_g is the grav. anomaly. In the heterotic case

$$\delta_{GS} = \frac{1}{192\pi^2}\mathrm{Tr}X$$

Suppose there is a gaugino condensation in a hidden sector, for concreteness SQCD with gauge group

$G_h = SU(N_c)$ and $N_f < N_c$ quark flavors $Q, \tilde{Q}$

The mixed gauge anomaly $U(1)_X [SU(N_c)]^2$ is

$$C_N = \frac{1}{4\pi^2} N_f (q + \tilde{q}) = k_N \delta_{GS} .$$

where $q, \tilde{q}$ are $U(1)_X$ charges of $Q, \tilde{Q}$

Degrees of freedom below the dynamical condensation scale Λ are the « mesons » $M^i_{\bar{i}} = Q^i \tilde{Q}_{\bar{i}}$

The dynamical scale Λ is not gauge invariant

$$\Lambda = M_P e^{-8\pi^2 k_N S/(3N_c - N_f)}$$

However the effective action

$$W = (N_c - N_f) \frac{\Lambda^{\frac{3N_c - N_f}{N_c - N_f}}}{(detM)^{\frac{1}{N_c - N_f}}} + (\frac{\phi}{M_P})^{q+\tilde{q}} m_i^{\bar{i}} M_{\bar{i}}^i$$

Quark « mass » terms

Nonperturbative (Afleck-Dine-Seiberg) term

is gauge invariant precisely due to the GS conditions.

This implies that the gaugino condensation scale

$$< \lambda\lambda > = \left(\Lambda^{3N_c - N_f} / det M \right)^{\frac{1}{N_c - N_f}}$$

is also gauge invariant.
There is also a D-term potential

$$V_D = \frac{g_X^2}{2} \left[(q + \tilde{q}) Tr (M^+ M)^{1/2} - \phi^+ \phi + \xi^2 \right]^2$$

with $\xi^2 = \frac{\delta_{GS}}{s + \bar{s}} > 0$

Suppose one « integrate-out » the hidden sector fields*.
One gets, adding also a constant W_0

$$W_{eff} = W_0 + aM_P^3 \left(\frac{\Phi}{M_P}\right)^{\frac{N_f(q+\tilde{q})}{N_c}} e^{-\frac{8\pi^2 k_N S}{N_c}}$$

Hidden sector produced a « fractional » instantonic effect, which respect the gauge invariance. Such terms were computed explicitly, both from gauge theory (fractional) and stringy instantons in type II/I strings, with S → geometric moduli

*Assume in what follows that S is stabilized.

2) The axions

There are three potential axions in the model :

$$a_S \ , \ a_\Phi \ , \ a_M$$

where

$$S = s + i a_S$$

$$\Phi = V e^{\frac{i a_\Phi}{\sqrt{2} V}}$$

$$M = M_0 \, \mathbb{I} \, e^{i \sqrt{\frac{2}{N_f M_0}} a_M}$$

One of them is unphysical: the goldstone absorbed by the massive « anomalous » gauge field

$$a_X \sim \frac{\delta_{GS}}{\sqrt{2}s} a_S + 2\sqrt{2} V a_\Phi - (q + \tilde{q}) \sqrt{2 N_f M_0 a_M}$$

The two physical ones correspond to the gauge invariant combinations

$$\frac{e^{-8\pi^2 k_N S}}{det M} \quad \text{and} \quad \left(\frac{\Phi}{M_P}\right)^{q+\tilde{q}} M$$

One of them is heavy, gets a mass from the hidden sector dynamics:

$$a_h \sim \frac{1}{N_c - N_f} \left(8\sqrt{2}\pi^2 k_N s a_S + N_c \sqrt{\frac{2}{N_f M_0}} a_M\right) + \frac{q+\tilde{q}}{\sqrt{2}V} a_\Phi$$

The second axion in massless in global SUSY and is a potential QCD axion or ALP.

The resulting PQ symmetry is accidental (…Svrcek,Witten)

In the limit $V << 1$ it is given by

$$a_l \sim 8\sqrt{2} k_N s a_S - \frac{N_f (q+\tilde{q})}{\sqrt{2}V} a_\Phi$$

It corresponds precisely to the gauge-invariant combination appearing in the fractional instanton effect.
It has no component on the hadronic axion a_M

Conclusion: one can « integrate-out » mesons and work only with Φ and S

One can introduce a small angle θ ,

$$\tan\theta = \frac{4sV}{\delta_{GS}}$$

such that
$$a_X = \cos\theta\, a_S + \sin\theta\, a_\Phi$$
$$a_l = -\sin\theta\, a_S + \cos\theta\, a_\Phi$$

In the unitary gauge $a_X = 0$ and therefore

$$a_S = -\sin\theta \;\, a_l \quad , \qquad a_\Phi = \cos\theta \;\, a_l$$

Supergravity couplings generate an explicit breaking of the Peccei-Quinn symmetry and an axion potential

$$V \sim m_{3/2} M_P^3 \, \epsilon^{\frac{N_f (q+\tilde{q})}{N_c}} e^{-\frac{8\pi^2 k_N s}{N_c}} \cos(\frac{2\pi a_l}{f_l})$$

where $\epsilon = \frac{V}{M_P} << 1$ and

$$f_l \simeq \frac{2\sqrt{2}\pi N_c}{N_f (q+\tilde{q})} V$$

For heterotic string, typically $f_l \sim 10^{16}$ GeV
Type II orientifolds: S replaced by :

Kahler moduli T_i , $V^2 = \xi^2 \sim \sum_i \frac{\delta_i}{T_i + \bar{T}_i}$

Twisted moduli M_α $V^2 = \xi^2 \sim \delta_\alpha M_\alpha$

In these cases, it is possible to obtain

$$f_l \sim V << M_P$$

- Other ways to naturally get $f_l << M_P$: large xtra dims or quiver models; see parallel talk of Quentin (with S. Pokorski)

The light axion can solve the strong CP problem if

$$m_{3/2}\langle\lambda\lambda\rangle < 10^{-5}\Lambda^4_{QCD}$$

where $\langle\lambda\lambda\rangle = M_P^3\left(\frac{V}{M_P}\right)^{\frac{N_f(q+\tilde{q})}{N_c}} e^{-\frac{8\pi^2 k_N S}{N_c}}$

Ex: $\langle\lambda\lambda\rangle = 1\ GeV^3$, $m_{3/2} = 10\ KeV$

Stringy instantons can lead easier to smaller axion masses

3) Axion low-energy couplings

After integrating-out all fermions, the gauge coupling should be manifestly gauge invariant

$$f_a = k_a S - C_a \ln \frac{\Phi}{M_P} \longrightarrow$$

axion coupling to gluons completely determined

$$\frac{C_3}{V} a_\Phi G \tilde{G} \longrightarrow \frac{k_3 \delta_{GS}}{V} a_l G \tilde{G}$$

Axion couplings to fermions proportional to their $U(1)_X$ charges

$$\frac{q_i}{V}\ \partial_m a_l \bar{\Psi}_i \gamma^m \gamma_5 \Psi_i$$

- Phenomenologically most interesting case is for Froggatt-Nielsen type flavor models with anomalous $U(1)_X$

Φ = flavon, $V = 0.1 - 0.01\ M_P$

In this case, charges q_i are related to fermion masses: first generation fermions have the largest charges/couplings: flavorful axion models.

In this case, axion decay constant is larger than the standard « axion window »

$$4 \times 10^8\,\mathrm{GeV} \lesssim f_a \lesssim 10^{12}\,\mathrm{GeV},$$

However upper bound not as solid as the lowest bound.

For $V << 10^{-2}$ these are not flavor models and it is increasingly difficult to charge SM fields under $U(1)_X$ (Yukawas)

Anomaly cancelation in this case require other (KSVZ-like) heavy colored fermions, which generate the couplings to gauge fields.

Conclusions

- Effective string models with anomalous U(1) have natural candidates for light axions.

- Gauge instantons/gaugino condensation or stringy instantons + SUGRA generate small axion masses.

- GUT scale axion decay constants go together with axiflavor models : axion couplings correlated to fermion masses and couplings
- Intermediate scale axion decays possible, correlated with small values of the FI terms after moduli stabilisation.

Thank you