Horava-Lifshitz cosmology revisited

Shinji Mukohyama
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Based on arXiv:1709.07084 (PRD97, 043512 (2018)) w/
S.Bramberger, A.Coates, J.Magueijo, R.Namba, Y.Watanabe
Also on CQG27 (2010) 223101 & JCAP0906 (2009) 001

Implication of GW170817 on gravity theories @ late time

- $|(c_{gw} - c_\gamma)/ c_\gamma| < 10^{-15}$
- Horndeski theoy (scalar-tensor theory with 2nd-order eom):
  Among 4 free functions, $G_4(\phi,X)$ & $G_5(\phi,X)$ are strongly constrained. Still $G_2(\phi,X)$ & $G_3(\phi,X)$ are free. $X = -\partial^\mu \phi \partial_\mu \phi$
- Generalized Proca theory (vector-tensor theory):
  Among 6 (or more) free functions, $G_4(X)$ & $G_5(X)$ are strongly constrained. Still $G_2(X,F,Y,U)$, $G_3(X)$, $G_6(X)$, $g_5(X)$ are free. $X = -A^\mu A_\mu$
- Horava-Lifshitz theory (renormalizable quantum gravity):
  The coefficient of $R^{(3)}$ is strongly constrained
  → IR fixed point with $c_{gw} = c_\gamma$? How to speed up the RG flow?
- Ghost condensation (simplest Higgs phase of gravity):
  No additional constraint
- Massive gravity (simplest modification of GR):
  Upper bound on graviton mass ≈ 10^{-22}eV
  Much weaker than the requirement from acceleration

c.f. "All" gravity theories (including general relativity):
The cosmological constant is strongly constrained ≈ 10^{-120}.

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Power counting

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- **Scaling dim of ϕ**

  $t \rightarrow b\,t$ ($E \rightarrow b^{-1}E$)

  $x \rightarrow b\,x$

  $\phi \rightarrow b^s\,\phi$

  $1+3-2+2s = 0$

  $s = -1$

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  $$n \leq 4$$

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- Gravity is highly non-linear and thus non-renormalizable

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- **Gravity becomes renormalizable!?**

Horava-Lifshitz gravity

- HL gravity realizes **z=3 scaling @ UV** and thus is power-counting renormalizable
- **Renormalizability was recently proved with any number of spacetime dimensions** [Barvinsky, et al. 2016]
- Ostrogradsky ghost is absent and thus HL gravity is **likely to be unitary**
- **In 2+1 dimensions HL gravity is asymptotically free.**
- Lorentz-invariance is broken @ UV
- **Lorentz-invariant IR fixed-point** is generic [Chadha & Nielsen 1983] (and may apply to GW as well; cf. $|c_{gw}^2 – c_\gamma^2| < 10^{-15}$ from GW170817) but running is slow (logarithmic)
- SUSY or/and strong dynamics can speed-up the RG running towards Lorentz-invariant IR fixed-point

Cosmological implications

Horava-Lifshitz Cosmology: A Review, arXiv: 1007.5199

- The z=3 scaling solves the horizon problem and leads to (almost) scale-invariant cosmological perturbations without inflation (Mukohyama 2009).
- Higher curvature terms lead to regular bounce (Calcagni 2009, Brandenberger 2009).
- Higher curvature terms ($1/a^6$, $1/a^4$) might make the flatness problem milder (Kiritsis&Kofinas 2009).
- The initial condition with z=3 scaling may actually solve the flatness problem. (Bramberger, Coates, Magueijo, Mukohyama, Namba and Watanabe 2017)
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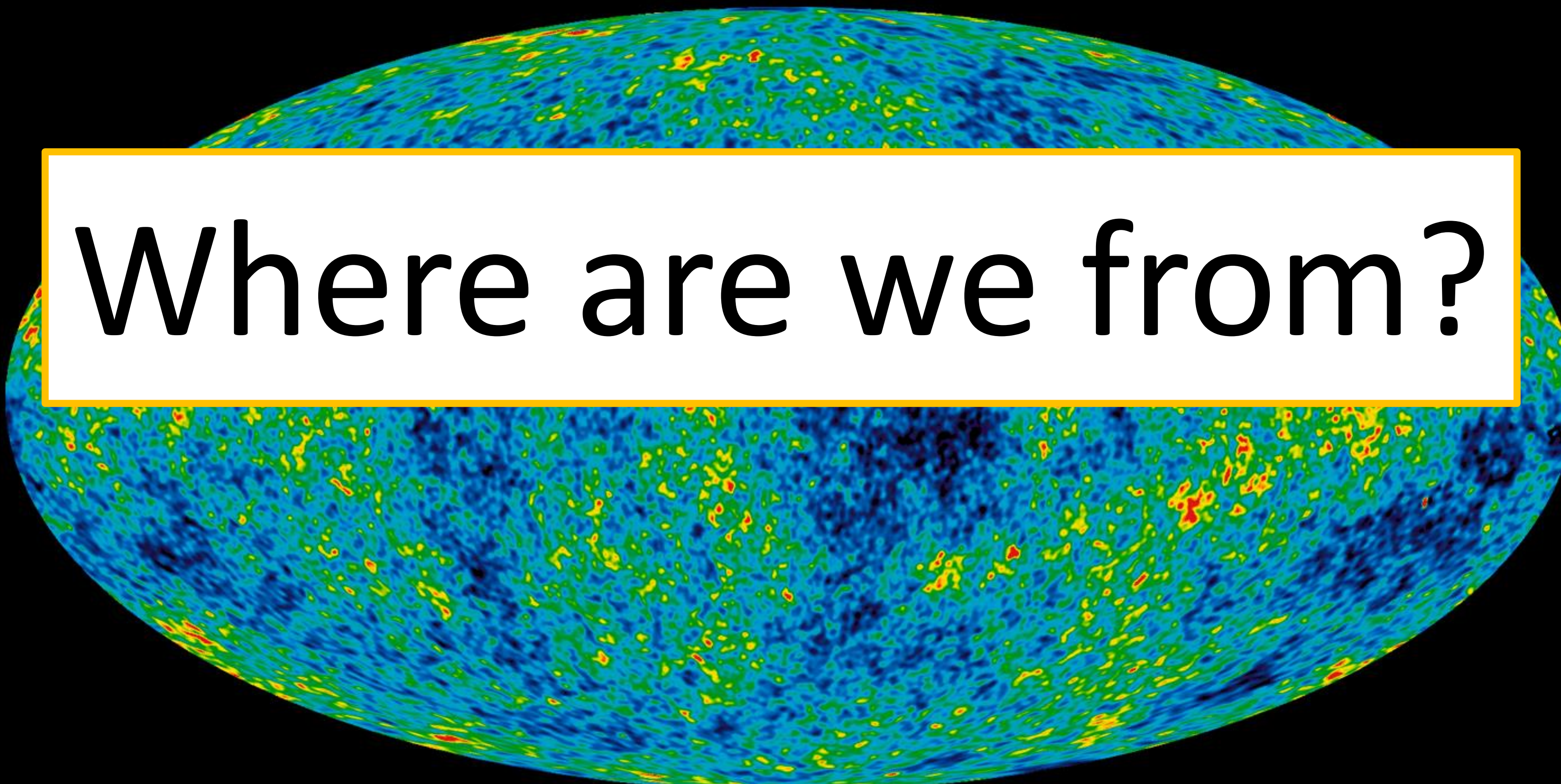
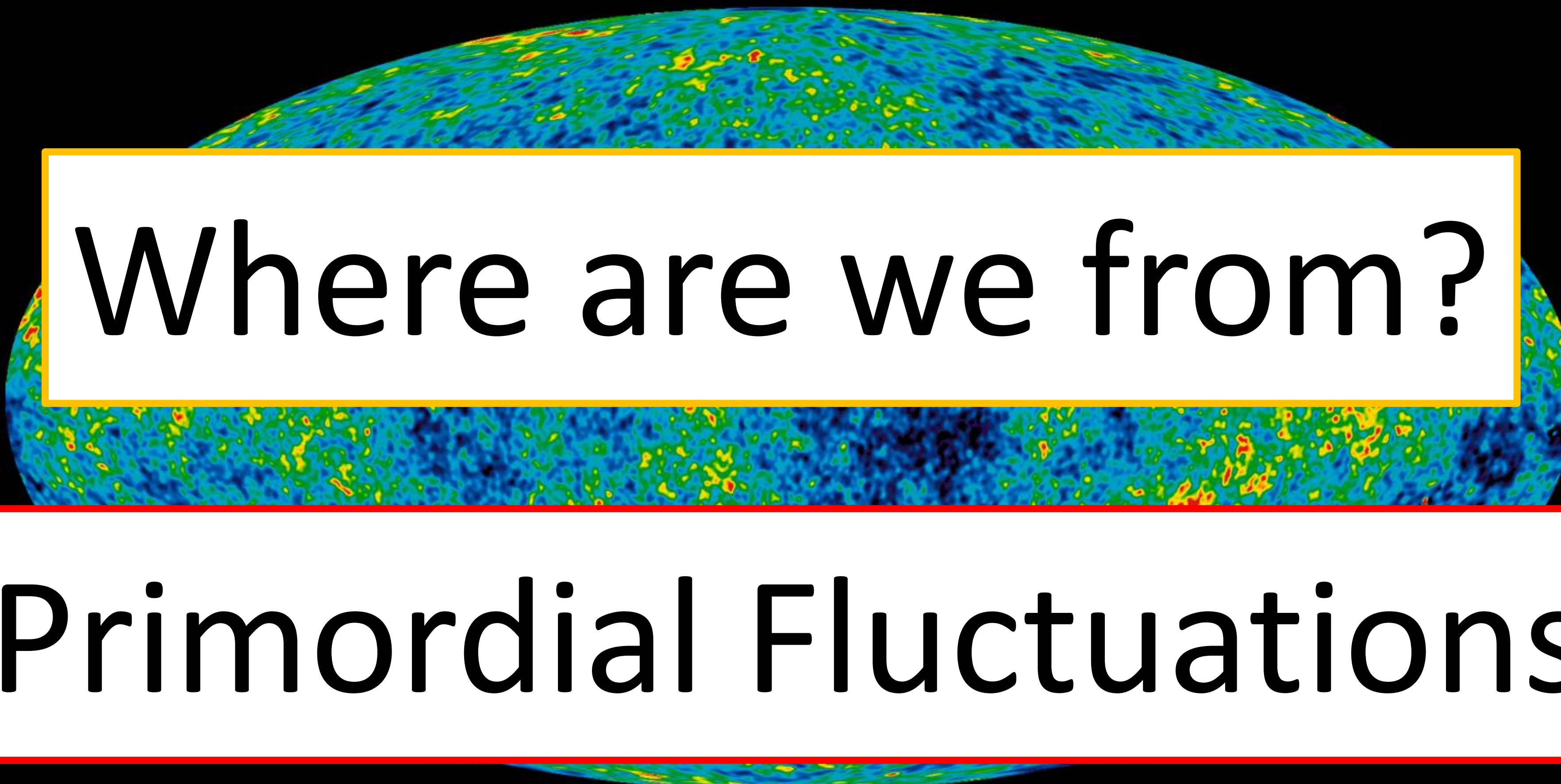
Where are we from?

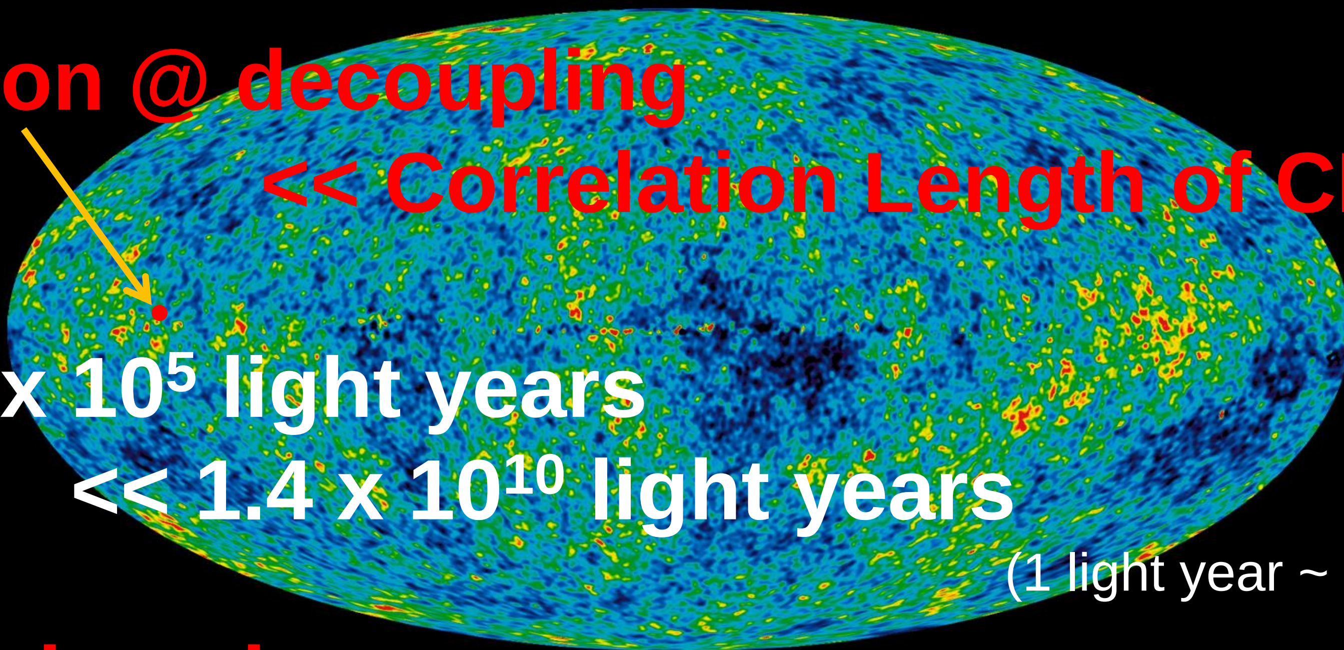




Where are we from?

Primordial Fluctuations

Horizon Problem
& Scale-Invariance



Horizon @ decoupling

<< Correlation Length of CMB

3.8 x 10^5 light years

<< 1.4 x 10^{10} light years

(1 light year ~ 10^{18} cm)

Scale-invariant spectrum

Δ ~ constant

$$\left\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \right\rangle = (2\pi)^3 \delta^3(\vec{k} + \vec{k}') \frac{\Delta}{|\vec{k}|^3}$$

Usual story

- $\omega^2 \gg H^2$: oscillate $\qquad$ $H = (da/dt) / a$

  $\omega^2 \ll H^2$: freeze $\qquad$ a : scale factor

  oscillation → freeze-out iff $d(H^2/\omega^2)/dt > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

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- Scaling law

  $t \rightarrow b\,t$ $\;(E \rightarrow b^{-1}E)$

  $x \rightarrow b\,x$ $\qquad \Rightarrow \qquad \delta\phi \propto E \sim H$

  $\phi \rightarrow b^{-1}\,\phi$

  Scale-invariance requires almost const. H, i.e. inflation.

New story with z=3

Mukohyama 2009

- oscillation → freeze-out iff $d(H^2/\omega^2)/dt > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
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  Scale-invariant fluctuations!

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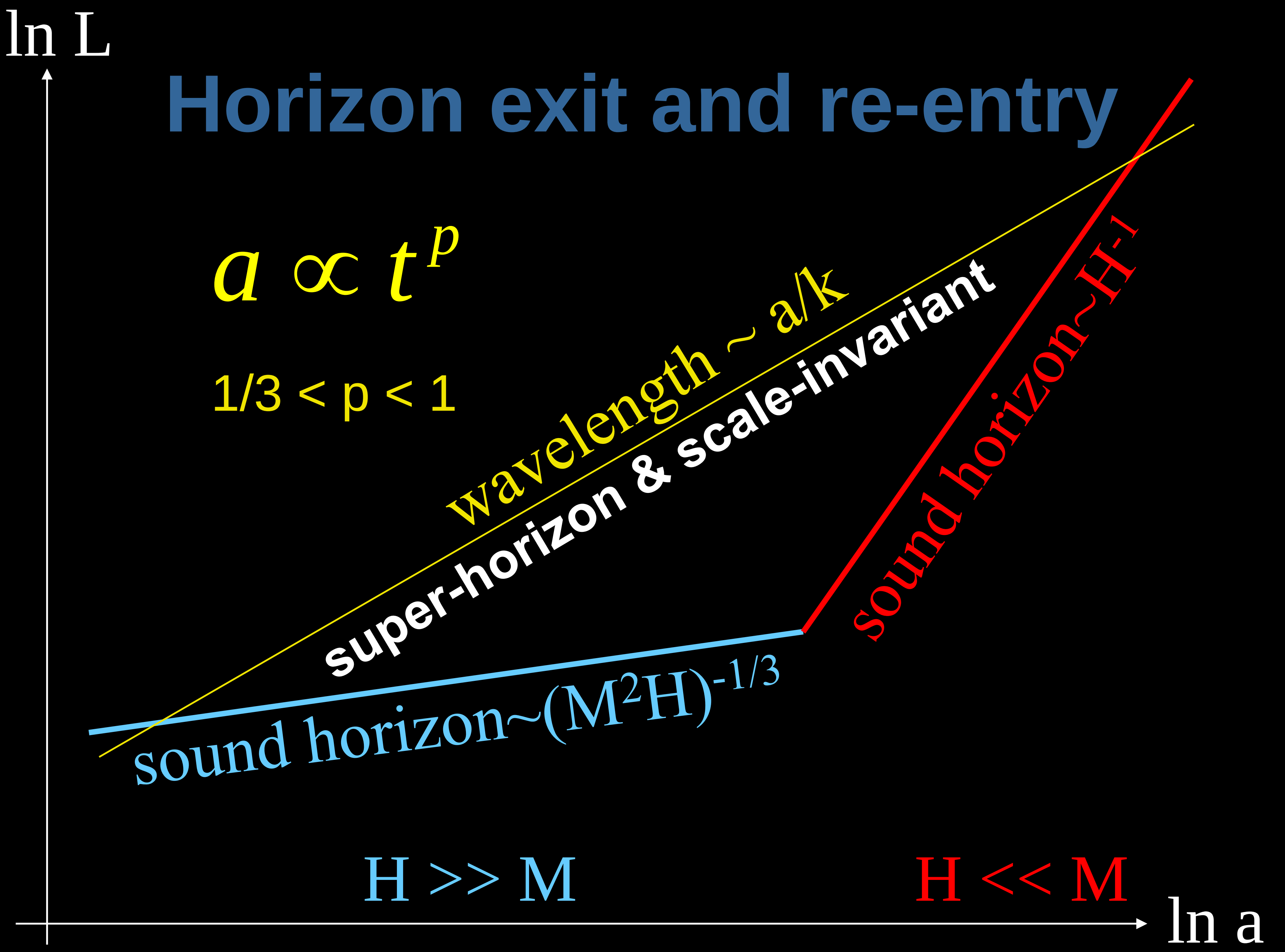
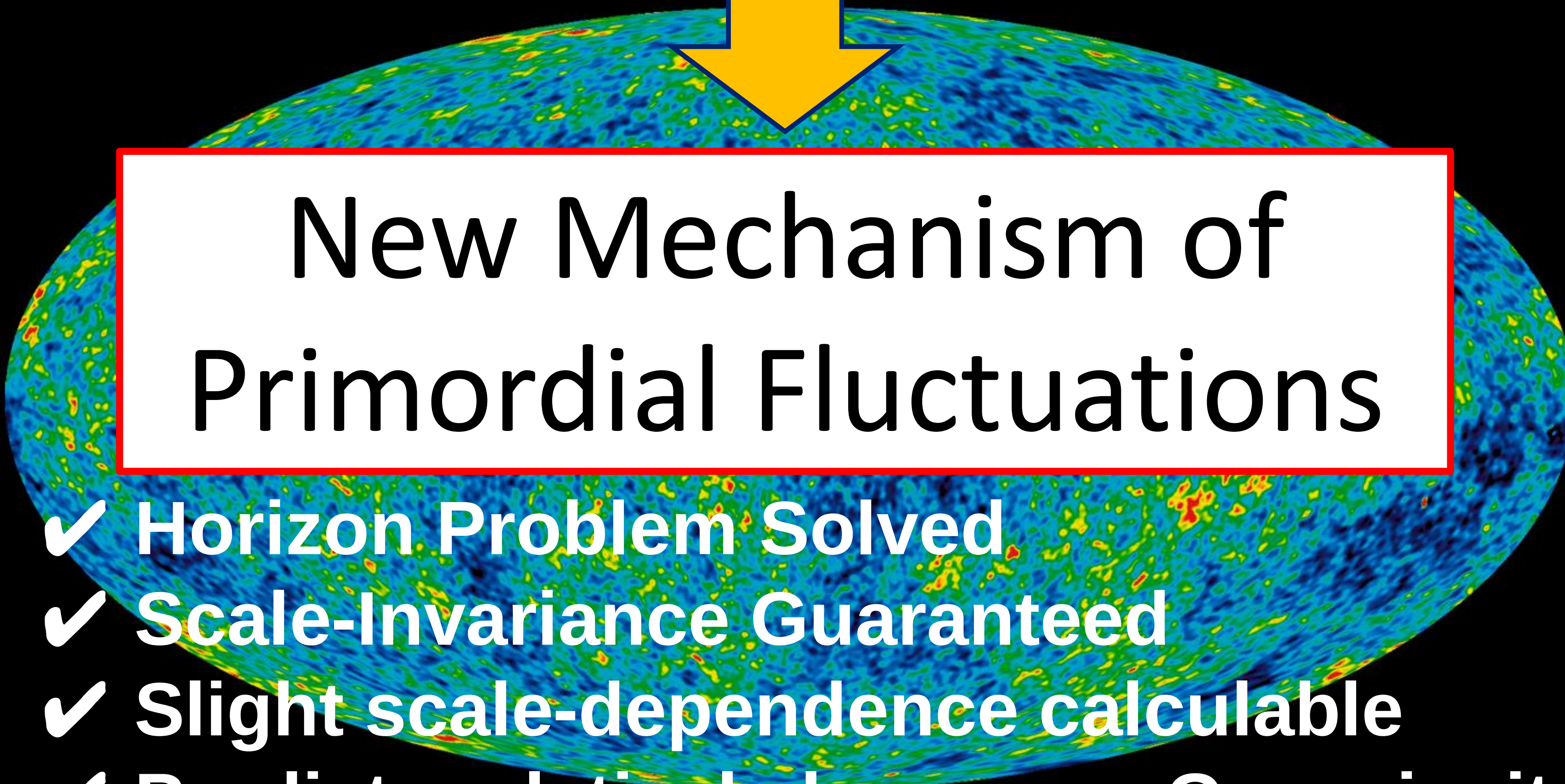
- Tensor perturbation $P_h \sim M^2/M_{Pl}^2$

Horizon exit and re-entry

$$a \propto t^{p}$$

$1/3 < p < 1$





New Quantum Gravity

New Mechanism of Primordial Fluctuations

- ✔ **Horizon Problem Solved**
- ✔ **Scale-Invariance Guaranteed**
- ✔ **Slight scale-dependence calculable**
- ✔ **Predicts relatively large non-Gaussianity**

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"Vainshtein screening" in projectable (N=N(t)) HL gravity

- Perturbative expansion breaks down in the $\lambda \rightarrow 1+0$ limit.

$$L_{kin} = K^{ij}K_{ij} - \lambda K^2$$

- Non-perturbative analysis shows continuity and GR is recovered in the $\lambda \rightarrow 1+0$ limit.

Screening scalar graviton

$$L = \left[f\left(\frac{\zeta}{\lambda - 1}\right) + g(\zeta, \lambda) \right] \frac{M_{Pl}^2 \dot{\zeta}^2}{\lambda - 1} - V(\zeta, D_i) \quad + \text{matter}$$

subleading

Independent of λ
No time derivative

Local in time, no time derivative
Non-local in space, each term has the same # of spatial derivatives in denominator and numerator

λ → 1

$$\boxed{L \sim \dot{\zeta}_c^2} \quad + \text{matter}$$

"Canonically normalized" scalar graviton decouples from the rest of the world.

Analogue of Vainshtein screening

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  - ✓Spherically-sym, static, vacuum (Mukohyama 2010)
  - ✓Spherically-sym, dynamical, vacuum (Mukohyama 201?)
  - ✓Spherically-sym, static, with matter (Mukohyama 201?)
  - ✓General super-horizon perturbations with matter (Izumi-Mukohyama 2011; Gumrukcuoglu-Mukohyama-Wang 2011)

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  ✓General super-horizon perturbations with matter (Izumi-Mukohyama 2011; Gumrukcuoglu-Mukohyama-Wang 2011)
- “Vainshtein radius” can be pushed to infinity in the $\lambda \rightarrow 1+0$ limit.

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cc & flatness problems

$$3H^2 = 8\pi G\rho - \frac{3K}{a^2} + \Lambda$$

- Λ does not decay → cc problem "Why is Λ as small as $8\pi G\rho$ now?"
- K/a^2 decays but only slowly → flatness problem "Why is K/a^2 smaller than $8\pi G\rho$ now?"

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We shall consider the flatness problem.

Two ways to tackle flatness problem

$$3H^2 = 8\pi G\rho - \frac{3K}{a^2}$$

- If ρ does not decay for an extended period then flatness problem solved → **Inflation**
- If $K/a^2 \ll 8\pi G\rho$ initially then flatness problem solved → **Quantum cosmology**

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We shall consider the second possibility.

Usual story

- Initial condition set by e.g. quantum tunneling
- O(4) symmetric instanton
  → **T ~ L, where T ~ 1/H, L ~ a/|K|$^{1/2}$**
- Three terms in $3H^2 = 8\pi G\rho - 3K/a^2$ are of the same order initially.
- **Flatness problem exists unless inflation occurs.**

New story with z=3

- Initial condition set by e.g. quantum tunneling
- Instanton with z=3 anisotropic scaling, which we call an anisotropic instanton
  → **$T \propto L^3$, where $T \sim 1/H$, $L \sim a/|K|^{1/2}$**
  → $T \sim M^2 L^3$
- $T \ll L$ if $L \ll 1/M$
- **Flatness problem may be solved if the anisotropic instanton is small.**

Summary

- Horava-Lifshitz gravity is renormalizable and likely to be unitary, and thus is a candidate for UV complete theory of quantum gravity.
- Lorentz-invariance can be restored at IR fixed-point. SUSY or/and strong dynamics can speed-up the RG running to match with phenomenology.
- It is likely that GR (+DM) is recovered in the $\lambda \rightarrow 1$ limit due to nonlinear effects. [c.f. Vainshtein effect]
- Horizon problem can be solved and (almost) scale-invariant cosmological perturbations can be generated without inflation.
- Flatness problem can be solved by equipartition in highly trans-Planckian regime.