# Moduli and obstructions of $\mathcal{N} = 1$ heterotic backgrounds

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# Motivation

Calabi–Yau compactifications have large numbers of moduli

Move away from Calabi–Yau and allow non-zero flux

- *Most* moduli can be stabilised
- Internal spaces are non-Kähler

Can we say anything about general heterotic compactifications?

**Goal:**

Understanding of *moduli spaces*

# Heterotic string

Work in heterotic string at $\mathcal{O}(\alpha')$

Want Minkowski compactifications that preserve minimal supersymmetry

$$M_{10} = \mathbb{R}^{1,3} \times X$$

$X$ is compact 6d space with vector bundle $V$

- Metric $g$
- Dilaton $\phi$
- Gauge fields $A$ with $G \subseteq \mathrm{E}_8 \times \mathrm{E}_8$
- 3-form flux $H$

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*H* satisfies a Bianchi identity

$$H = \mathrm{i}(\partial - \bar{\partial})\omega, \qquad \mathrm{d}H = \frac{\alpha'}{4}(\operatorname{tr} F \wedge F - \operatorname{tr} R \wedge R)$$

# Moduli

Difficult to find solutions! [Goldstein, Prokushkin; Fu, Yau; Becker, Sethi; Becker$^2$ et al.;. . .]

- Torsional geometries not well understood

What are the moduli of these solutions?

- Deformations of $X$ and $V$ that preserve SUSY
- Hermitian, complex structure and bundle moduli
- No systematic understanding until recently [Anderson, Gray, Sharp ’14; Garcia-Fernandez ’13; Baraglia, Hekmati ’13; de la Ossa, Svanes ’14]

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$$(\bar{D}^2 = 0) + \text{polystability} + \text{conformally balanced}$$
$$\Updownarrow$$
$$(X, V) \text{ gives } \mathcal{N} = 1 \text{ solution}$$

[Anderson, Gray, Sharp ’14; de la Ossa, Svanes ’14]

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Analogous to complex structure defs

- Infinitesimally: $\bar{\partial}\mu = 0$
- Higher order: $\bar{\partial}\mu - \frac{1}{2}[\mu, \mu] = 0$

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Physics guides us

- $\mathcal{N} = 1$ theory $\Rightarrow$ 4d superpotential is holomorphic [McOrist '16]
- Field space is complex with Kähler metric [Candelas et al. '15]
- Superpotential sees only holomorphic deformations

# The heterotic superpotential

4d heterotic theory has a GVW-like superpotential [Gukov et al. '99; Becker et al. '03; Cardoso et al. '03, Lukas et al. '05; McOrist '16]

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Minkowski vacuum $\Leftrightarrow$ $W = \delta W = 0$ on solution

- Recover $F$-term conditions
- $D$-term conditions are polystability and conformal balance – *not relevant for moduli*

[de la Ossa, Hardy, Svanes '14]

(Suppress $TX$ for now)

# Change of superpotential

Holomorphic deformations are

$$\begin{aligned} \Delta\Omega &= \imath_\mu\Omega + \tfrac{1}{2}\imath_\mu\imath_\mu\Omega + \tfrac{1}{3!}\imath_\mu\imath_\mu\imath_\mu\Omega, \\ \Delta(B + \mathrm{i}\,\omega) &= x_{(1,1)} + b_{(0,2)} \\ \Delta A &= \alpha_{(0,1)} \end{aligned}$$

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Generic holomorphic deformation gives

$$\begin{aligned} \Delta W = 2 \int_X &(-\imath_\mu\bar{\partial}x + \tfrac{1}{2}\mathrm{i}\,\imath_\mu\imath_\mu\partial\omega + \ldots - \tfrac{1}{2}\imath_\mu\partial b) \wedge \Omega \\ &+ \int_X \mathrm{tr}(\alpha \wedge \bar{\partial}_A\alpha - 2\,\imath_\mu F \wedge \alpha + \tfrac{2}{3}\alpha \wedge \alpha \wedge \alpha + \ldots) \wedge \Omega \end{aligned}$$

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Now want $\Delta W = \delta\Delta W = 0$ for $\mathcal{N} = 1$ Minkowski vacuum

- Is there some structure hiding here?

# $\bar{D}$ and brackets

Looking for a Maurer–Cartan equation – need a differential and a bracket

Package deformation as

$$y = (x, \alpha, \mu)$$
$$y \in \Omega^{(0,1)}(\mathcal{Q}) \simeq \Omega^{(0,1)}(T^{*(1,0)}X \oplus \operatorname{End} V \oplus T^{(1,0)}X)$$

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Already have a candidate for the differential: $\bar{D}$

$$(\bar{D}y)_a = \bar{\partial}x_a + \mathrm{i}(\partial\omega)_{ea\bar{c}}\mathrm{d}\bar{z}^{\bar{c}} \wedge \mu^e - \operatorname{tr}(F_{a\bar{b}}\mathrm{d}\bar{z}^{\bar{b}} \wedge \alpha)$$
$$(\bar{D}y)_\alpha = \bar{\partial}_A\alpha + F_{b\bar{c}}\mathrm{d}\bar{z}^{\bar{c}} \wedge \mu^b$$
$$(\bar{D}y)^a = \bar{\partial}\mu^a$$

[Anderson–Gray–Sharp ’14; de la Ossa–Svanes ’14]

# $\bar{D}$ and brackets

Appearance of $TX \oplus T^*X$ in $\mathcal{Q}$ suggests form of bracket

$$
\begin{aligned}
[y, y]_a &= 2\,\mu^b \wedge \partial_b x_a - \mu^b \wedge \partial_a x_b + \dots \\
[y, y]_\alpha &= -2\,\alpha \wedge \alpha + \dots \\
[y, y]^a &= 2\,\mu^b \wedge \partial_b \mu^a
\end{aligned}
$$

Also have a natural pairing on sections

$$
\langle y, y \rangle = 2\,\mu^a \wedge x_a + \operatorname{tr} \alpha \wedge \alpha
$$

$\bar{D}$ and $[\cdot,\cdot]$ satisfy *Leibniz identity*, and bracket satisfies *Jacobi identity* up to $\partial$-exact terms

# Superpotential

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Solutions $(y, b)$ are moduli

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- Generalisation of holomorphic Chern–Simons theory
- Can recast as an $L_3$ algebra

# Summary and outlook

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Still to do

- Specific examples? Can we *compute* the cohomologies?
- Are there conditions for moduli to be unobstructed?
- Quantum corrections?
- Topological theory? [Witten '91]
- New invariants? [Donaldson, Thomas '98]